

ALGEBRA PROBLEMS

DATE OF SOLUTIONS: 30/05/2018

MAXIMUM MARK: 95

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1 [Edexcel, 2013]

Generating Algebraic Expressions [4 Marks]

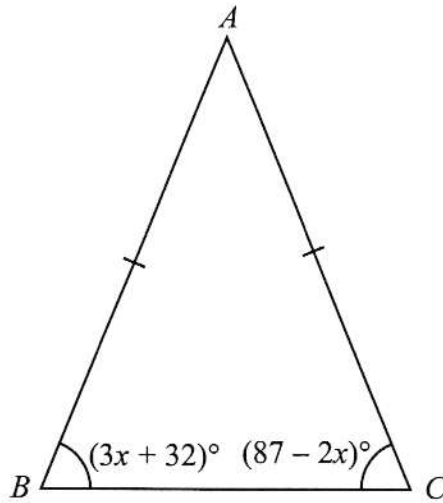


Diagram NOT
accurately drawn

TWO EQUAL
ANGLES

In the isosceles triangle ABC ,
 $AB = AC$
angle $B = (3x + 32)^\circ$
angle $C = (87 - 2x)^\circ$

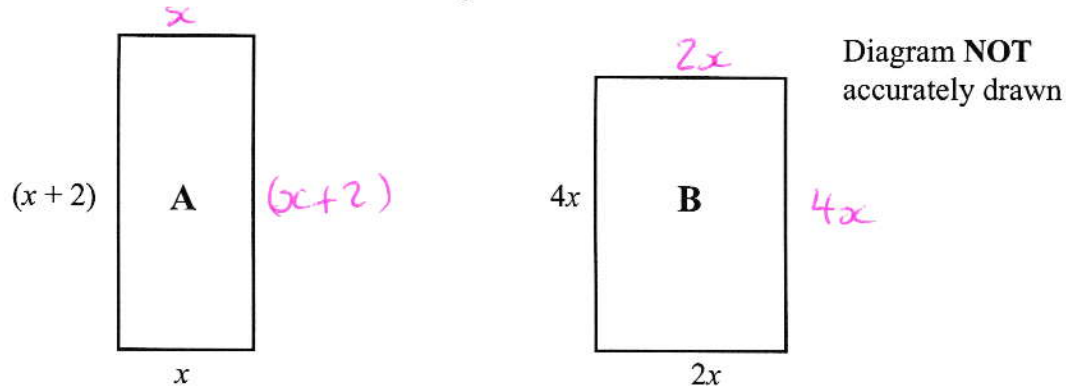
Work out the value of x .
Show clear algebraic working.

$$\begin{aligned}3x + 32 &= 87 - 2x && \text{(M1) [EQUATION]} \\5x + 32 &= 87 && \text{(M1) [SIMPLIFYING]} \\5x &= 55 && \text{(M1) [ISOLATING 'x']}\end{aligned}$$
$$x = \frac{55}{5}$$
$$= \underline{\underline{11}}$$

$x = \dots\dots\dots 11$ (A1)

Rectangle **A** has a width of x metres and a height of $(x + 2)$ metres.

Rectangle **B** has a width of $2x$ metres and a height of $4x$ metres.



The perimeter of rectangle **A** is equal to the perimeter of rectangle **B**.

KEY!

(i) Use this information to write down an equation in x .

(M1) [FOR GETTING AT LEAST ONE CORRECT PERIMETER]

$$4x + 4 = 12x$$

(ii) Find the value of x .

(A1) [EQUATION]

$$12x = 4x + 4$$

$$8x = 4 \quad \text{(M1) [ISOLATING 'x']}$$

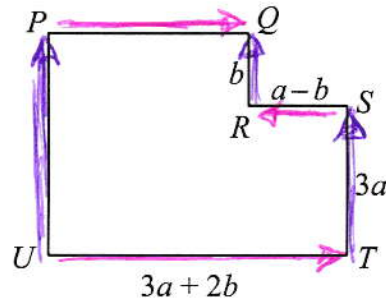
$$x = \frac{1}{2}$$

$$x = \frac{1}{2} \quad \text{(A1)}$$

The diagram shows a shape, $PQRSTU$.

All the corners are right angles.

The lengths of four of the sides are given in terms of a and b .



Find an expression, in terms of a and b , for

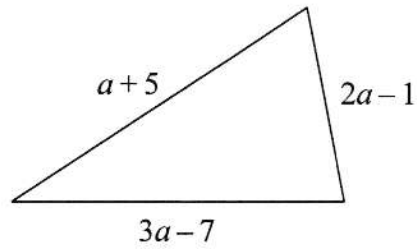
(i) PU ,

$$3a + b \quad (A1)$$

(ii) PQ .

$$(3a + 2b) - (a - b) \quad (M1)$$

$$= 3a + 2b - a + b \quad \rightarrow \quad 2a + 3b \quad (A1)$$



The lengths, in cm, of the sides of a triangle are $(a+5)$, $(3a-7)$ and $(2a-1)$.

The perimeter of the triangle is 24 cm.

Work out the value of a .

$$(a+5) + (3a-7) + (2a-1) = 24 \quad \text{(M1) [EQUATION]}$$

$$6a - 3 = 24$$

$$6a = 27 \quad \text{(M1) [ISOLATE 'a']}$$

$$a = \frac{27}{6}$$

$$a = \dots\dots\dots 4.5 \quad \text{(A1)}$$

Angelou has x sweets.

He eats 5 of these sweets. $\rightarrow (x - 5)$

He puts all the sweets he has left into a bag.

- (i) Nina has 3 times as many sweets as the number that Angelou put into the bag.
Nina has 39 sweets.

Use this information to write down an equation in x .

$$\textcircled{B1} \rightarrow \textcircled{B1} \underline{3(x - 5) = 39}$$

- (ii) Solve your equation to find the value of x .

$$3x - 15 = 39$$

$$3x = 54$$

$$x = \frac{54}{3}$$

$\textcircled{M1}$ [ISOLATING 'x']

$\textcircled{A1}$ [CORRECT EQUATION]

$$x = \underline{18} \quad \textcircled{A1}$$

Paper clips are sold in small boxes and in large boxes.

There is a total of 1115 paper clips in 4 small boxes and 5 large boxes.

There is a total of 530 paper clips in 3 small boxes and 2 large boxes.

Work out the number of paper clips in each small box and in each large box.

$$\textcircled{mi} \begin{cases} 4s + 5L = 1115 & \text{--- ①} \times 2 \\ 3s + 2L = 530 & \text{--- ②} \times 5 \end{cases}$$

$$\textcircled{mi} \begin{cases} 8s + 10L = 2230 \\ 15s + 10L = 2650 \end{cases} \text{ SUBTRACT}$$

$$-7s \qquad = -420 \quad \textcircled{mi}$$

$$s = \frac{-420}{-7}$$

$$= \underline{\underline{60}} \quad \textcircled{A1}$$

SUBSTITUTE INTO ①

$$4(60) + 5L = 1115$$

$$240 + 5L = 1115$$

$$5L = 875$$

$$L = \underline{\underline{175}} \quad \textcircled{A1}$$

Rectangular tiles have width x cm and height $(x + 7)$ cm.

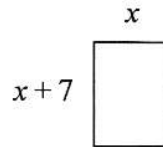


Diagram NOT accurately drawn

Some of these tiles are used to form a shape.
The shape is 6 tiles wide and 4 tiles high.

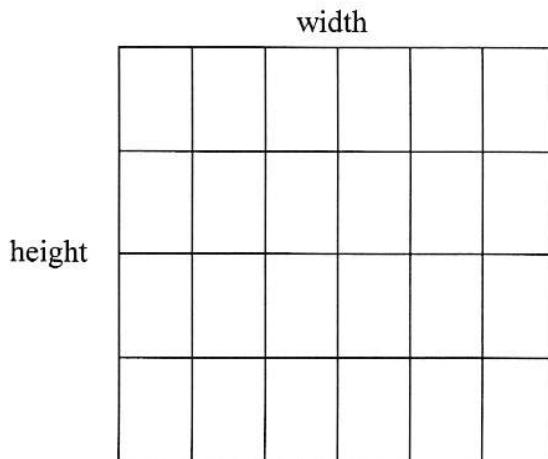


Diagram NOT accurately drawn

(a) Write down expressions, in terms of x , for the width and height of this shape.

width = $6x$ (A1) cm

height = $4(x+7)$ (A1) cm
(2)

(b) The width and the height of this shape are equal.

(i) Write down an equation in x .

..... $6x = 4(x+7)$ (A1)

(ii) Solve your equation to find the value of x .

$$6x = 4(x+7)$$

$$6x = 4x + 28 \quad \text{(M1) [NO BRACKETS]}$$

$$2x = 28 \quad \text{(M1) [ISOLATING 'x']}$$

$$x = \underline{\underline{14}}$$

$x = \underline{\underline{14}}$ (A1)
(4)

Arul had x sweets.

Nikos had four times as many sweets as Arul.

(a) Write down an expression, in terms of x , for the number of sweets Nikos had.

$$\frac{4x}{\dots\dots\dots} \quad \text{(B1)}$$

(1)

Nikos gave 6 of his sweets to Arul.

Now they both have the same number of sweets.

(b) Use this information to form an equation in x .

$$\frac{\text{(M1)}}{4x - 6} = \frac{\text{(B1)}}{x + 6}$$

(2)

(c) Solve your equation to find the number of sweets that Arul had at the start.

$$4x - 6 = x + 6$$

$$3x - 6 = 6$$

$$3x = 12 \quad \text{(M1) [ISOLATE 'x']}$$

$$x = \underline{\underline{4}}$$

$$\frac{4}{\dots\dots\dots} \quad \text{(A1)}$$

(2)

Cups cost x dollars each.

Mugs cost $(x + 2)$ dollars each.

- (a) Write down an expression, in terms of x , for the total cost of 12 cups and 6 mugs.

$$\frac{12x + 6(x+2)}{\dots\dots\dots} \text{ dollars}$$

(2)

(B1) (B1)

- (b) The total cost of 12 cups and 6 mugs is 57 dollars.
Work out the cost of 1 cup.

$$\left. \begin{aligned} 12x + 6(x+2) &= 57 \\ 12x + 6x + 12 &= 57 \\ 18x + 12 &= 57 \\ 18x &= 45 \\ x &= \frac{45}{18} \end{aligned} \right\} \text{(M1) [ANY CORRECT EQUATION]}$$

$$\frac{2.5}{\dots\dots\dots} \text{ dollars}$$

(2)

(A1)

Rectangular tiles have width $(x + 1)$ cm and height $(5x - 2)$ cm.

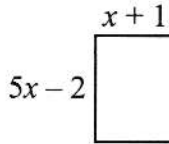


Diagram **NOT**
accurately drawn

Some of these tiles are used to form a large rectangle.
The large rectangle is 7 tiles wide and 3 tiles high.

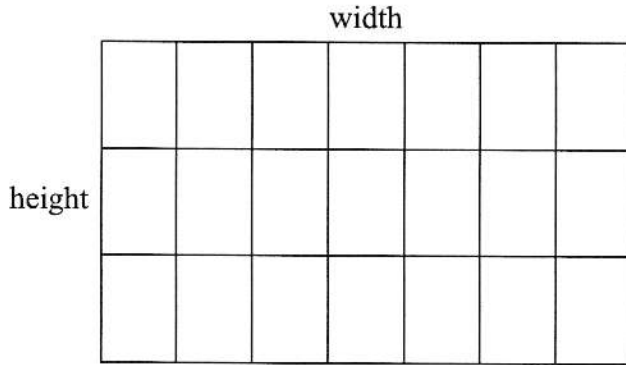


Diagram **NOT**
accurately drawn

The perimeter of the large rectangle is 68 cm.

(a) Write down an equation in x .

$$\overbrace{14(x+1)}^{(B1)} + \overbrace{6(5x-2)}^{(B1)} = \overbrace{68}^{(B1)}$$

(3)

(b) Solve this equation to find the value of x .

$$14x + 14 + 30x - 12 = 68 \quad (M1) \text{ [NO BRACKETS]}$$

$$44x + 2 = 68$$

$$44x = 66 \quad (M1) \text{ [ISOLATING 'x']}$$

$$x = \frac{66}{44}$$

$$= \underline{\underline{1.5}} \quad (A1)$$

The diagram shows a parallelogram $ABCD$.
In the diagram, all the angles are in degrees.

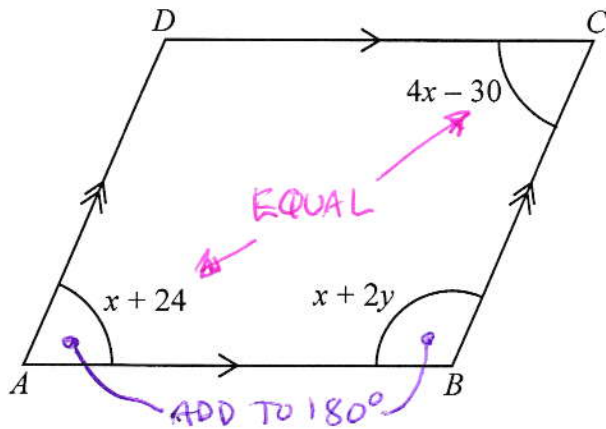


Diagram **NOT**
accurately drawn

Work out the value of x and the value of y .

1ST

$$4x - 30 = x + 24 \quad (m)$$

$$3x - 30 = 24$$

$$3x = 54$$

$$x = \underline{18^\circ} \quad (A1)$$

2ND

$$(x + 2y) + (x + 24) = 180 \quad (m)$$

$$2x + 2y + 24 = 180 \quad x = 18^\circ$$

$$2y = 180 - 24 - 2x$$

$$2y = 180 - 24 - 2(18)$$

$$2y = 120$$

$$y = \underline{60^\circ} \quad (A1)$$

Ben is x cm tall.

Kieran is 8 cm taller than Ben. $\rightarrow x+8$

Bianca is 2 cm shorter than Ben. $\rightarrow x-2$

Write an expression, in terms of x , for the mean of their heights in centimetres.

Give your answer in its simplest form.

$$\begin{aligned} \textcircled{m} \quad \left\{ \frac{x + (x+8) + (x-2)}{3} \right. &= \frac{3x+6}{3} \\ &= \underline{\underline{x+2}} \quad \textcircled{A} \end{aligned}$$

$\textcircled{B1}$ [FOR $\div 3$]

The diagram shows a right-angled triangle and a rectangle.

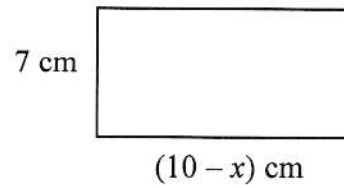
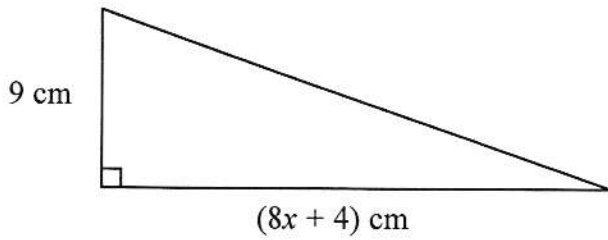


Diagram NOT accurately drawn

The area of the triangle is twice the area of the rectangle.

(i) Write down an equation for x .

$$\frac{9 \times (8x + 4)}{2} = 2 \times [7 \times (10 - x)] \quad \text{(M1) [EQUATION]}$$

x2!

$$\Rightarrow \underline{9(8x + 4) = 28(10 - x)}$$

(ii) Find the area of the rectangle.
Show clear algebraic working.

[1ST]

$$9(8x + 4) = 28(10 - x)$$

$$72x + 36 = 280 - 28x \quad \text{(M1) [EXPANDING]}$$

$$100x + 36 = 280$$

$$100x = 244 \quad \text{(M1) [ISOLATE 'x']}$$

$$x = \underline{\underline{2.44}} \quad \text{(A1)}$$

[2ND]

$$\begin{aligned} \text{AREA OF RECTANGLE} &= 7 \times (10 - x) \\ &= 7 \times (10 - 2.44) \quad \text{(M1)} \\ &= \underline{\underline{52.92 \text{ cm}^2}} \quad \text{(A1)} \end{aligned}$$

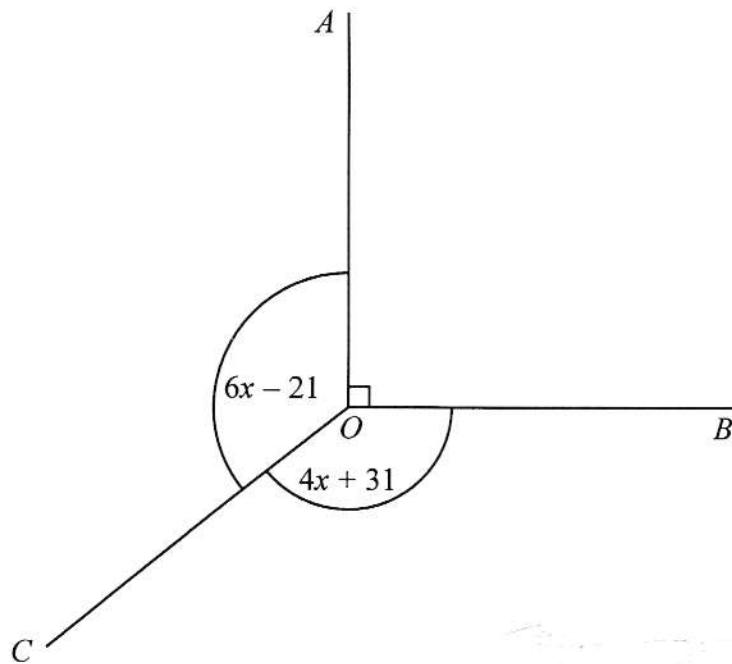


Diagram **NOT**
accurately drawn

In the diagram, all angles are in degrees.

Angle AOB is a right angle.

Angle $AOC =$ Angle BOC .

KEY!

Work out the value of x .

$$6x - 21 = 4x + 31 \quad \text{(M1) [EQUATION]}$$

$$2x - 21 = 31$$

$$2x = 52 \quad \text{(M1) [ISOLATING 'x']}$$

$$x = \frac{52}{2}$$

$$x = \dots 26^\circ \quad \text{(A1)}$$

Tara has 3 dogs and 4 cats.

The dogs have a mean weight of x kg.

The cats have a mean weight of y kg.

Write down an expression, in terms of x and y , for the mean weight of all 7 of Tara's pets.

$$\frac{3x + 4y}{7} \text{ kg}$$

(B1) (B1)

Here is a rectangle.

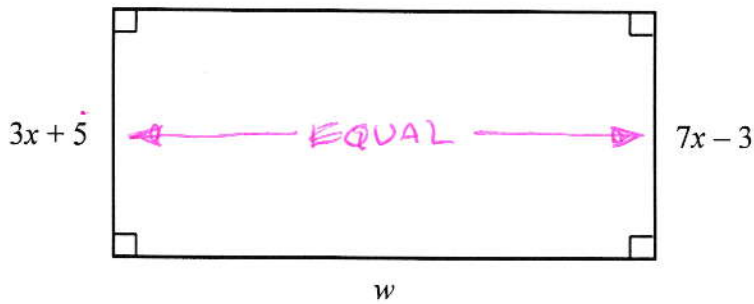


Diagram **NOT**
accurately drawn

All measurements are in centimetres.

The area of the rectangle is 242 cm^2 .

Find the value of w .

1ST FIND x

$$7x - 3 = 3x + 5 \quad (\text{m1})$$

$$4x - 3 = 5$$

$$4x = 8$$

$$x = \frac{8}{4}$$

$$= \underline{2} \quad (\text{B1})$$

2ND

$$w(3x + 5) = 242 \quad (\text{m1})$$

$$w(3 \times 2 + 5) = 242 \quad (\text{m1})$$

$$x = 2 \quad \nearrow \quad 11w = 242$$

$$w = \frac{242}{11}$$

$$= \underline{\underline{22 \text{ cm}}} \quad (\text{A1})$$

Barney has the same number of sweets as Millie.

[x EACH]

Barney gives 15 of his sweets to Millie.

Millie now has 4 times as many sweets as Barney.

Work out the total number of sweets that Barney and Millie have.

→ [$x - 15$] AND [$x + 15$]

$$x + 15 = 4(x - 15) \quad (m1) \quad \text{[EQUATION]}$$

$$\Rightarrow x + 15 = 4x - 60$$

$$4x - 60 = x + 15 \quad \# \quad \text{[I PREFER TO HAVE LARGEST } x\text{-TERM ON THE LEFT]}$$

$$3x - 60 = 15$$

$$3x = 75 \quad (m1)$$

$$x = \frac{75}{3}$$

$$= \underline{\underline{25}} \quad (m1) \quad \text{[EACH]}$$

$$\xrightarrow{\quad (A1) \quad} \underline{\underline{50}} \quad \text{[TOTAL]}$$

Vicky makes 8 purses and 9 key rings to sell for charity.

The price of a purse will be twice as much as the price of a key ring.

Vicky wants to get a total of exactly £40 when she sells all the purses and all the key rings.

Work out the price Vicky needs to charge for each purse and for each key ring.

$$8p + 9k = 40 \text{ (ml)}, \text{ BUT } p = 2k$$

$$\text{so } 16k + 9k = 40 \text{ (ml)}$$

$$\Rightarrow 25k = 40$$

$$k = \frac{40}{25}$$

$$= \underline{\underline{1.6}}$$

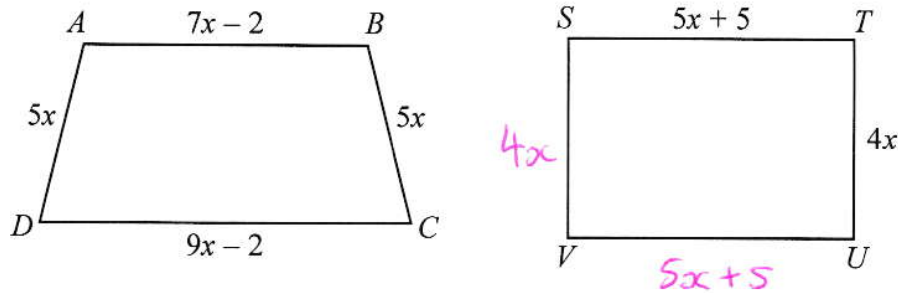
$$\longrightarrow p = 2 \times 1.6$$

$$= \underline{\underline{3.2}}$$

purse $\underline{\underline{£3.20}}$ (AI) key ring $\underline{\underline{£1.60}}$ (AI)

$ABCD$ is a trapezium.
 $STUV$ is a rectangle.

Diagram **NOT**
 accurately drawn



All measurements are in centimetres.

The two shapes have the same perimeter.

Work out the length of ST .

(M1) [ANY CORRECT EQ.]

$$5x + (7x - 2) + 5x + (9x - 2) = 4x + (5x + 5) + 4x + (5x + 5)$$

$$26x - 4 = 18x + 10 \quad \text{(M1) [SIMPLIFIED]}$$

$$8x - 4 = 10$$

$$8x = 14$$

$$x = \frac{14}{8}$$

$$= \underline{\underline{1.75}} \quad \text{(A1)}$$

2ND

$$ST = 5x + 5$$

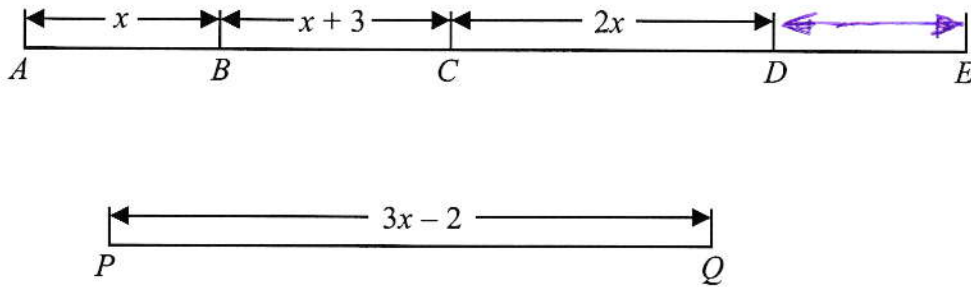
$$= 5(1.75) + 5 \quad \text{(M1) [SUBSTITUTION]}$$

$$= \underline{\underline{13.75}}$$

$$\underline{\underline{13.75}} \text{ cm} \quad \text{(A1)}$$

Here are two straight lines, $ABCDE$ and PQ

Diagram NOT
accurately drawn



In the diagrams all the lengths are in cm.

$$AE = 2PQ.$$

Find an expression, in terms of x , for the length of DE .

Give your answer in its simplest form.

1ST

$$\begin{aligned}
 DE &= AE - AD \\
 &= 2PQ - AD \\
 &= 2(3x-2) - [x + x+3 + 2x] \\
 &= 6x-4 - [4x+3] \\
 &= 6x-4 - 4x-3 \\
 &= \underline{\underline{2x-7}}
 \end{aligned}$$

$$\underline{\underline{2x-7}} \text{ cm}$$

There are 30 sweets in a box.

x of the sweets are blue.

The rest of the sweets are green. $\rightarrow (30 - x)$

Aaron takes at random a sweet from the box.

Write down an expression, in terms of x , for the probability that Aaron takes a green sweet.

$$\textcircled{B1} \left[\frac{\textcircled{B1} \overbrace{30-x}}{30} \dots \right]$$

Amita, Monica and Rita are three sisters.

Monica is x years old.

Amita is 3 years older than Monica. $\rightarrow x+3$

Rita is twice the age of Amita. $\rightarrow 2(x+3)$

If the mean age of the three sisters is 15, how old is Amita?

$$\frac{x + (x+3) + 2(x+3)}{3} = 15 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \textcircled{M1} \text{ [EITHER]}$$

$$x + (x+3) + 2(x+3) = 45$$

$$x + (x+3) + 2x+6 = 45$$

$$4x + 9 = 45$$

$$4x = 36$$

$$x = 9 \quad \textcircled{M1}$$

AMITA IS

$$9+3 = \underline{\underline{12}}$$

12 $\textcircled{A1}$

..... years

Asha and Lucy are selling pencils in a school shop.

They sell boxes of pencils and single pencils.

Asha sells 7 boxes of pencils and 22 single pencils.

Lucy sells 5 boxes of pencils and 2 single pencils.

Asha sells twice as many pencils as Lucy.

Work out how many pencils there are in a box.

$$7b + 22 = 2 \times (5b + 2) \quad \text{(M1) [EQUATION]}$$

$$7b + 22 = 10b + 4$$

$$10b + 4 = 7b + 22$$

$$3b + 4 = 22$$

$$3b = 18 \quad \text{(M1) [ISOLATING 'b']}$$

$$b = \frac{18}{3}$$

$$= \underline{\underline{6}} \quad \text{(A1)}$$

Disclaimer

While reasonable endeavours have been used to verify the accuracy of these solutions, these solutions are provided on an “as is” basis and no warranties are made of any kind, whether express or implied, in relation to these solutions.

There is no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

Please report any errors or omissions that You find*. These solutions will be updated to correct errors that are discovered. It is recommended that You always check that You have the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or ‘best’ method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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