3D TRIGONOMETRY

DATE OF SOLUTIONS: 14/05/2018
MAXIMUM MARK: 77

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2016] — 3D Trigonometry (Inc Pythagoras) **[4 Marks]**

ABCDEFGH is a cuboid.

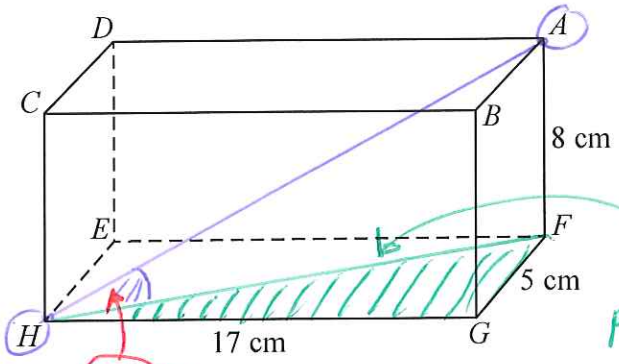


Diagram **NOT** accurately drawn

1ST

$FH = \sqrt{17^2 + 5^2}$

$= 17.7200...$ B1

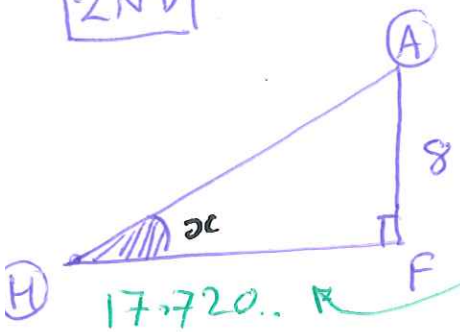
M1 [CORRECT ANGLE IDENTIFIED – ANYWHERE!]

The cuboid has

length 17 cm
width 5 cm
height 8 cm

Work out the size of the angle that *AH* makes with the plane *EFGH*.
Give your answer correct to 1 decimal place.

2ND



$\tan x = \frac{OPP}{ADJ}$

$\tan x = \frac{8}{17.72...}$ M1

$x = \tan^{-1}\left(\frac{8}{17.72...}\right)$

$= 24.297...$

$= 24.3°$ A1

2. [Edexcel, 2005] 3D Trigonometry (Inc Pythagoras) **[6 Marks]**

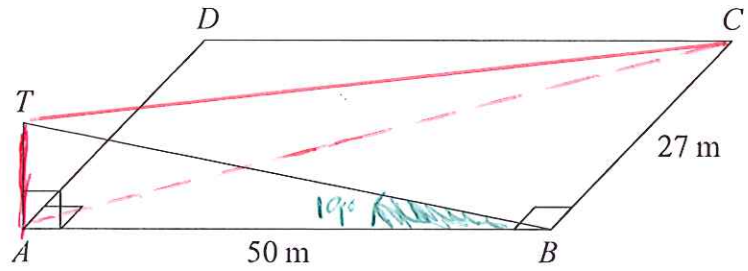
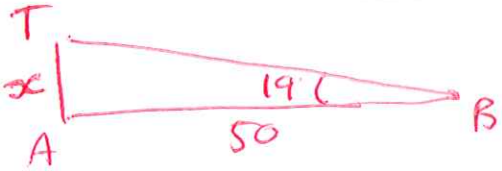


Diagram **NOT** accurately drawn

ABCD is a horizontal rectangular field.
AB = 50 m.
BC = 27 m.
AT is a vertical mast.

(a) The angle of elevation of *T* from *B* is 19°.
Calculate the length of *AT*.
Give your answer correct to 3 significant figures.



$\tan 19° = \frac{x}{50}$

$\Rightarrow x = 50 \times \tan 19°$

$= 17.216...$

17.2 m
(3)

(b) Calculate the distance from *C* to *T*.
Give your answer correct to 3 significant figures.

$AC^2 = 27^2 + 50^2$

$= 3229$

$CT^2 = AC^2 + AT^2$

$= 3229 + 17.216...^2$

$= 3525.39...$

$CT = \sqrt{3525.39...}$

$= 59.375...$

59.4 m
(3)

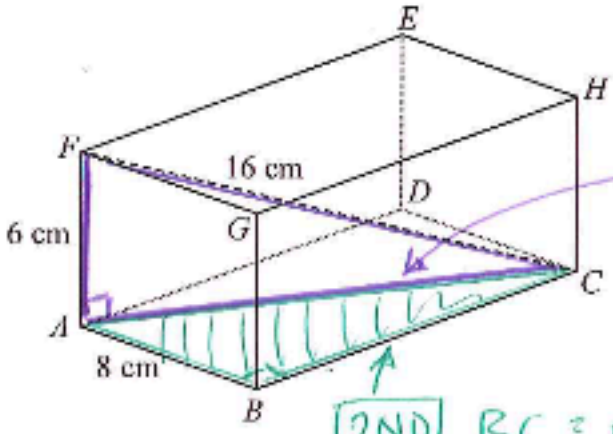


Diagram **NOT** accurately drawn

1ST $AC = \sqrt{16^2 - 6^2}$ (M1)
$= 14.83$ (A1)

2ND $BC = \sqrt{14.83^2 - 8^2}$
$= 12.489\ldots$
$= 12.5$ (A1)

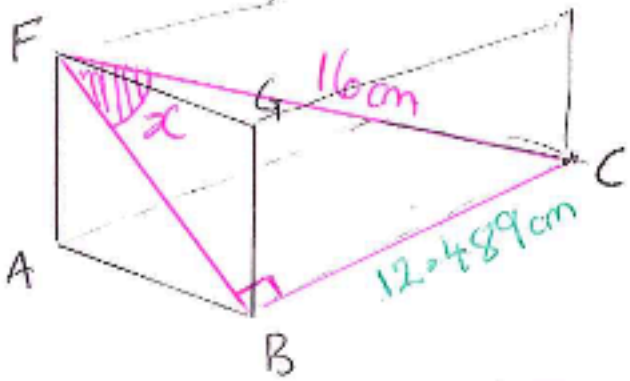
The diagram shows a cuboid $ABCDEFGH$.
$AB = 8$ cm, $AF = 6$ cm and $FC = 16$ cm.

(a) Find the length of BC.
Give your answer correct to 3 significant figures.

WORKING OUT IS BY DIAGRAM

$BC =$ 12.5 cm
(3)

(b) Find the size of the angle between the line FC and the plane $ABGF$. } DIFFICULT !!
Give your answer correct to 1 decimal place.



$\sin x = \frac{OPP}{HYP}$

$\sin x = \frac{12.489}{16}$ (M1)

$x = \sin^{-1}\left(\frac{12.489}{16}\right)$

$= 51.3°$ (A1)

4. [Edexcel, 2016] 3D Trigonometry (Inc Pythagoras) **[5 Marks]**

The diagram shows a prism.

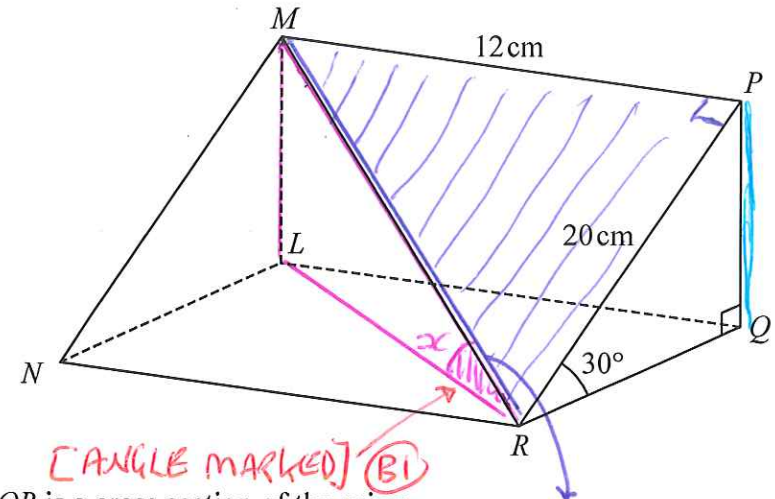


Diagram **NOT** accurately drawn

2ND
$ML = PQ = 20 \sin 30$
$= 10\text{ cm}$ B1

[ANGLE MARKED] B1

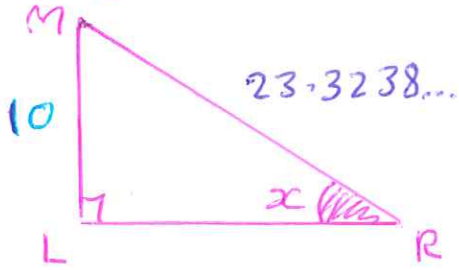
Triangle PQR is a cross section of the prism.

$PR = 20$ cm
$MP = 12$ cm
Angle $PRQ = 30°$
Angle $PQR = 90°$

1ST
$MR = \sqrt{20^2 + 12^2}$
$= 23.3238...$ B1

Calculate the size of the angle that the line MR makes with the plane $RQLN$.
Give your answer correct to 1 decimal place.

3RD



$\sin x = \frac{OPP}{HYP}$

$\sin x = \frac{10}{23.3238}$ M1

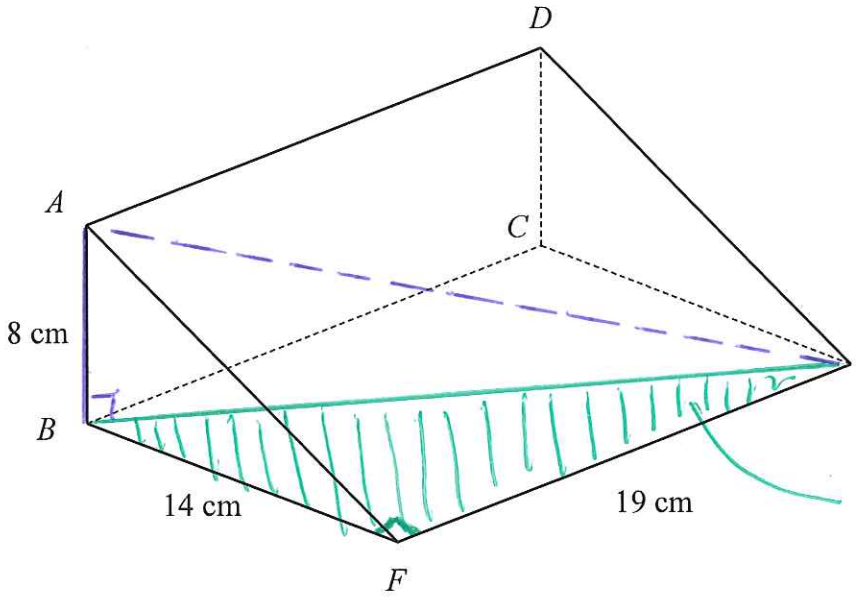
$\Rightarrow x = \sin^{-1}\left(\frac{10}{23.3238}\right)$

$= 25.388...$

$25.4°$ A1

5. [New Question, by Maths4Everyone.com] 3D Trigonometry (Inc Pythagoras) **[6 Marks]**

The diagram shows a triangular prism *ABCDEF*
AB = 8 cm
BF = 14 cm
EF = 19 cm

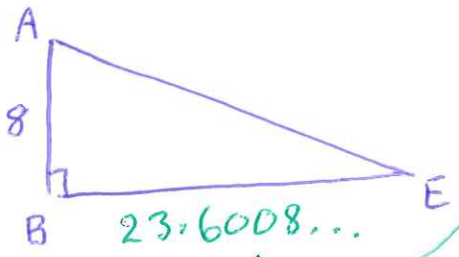


1ST

$BE = \sqrt{19^2 + 14^2}$

$= 23.6008...$ B1

(a) Calculate the distance between *A* and *E*.



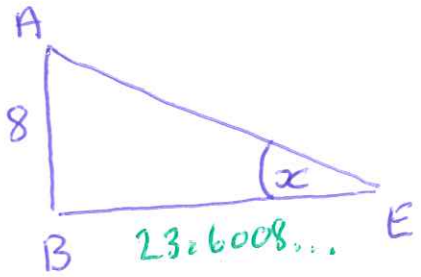
2ND

$AE = \sqrt{23.6^2 + 8^2}$ M1

$= 24.9198...$

$= 24.9\text{cm}$ A1

(b) Calculate the angle between *AE* and the plane *BCEF*.



$\tan x = \frac{8}{23.600..}$ M1

$x = \tan^{-1}\left(\frac{8}{23.600..}\right)$ M1

$= 18.725...$

$= 18.7°$ A1

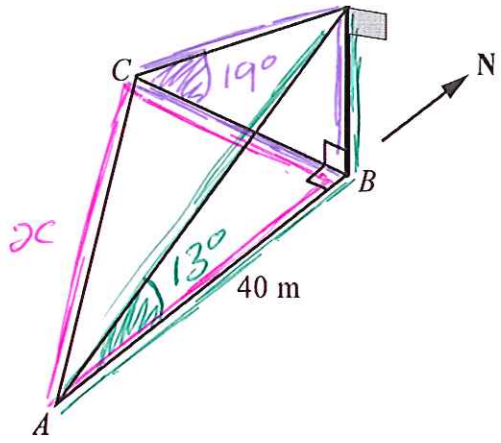
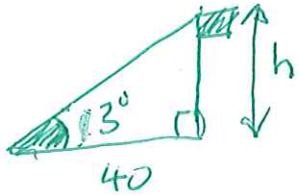


Diagram **NOT** accurately drawn

A, *B* and *C* are points on horizontal ground.
C is due West of *B*.
A is due South of *B* and *AB* = 40 m.
There is a vertical flagpole at *B*.
From *A*, the angle of elevation of the top of the flagpole is 13°.
From *C*, the angle of elevation of the top of the flagpole is 19°.

Calculate the distance *AC*. (x)
Give your answer correct to 3 significant figures.

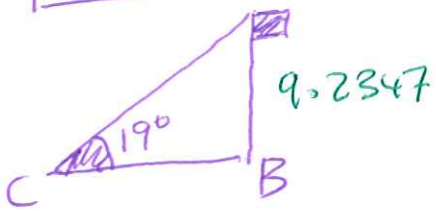
1ST



$\tan 13° = \frac{h}{40} \Rightarrow h = 40 \tan 13$ (M1)

$= 9.2347...$ (A1)

2ND

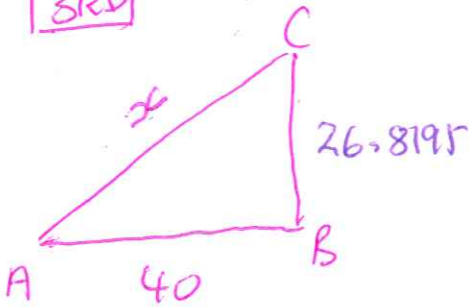


$\tan 19° = \frac{9.2347}{CB}$

$\Rightarrow CB = \frac{9.2347}{\tan 19}$ (M1) EITHER

$= 26.8195...$ (A1)

3RD



$x = \sqrt{40^2 + 26.8195^2}$ (M1)

$= 48.2\text{ m}$ (A1)

The diagram shows a pyramid with a horizontal rectangular base $PQRS$.
$PQ = 16$ cm.
$QR = 10$ cm.
M is the midpoint of the line PR.
The vertex, T, is vertically above M.
$MT = 15$cm.

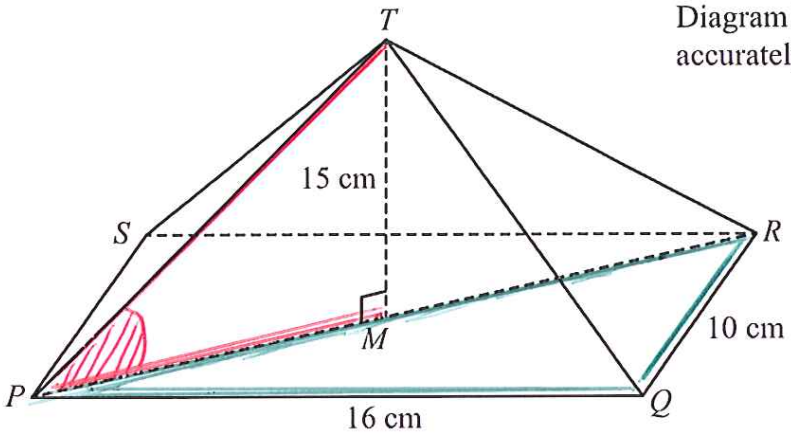
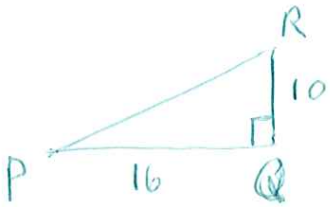


Diagram **NOT** accurately drawn

Calculate the size of the angle between TP and the base $PQRS$.
Give your answer correct to 1 decimal place.

(NEED TO KNOW PM)



$PR^2 = 10^2 + 16^2$

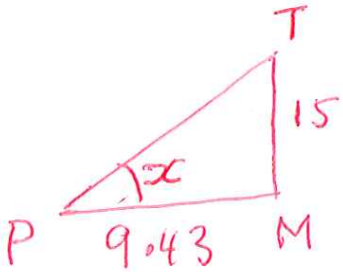
$= 356$ (M1)

$PR = \sqrt{356}$

$= 18.867...$

$PM = \frac{18.867...}{2}$

$= 9.4339...$ (B1)



$\tan x = \frac{15}{9.43...}$ (M1)

$x = \tan^{-1}\left(\frac{15}{9.43...}\right)$

$= 57.832...$

57.8° (A1)

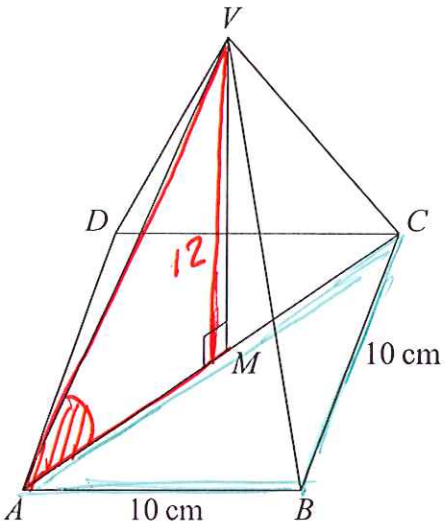
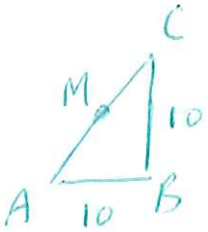


Diagram **NOT** accurately drawn

The diagram shows a pyramid.
The base, $ABCD$, is a horizontal square of side 10 cm.
The vertex, V, is vertically above the midpoint, M, of the base.
$VM = 12$ cm.

Calculate the size of angle VAM.

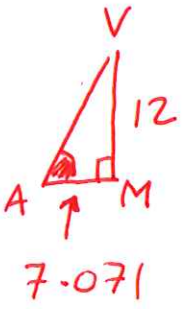


$AC^2 = 10^2 + 10^2$ (M1)

$= 200$

$AC = \sqrt{200}$

$= 14.142\ldots \Rightarrow AM = 7.071\ldots$ (B1)



$\text{TAN } A = \frac{\text{OPP}}{\text{ADJ}}$

$\Rightarrow \text{TAN } A = \frac{12}{7.071}$ (M1)

$A = \text{TAN}^{-1}\left(\frac{12}{7.071}\right)$

$59.5°$ (A1)

ABCDE is a square-based pyramid.

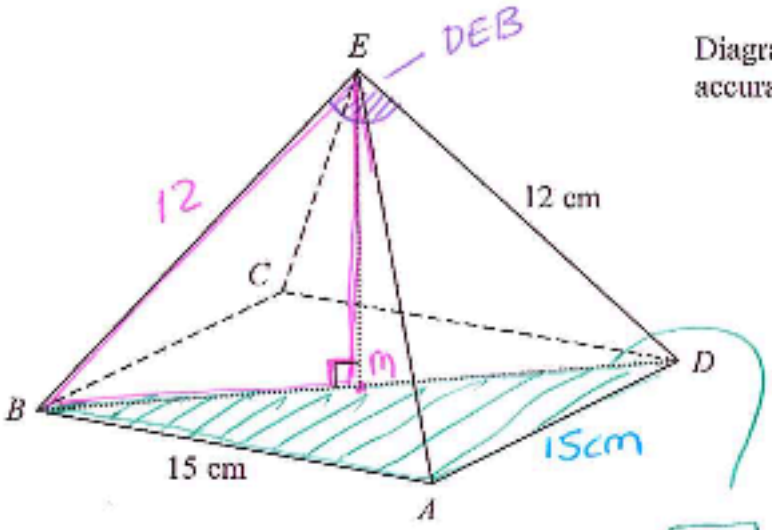


Diagram **NOT** accurately drawn

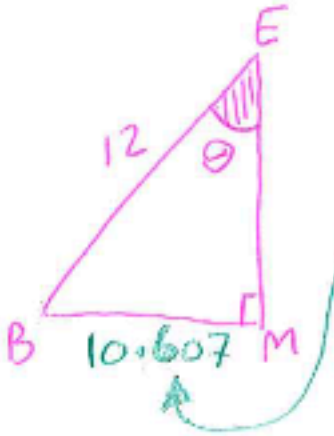
$AE = BE = CE = DE = 12$ cm
$AB = 15$ cm

Calculate the size of angle *DEB*.
Give your answer to the nearest degree.

1ST

$BD = \sqrt{15^2 + 15^2}$

$= 21.213...$ (M1)



$\div 2$

$\sin\theta = \frac{10.607}{12}$ (M1)

$\Rightarrow \theta = \sin^{-1}\left(\frac{10.607}{12}\right)$

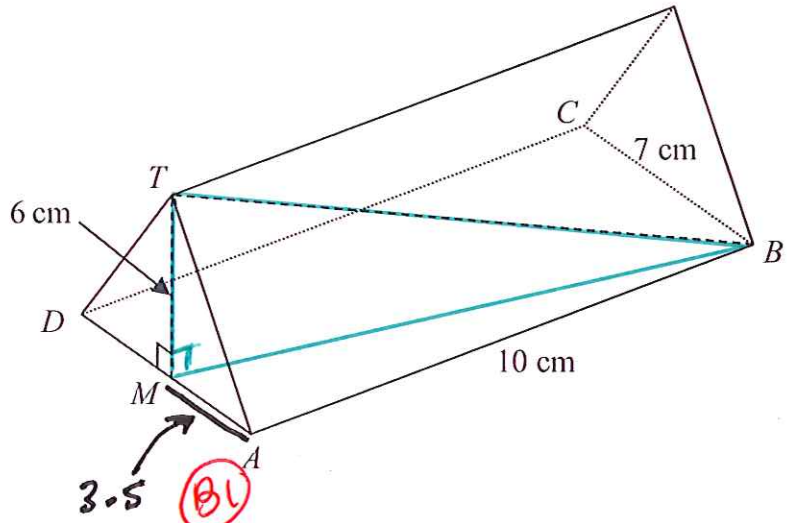
$= 62.113...$ (M1)

$\therefore DEB = 2 \times 62.113...$

$= 124.226...$

$= 124^\circ$ (A1)

The diagram shows a triangular prism with a horizontal rectangular base *ABCD*.
AB = 10 cm. *BC* = 7 cm.
M is the midpoint of *AD*.
The vertex *T* is vertically above *M*.
MT = 6 cm.

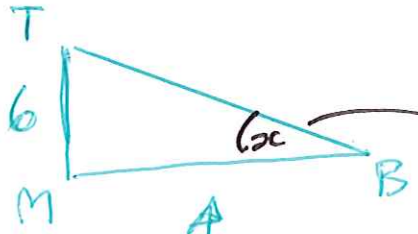


B1

Diagram **NOT** accurately drawn

Calculate the size of the angle between *TB* and the base *ABCD*.

Give your answer correct to 1 decimal place.



$MB^2 = 10^2 + 3.5^2$ M1

$= 112.25$

$MB = \sqrt{112.25}$

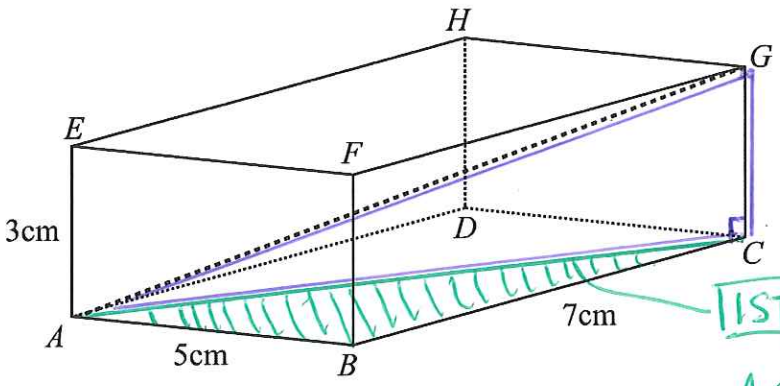
$= 10.5948...$ A1

$\tan x = \frac{6}{10.5948}$ M1

$x = \tan^{-1}\left(\frac{6}{10.5948}\right)$

$= 29.5°$ A1

Diagram **NOT** accurately drawn



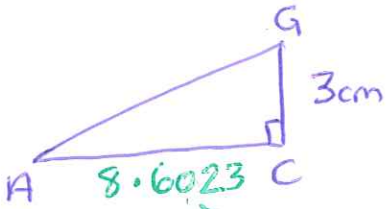
IST

The diagram shows a cuboid *ABCDEFGH*.
AB = 5cm
BC = 7cm
AE = 3cm

(a) Calculate the length of *AG*.
Give your answer correct to 3 significant figures.

$AC = \sqrt{7^2 + 5^2}$

$= 8.6023...$ (A1)



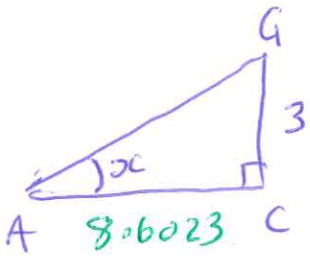
$AG = \sqrt{8.6023^2 + 3^2}$ (M1)

$= 9.11043...$

9.11 (A1) cm

(3)

(b) Calculate the size of the angle between *AG* and the plane *ABCD*.
Give your answer correct to 1 decimal place.



$\tan x = \frac{3}{8.6023}$

$x = \tan^{-1}\left(\frac{3}{8.6023}\right)$ (M1)

$= 19.2259...$

$= 19.2$ cm (A1)

12. [Edexcel, 2015] 3D Trigonometry (Inc Pythagoras) **[4 Marks]**

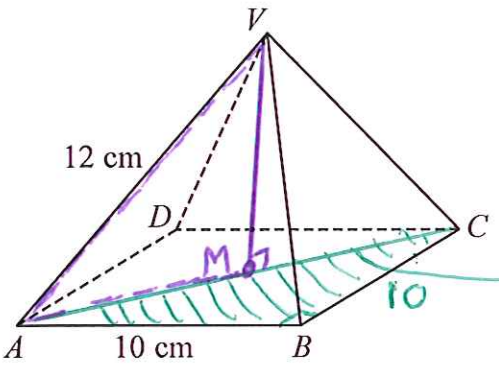


Diagram **NOT** accurately drawn

$AC = \sqrt{10^2+10^2}$

ABCD is the square base of the pyramid *VABCD*.

$AB = BC = CD = DA = 10$ cm.
$VA = VB = VC = VD = 12$ cm.

IF 'M' IS MIDPOINT OF AC
THEN HEIGHT IS MV

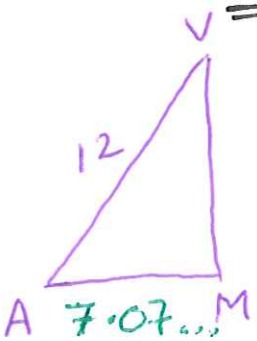
Calculate the height of the pyramid.
Give your answer correct to 3 significant figures.

$AC = \sqrt{10^2+10^2}$

$= 14.142...$ (m1)

$AM = \frac{14.142...}{2}$

$= 7.07106...$ (m1)



$VM = \sqrt{12^2 - 7.07...^2}$ (m1)

$= 9.695...$

$= 9.70$ cm (A1)

13. [Edexcel, 2014] 3D Trigonometry (Inc Pythagoras) **[5 Marks]**

A pyramid has a horizontal square base *ABCD* with sides of length 230 metres.
M is the midpoint of *AC*.
The vertex, *T*, is vertically above *M*.
The slant edges of the pyramid are of length 218 metres.

Calculate the height, *MT*, of the pyramid.
Give your answer correct to 3 significant figures.

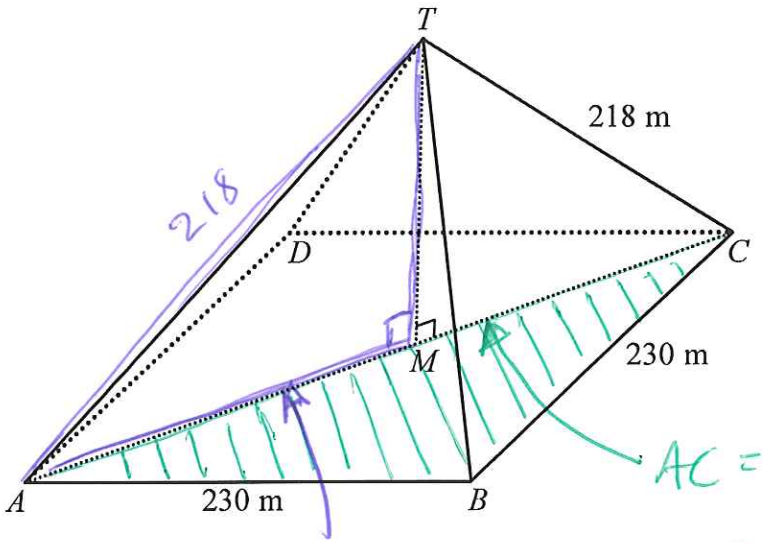


Diagram **NOT** accurately drawn

1ST

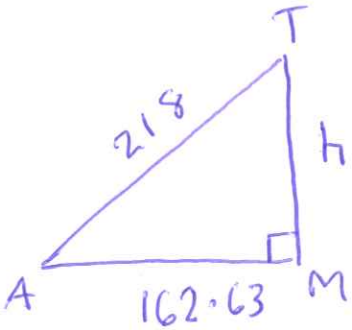
$AC = \sqrt{230^2 + 230^2}$ (M1)

$= 325.269...$ (M1)

2ND

$AM = \frac{325.269}{2}$

$= 162.63...$ (M1)



3RD

$h = \sqrt{218^2 - 162.63^2}$ (M1)

$= 145.168...$

$= 145\text{ m}$ (A1)

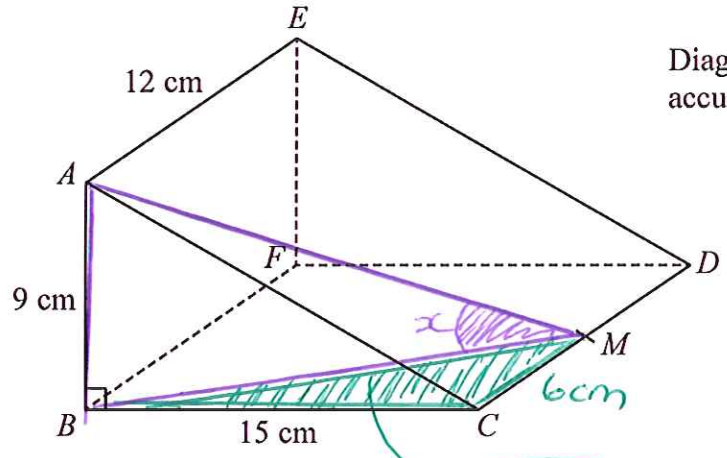


Diagram **NOT** accurately drawn

ABCDEF is a triangular prism.
AB = 9 cm, *BC* = 15 cm and *AE* = 12 cm.
Angle *ABC* = 90°
M is the midpoint of *CD*.

Calculate the size of the angle between *AM* and the plane *BCDF*.
Give your answer correct to 1 decimal place.

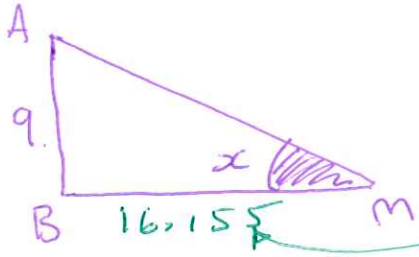
1ST

$BM = \sqrt{15^2 + 6^2}$ (M1)

$= 16.155\ldots$

(A1)

2ND



$\tan x = \frac{9}{16.155}$ (M1)

$\Rightarrow x = \tan^{-1}\left(\frac{9}{16.155}\right)$ (M1)

$= 29.122\ldots$

$= 29.1°$ (A1)

15. [Edexcel, 2015] 3D Trigonometry (Inc Pythagoras) **[6 Marks]**

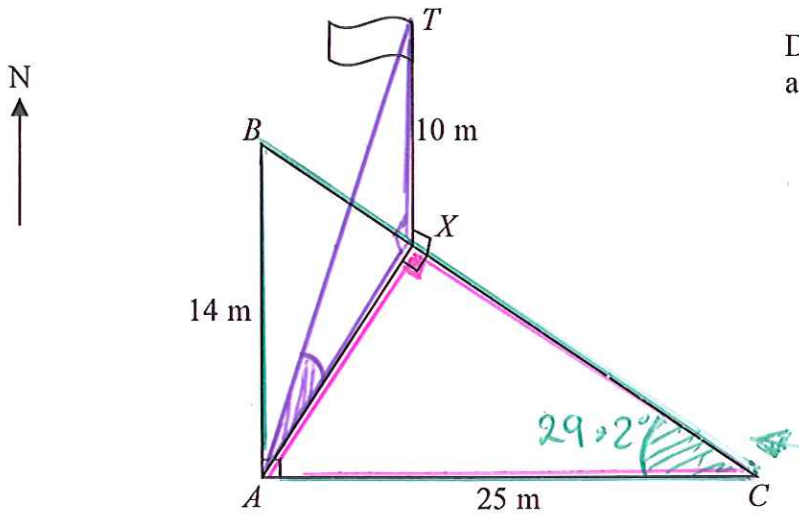


Diagram **NOT** accurately drawn

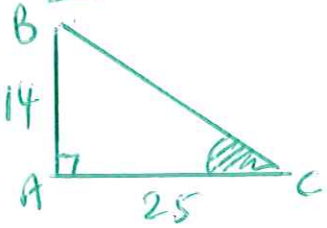
A, *B* and *C* are points on horizontal ground.
B is due North of *A* and *AB* is 14 m.
C is due East of *A* and *AC* is 25 m.

A vertical flagpole, *TX*, has its base at the point *X* on *BC* such that the angle *AXC* is a right angle.

The height of the flagpole, *TX*, is 10 m.

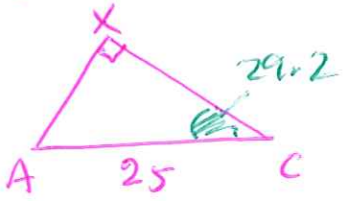
Calculate the size of the angle of elevation of *T* from *A*.
Give your answer correct to 1 decimal place.

1ST



$\tan C = \frac{14}{25} \Rightarrow C = \tan^{-1}\left(\frac{14}{25}\right) = 29.249^\circ$ (M1) (A1)

2ND

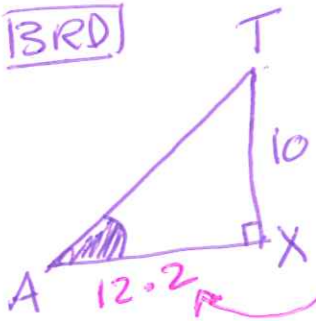


$\sin 29.249 = \frac{AX}{25}$

$\Rightarrow AX = 25 \times \sin 29.249...$ (M1)

$= 12.215...$ (A1)

3RD



$\tan A = \frac{10}{12.2}$

$\Rightarrow A = \tan^{-1}\left(\frac{10}{12.2}\right)$ (M1) $= 39.3058...$

$= 39.3^\circ$ (A1)

The diagram shows a cube *ABCDEFGH*.
The sides of the cube are of length 5 cm.

Calculate the size of the angle between the diagonal *AH* and the base *EFGH*.
Give your answer correct to 1 decimal place.

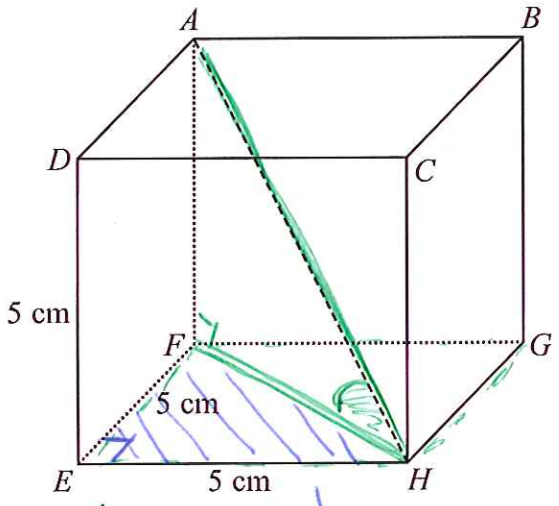
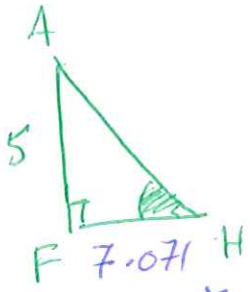


Diagram **NOT** accurately drawn



1ST $FH^2 = 5^2 + 5^2$ M1

$= 50$

$FH = \sqrt{50}$

A1

$= 7.07106...$

2ND

$\tan H = \frac{OPP}{ADJ}$

$\tan H = \frac{5}{7.071}$

M1 EITHER

$H = \tan^{-1}\left(\frac{5}{7.071}\right)$

$= 35.264$

A1

$= 35.3$ cm

Disclaimer

While reasonable endeavours have been used to verify the accuracy of these solutions, these solutions are provided on an "as is" basis and no warranties are made of any kind, whether express or implied, in relation to these solutions.

There is no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

Please report any errors or omissions that You find*. These solutions will be updated to correct errors that are discovered. It is recommended that You always check that You have the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or 'best' method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners' mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners' mark scheme.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site.

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words 'show that', 'show your working out', or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

* The best way to inform of errors or omissions is a direct Twitter message to @Maths4Everyone