

# ALGEBRAIC FRACTIONS

(SIMPLIFYING) 15/05/2018

MAXIMUM MARK: 74

# SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2010]

Algebraic Fractions (Simplifying) [3 Marks]

Simplify fully

$$\frac{x}{6} + \frac{3x}{4}$$

$$= \frac{4x + 6 \times 3x}{24} \quad \leftarrow \textcircled{M1} \quad \leftarrow \textcircled{M1} \quad \left[ \text{COMMON DENOMINATOR OF 12 WOULD BE BETTER!} \right]$$

$$= \frac{4x + 18x}{24}$$

$$= \frac{22x}{24}$$

$$\frac{11x}{12} \quad \textcircled{A1}$$

Simplify  $\frac{x^2 - 25}{2x^2 - 9x - 5}$

FACTORISE AND CANCEL

$$\frac{(x+5)(x-5) \text{ (M1)}}{(2x+1)(x-5) \text{ (M1)}} = \frac{x+5 \text{ (A1)}}{\underline{\underline{2x+1}}}$$

Express the algebraic fraction  $\frac{2x^2 - 3x - 20}{x^2 - 16}$  as simply as possible.

FACTORISE!

$$\frac{(2x+5)(x-4)}{(x+4)(x-4)} = \frac{(2x+5)}{(x+4)}$$

$$\frac{2x+5}{x+4}$$

Simplify

$$(a) \frac{x^2 - 3x}{2x - 6} = \frac{x(x-3)}{2(x-3)} \leftarrow \text{(M1)}$$

$$\leftarrow \text{(M1)}$$

$$= \frac{x}{2}$$

$$\frac{x}{2} \quad \text{(A1)}$$


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(3)

$$(b) \frac{2}{x-1} - \frac{3}{x}$$

$$= \frac{2x - 3(x-1)}{x(x-1)} \quad \text{(M1)}$$

$$= \frac{2x - 3x + 1}{x(x-1)} \quad \text{(M1)}$$

$$= \frac{-x + 1}{x(x-1)}$$

$$= \frac{-(x-1)}{x(x-1)} = \frac{-1}{x}$$

$$-\frac{1}{x} \quad \text{(A1)}$$


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(3)

Simplify fully  $\frac{4x^2 - 25}{6x^2 + 13x - 5}$

$$= \frac{(2x+5)(2x-5) \leftarrow \textcircled{M1} \text{ FACTORISING.}}{(2x+5)(3x-1) \leftarrow \textcircled{M1} \text{ FACTORISING.}}$$

$$= \frac{2x-5}{3x-1}$$

$$\frac{2x-5}{3x-1} \textcircled{A1}$$

Simplify fully  $\frac{5x^2 + 14x - 3}{50x^2 - 2}$

$$= \frac{(5x-1)(x+3)}{2(25x^2-1)}$$

← FACTORISE  
(M1)

$$= \frac{(5x-1)(x+3)}{2(5x+1)(5x-1)}$$

← DOTS

$$= \frac{x+3}{2(5x+1)}$$

(A1)  
(A1)

Simplify fully  $\frac{x^2 + 6x}{x^2 - 36}$

$$= \frac{x \cancel{(x+6)}}{(x-6) \cancel{(x+6)}} \quad \text{(M1)} \quad \text{(M1)}$$

$$\frac{x}{x-6} \quad \text{(A1)}$$

Simplify fully  $\frac{2}{x+2} + \frac{x}{x^2+5x+6}$

$$= \frac{2}{x+2} + \frac{x}{(x+2)(x+3)}$$

← FACTORISE (M1)

$$= \frac{2(x+3) + x}{(x+2)(x+3)}$$

COMMON DENOMINATOR (M1)

$$= \frac{2x+6+x}{(x+2)(x+3)}$$

SIMPLIFY NUMERATOR (M1)

$$= \frac{3x+6}{(x+2)(x+3)}$$

$$= \frac{3(x+2)}{(x+2)(x+3)}$$

← FACTORISE (M1)

$$\frac{3}{x+3}$$



(a) Simplify  $\frac{x^2}{x^2 - 2x}$

$$= \frac{\cancel{x} \times \cancel{x}}{\cancel{x}(x-2)} \quad (M1)$$

$$= \frac{x}{x-2}$$

$$\frac{x}{x-2} \quad (A1)$$


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(2)

(b) Simplify  $\frac{2}{2x-1} - \frac{1}{x+1}$

$$= \frac{2(x+1) - (2x-1)}{(2x-1)(x+1)} \quad (M1)$$

$$= \frac{2x+2 - 2x+1}{(2x-1)(x+1)} \quad (M1)$$

$$\frac{3}{(2x-1)(x+1)} \quad (A1)$$


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(4)

Simplify fully  $\frac{4}{x} + \frac{3}{2-x}$ 

$$= \frac{4(2-x) + 3x}{x(2-x)} \quad (M1)$$

$$= \frac{8 - 4x + 3x}{x(2-x)} \quad (M1)$$

$$= \frac{8-x}{x(2-x)}$$

$$\frac{8-x}{x(2-x)} \quad (A1)$$

Simplify fully  $\frac{x^2 - 16}{x^2 - 6x + 8}$

$$= \frac{(x-4)(x+4)}{(x-4)(x-2)}$$

$$= \frac{x+4}{x-2}$$

Express  $\frac{4}{x-1} - \frac{3}{x+1}$  as a single fraction.

Give your answer as simply as possible.

$$\begin{aligned}
 \frac{4}{x-1} - \frac{3}{x+1} &= \frac{4(x+1) - 3(x-1)}{(x-1)(x+1)} && \text{CROSS MULTIPLY} \\
 & && \text{(M)} \text{ [COMMON DENOMINATOR]} \\
 &= \frac{4x + 4 - 3x + 3}{(x-1)(x+1)} && \text{(M)} \text{ [EXPAND NUMERATOR]} \\
 &= \frac{x + 7}{(x-1)(x+1)} && \text{(A)} \\
 & \quad \swarrow
 \end{aligned}$$

NOTE

DENOMINATOR IS FACTORISED  
SO I HAVE LEFT IT ALONE.

HOWEVER, ON THIS OCCASION,  
YOU MAY SPOT THAT IT'S "THE  
DIFFERENCE OF TWO SQUARES"  
SO MAY LIKE TO WRITE IT  
AS SUCH

$$\rightarrow \frac{x+7}{x^2-1}$$

[NOT NECESSARY THOUGH!]

Express  $\frac{3}{x+2} - \frac{6}{2x+5}$  as a single fraction.

Simplify your answer.

$$\frac{3}{x+2} - \frac{6}{2x+5}$$

$$= \frac{6x+15 - 6x-12}{(x+2)(2x+5)} \quad (M1) \quad (B1)$$

$$= \frac{3}{(x+2)(2x+5)} \quad (A1)$$

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Simplify fully  $\frac{6x^2 + x - 15}{12x^2 - 27}$

Show clear algebraic working.

$$\begin{aligned}
 \frac{6x^2 + x - 15}{12x^2 - 27} &= \frac{(2x-3)(3x+5)}{3(4x^2-9)} \quad \text{(M1)} \\
 &= \frac{(2x-3)(3x+5)}{3(2x-3)(2x+3)} \quad \text{(M1)} \\
 &= \frac{3x+5}{3(2x+3)} \quad \text{(A1)}
 \end{aligned}$$

1x15  
/ 3x5

Simplify fully  $1 + \frac{x^2 + x - 6}{(x+4)(x-2)}$

$$= \frac{(x+4)(x-2) + x^2 + x - 6}{(x+4)(x-2)} \quad \text{(M1) [SINGLE FRACTION]}$$

$$= \frac{x^2 - 2x + 4x - 8 + x^2 + x - 6}{(x+4)(x-2)}$$

$$= \frac{2x^2 + 3x - 14}{(x+4)(x-2)} \quad \text{(M1) [SIMPLIFIED NUMERATOR]}$$

$$= \frac{(2x+7)\cancel{(x-2)}}{(x+4)\cancel{(x-2)}} \quad \text{(M1) [FACTORISING]}$$

$$= \frac{2x+7}{x+4} \quad \text{(AG)}$$

Simplify fully  $\frac{2}{x-1} + \frac{x-11}{x^2+3x-4}$

$$= \frac{2}{x-1} + \frac{x-11}{(x-1)(x+4)} \quad \text{(B1) [FACTORISING]}$$

$$= \frac{2x+8}{(x-1)(x+4)} + \frac{x-11}{(x-1)(x+4)} \quad \text{(B1) [COMMON DENOMINATOR]}$$

$$= \frac{3x-3}{(x-1)(x+4)} \quad \text{(B1) [SIMPLIFYING NUMERATOR]}$$

$$= \frac{3(x-1)}{(x-1)(x+4)} \quad \text{(B1) [FACTORISING NUMERATOR]}$$

$$= \frac{3}{x+4} \quad \text{(A1)}$$



Simplify fully  $\frac{2x^2 - 5x - 12}{4x^2 - 9}$

$$= \frac{(2x+3)(x-4)}{(2x+3)(2x-3)}$$

(B1) FACTORISE NUMERATOR  
(B1) FACTORISE DENOMINATOR

$$= \frac{x-4}{2x-3}$$

(A1)

Write  $5 - (x + 2) \div \left(\frac{x^2 - 4}{x - 3}\right)$  as a single fraction.

Simplify your answer fully.

$$\begin{aligned}
 & 5 - (x+2) \times \frac{(x-3)}{x^2-4} & = & 5 - \frac{(x+2) \times (x-3)}{(x+2)(x-2)} \\
 & & & \text{(B1) [FACTORISING]} \\
 & & = & 5 - \frac{x-3}{x-2} \\
 & \text{(M1) [CHANGING TO MULTIPLY]} & = & \frac{5(x-2) - (x-3)}{x-2} \\
 & & = & \frac{5x-10-x+3}{x-2} & \text{(M1) [SINGLE FRACTION]} \\
 & & = & \frac{4x-7}{x-2} & \text{(A1)}
 \end{aligned}$$



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There is no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

Please report any errors or omissions that You find\*. These solutions will be updated to correct errors that are discovered. It is recommended that You always check that You have the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or ‘best’ method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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