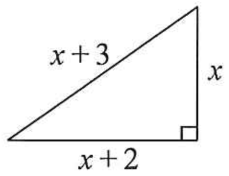
1. [Edexcel, 2005] Algebraic Proof (Quadratic Equations) **[6 Marks]**

11. A right-angled triangle has sides of length x cm, $(x + 2)$ cm and $(x + 3)$ cm.



(a) Use Pythagoras' theorem to write down an equation in x.

$x^2 + (x+2)^2 = (x+3)^2$ (A1)

(1)

(b) Show that your equation simplifies to $x^2 - 2x - 5 = 0$

$x^2 + (x+2)(x+2) = (x+3)(x+3)$

$\Rightarrow x^2 + x^2 + 4x + 4 = x^2 + 6x + 9$ (M1) [EXPANDING BRACKETS]

$\Rightarrow x^2 + x^2 - x^2 + 4x - 6x + 4 - 9 = 0$

$\Rightarrow x^2 - 2x - 5 = 0$ (M1) [RHS = 0]

(2)

(c) By solving the equation $x^2 - 2x - 5 = 0$, find the length of each side of the triangle.
Give your answers correct to one decimal place.

$a = 1,\ b = -2,\ c = -5$

USE $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$ (M1)

$= \dfrac{2 \pm \sqrt{4 + 20}}{2} \rightarrow \underline{3.4494\ldots}$ (A1)

$-1.4494\ldots$ [NOT POSSIBLE!]

(A1)

3.4 cm, 5.4 cm, 6.4 cm

x $\quad x+2$ $\quad x+3$

(3)

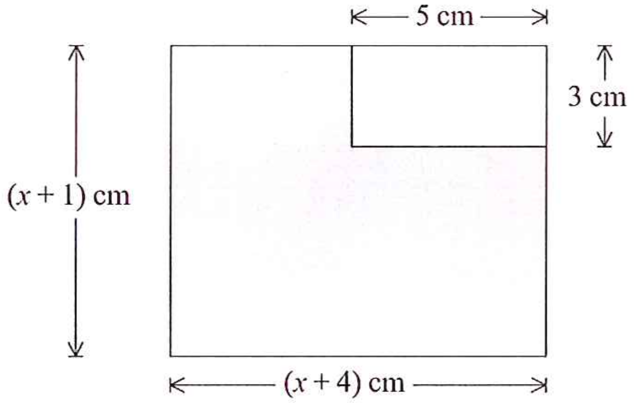


Diagram **NOT** accurately drawn

A rectangular piece of card has length $(x + 4)$ cm and width $(x + 1)$ cm.
A rectangle 5 cm by 3 cm is cut from the corner of the piece of card.
The remaining piece of card, shown shaded in the diagram, has an area of 35 cm^2.

(a) Show that $x^2 + 5x - 46 = 0$

FROM DIAGRAM AND INFORMATION IN QUESTION:-

$(x+1)(x+4) - 5\times 3 = 35$ B1

$x^2 + 4x + 1x + 4 - 15 = 35$ M1

$x^2 + 5x - 11 = 35$ M1

$x^2 + 5x - 46 = 0$ QED

(3)

(b) Solve $x^2 + 5x - 46 = 0$ to find the value of x.
Give your answer correct to 3 significant figures.

$a = 1 \quad b = 5 \quad c = -46$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(5) \pm \sqrt{(5)^2 - 4\times(1)\times(-46)}}{2}$$ M1

$$= \frac{-5 \pm \sqrt{25 + 184}}{2}$$

$$\frac{-5 + \sqrt{209}}{2} = 4.73$$ M1

$$\frac{-5 - \sqrt{209}}{2} = -9.73$$

NOT POSSIBLE!

$x =$ 4.73 A1

(3)

3. [Edexcel, 2015] Algebraic Proof (Quadratic Equations) **[6 Marks]**

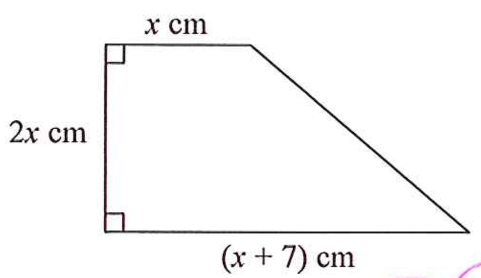


Diagram **NOT** accurately drawn

The diagram shows a trapezium.
The trapezium has an area of 17 cm²

$A = \frac{1}{2}(a+b)h$

(a) Show that $2x^2 + 7x - 17 = 0$

$\frac{1}{2}(x + x + 7) \times 2x = 17$ B1 [EQUATION]

$x(2x+7) = 17$ M1 [SIMPLIFY]

$2x^2 + 7x = 17$ M1 [EXPAND BRACKETS]

$\Rightarrow 2x^2 + 7x - 17 = 0$ QED!

(b) Work out the value of x.
Give your answer correct to 3 significant figures.
Show your working clearly.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 2, \; b = 7, \; c = -17$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(2)(-17)}}{2(2)}$$ M1 [CORRECT SUBSTITUTIONS]

$$= \frac{-7 \pm \sqrt{49 + 136}}{4}$$

1.65 $\quad -5.15$ M1 [TWO ANSWERS]

[-VE ANSWER IS NOT POSSIBLE!]

$x =$ 1.65 A1

(3)

4. [Edexcel, 2016] Algebraic Proof (Quadratic Equations) **[6 Marks]**

The diagram shows a trapezium.

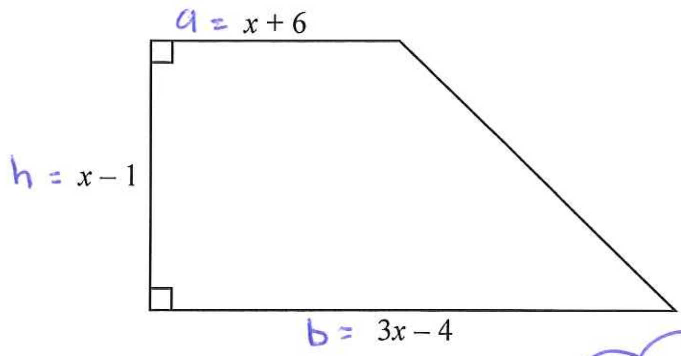


Diagram **NOT** accurately drawn

All measurements on the diagram are in centimetres.

The area of the trapezium is 119 cm^2

$A = \frac{1}{2}(a+b)h$

(i) Show that $2x^2 - x - 120 = 0$

$\frac{1}{2}(x + 6 + 3x - 4) \times (x - 1) = 119$ (M1)

$\Rightarrow (4x + 2)(x - 1) = 238$

$\Rightarrow 4x^2 - 4x + 2x - 2 = 238$

$\Rightarrow 4x^2 - 2x - 240 = 0$

(M2) [ANY TWO STEPS OF WORKING]

$\Rightarrow 2x^2 - x - 120 = 0$ QED!

(ii) Find the value of x.
Show your working clearly.

$2x^2 - x - 120 = 0$

$(2x + 15)(x - 8) = 0$ (M1)

$x = -\frac{15}{2}$ $\quad$ $x = 8$ (M1) [TWO ANSWERS]

[NEGATIVE x IS NOT POSSIBLE SO... $\longrightarrow$ $x =$ 8 (A1) [DISCOUNTS NEGATIVE]

A rectangular lawn has a length of $3x$ metres and a width of $2x$ metres.
The lawn has a path of width 1 metre on three of its sides.

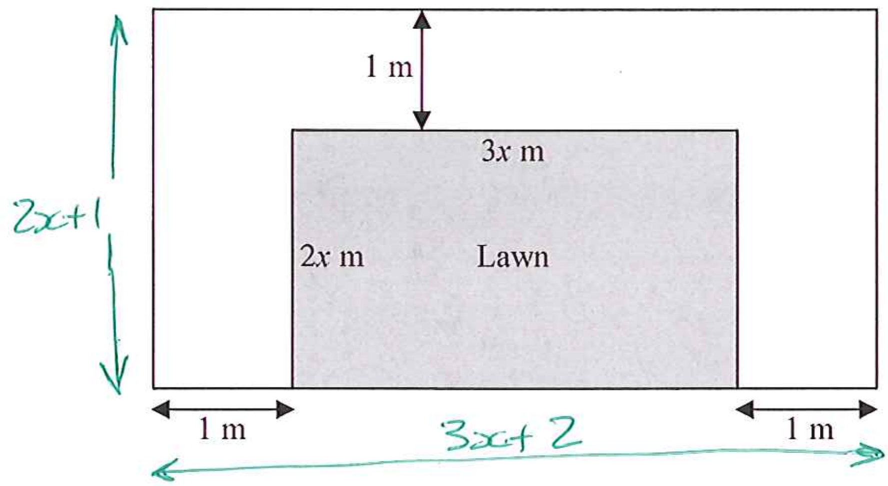


Diagram **NOT** accurately drawn

The total area of the lawn and the path is 100 m^2

(a) Show that $6x^2 + 7x - 98 = 0$

$(2x+1)(3x+2) = 100$ M1

$6x^2 + 4x + 3x + 2 = 100$ M1

$6x^2 + 7x - 98 = 0$

(b) Calculate the area of the lawn.
Show clear algebraic working.

$(3x+14)(2x-7) = 0$ M1

$3x + 14 = 0$
$3x = -14$
$x = -\frac{14}{3}$

[NOT POSSIBLE]

$2x - 7 = 0$
$2x = 7$
$x = \underline{3.5}$ A1

AREA OF LAWN $= 2x \times 3x$
$= 6x^2$ M1
$= 6 \times 3.5^2$ M1
$= \underline{73.5\text{ m}^2}$ A1

The diagram shows a rectangular playground of width x metres and length $3x$ metres.

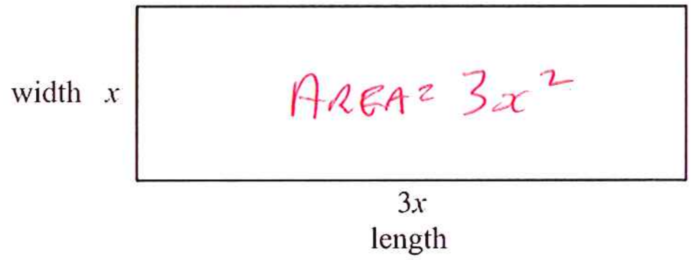
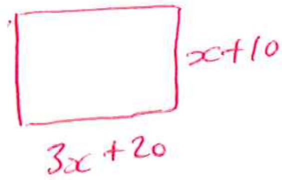


Diagram **NOT** accurately drawn

The playground is extended, by adding 10 metres to its width and 20 metres to its length, to form a larger rectangular playground.



The area of the larger rectangular playground is double the area of the original playground.

AREA = $6x^2$

(a) Show that $3x^2 - 50x - 200 = 0$

B1 $(3x+20)(x+10) = 6x^2$ B1

$3x^2 + 30x + 20x + 200 = 6x^2$

$6x^2 = 3x^2 + 50x + 200$ B1

$\Rightarrow$

$\Rightarrow 3x^2 - 50x - 200 = 0$

(3)

(b) Calculate the area of the original playground.

$3x^2 - 50x - 200 = 0$

$(3x+10)(x-20) = 0$

M1

$3x+10=0$

$3x = -10$

$x = -\frac{10}{3}$

$x - 20 = 0$ A1

$x = 20$

$\Rightarrow$ AREA $= 3x^2$

$= 3 \times 20^2$ M1

A1

$= 1200\text{ m}^2$

–VE VALUE IS NOT POSSIBLE

7. [Edexcel, 2015] Algebraic Proof (Quadratic Equations) **[8 Marks]**

The diagram shows a circular pond, of radius r metres, surrounded by a circular path.
The circular path has a constant width of 1.5 metres.

$A = \pi r^2$

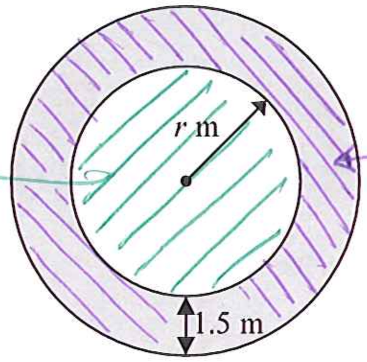


Diagram **NOT** accurately drawn

$A = \pi(r+1.5)^2 - \pi r^2$

The area of the path is $\boxed{\frac{1}{10}}$ the area of the pond. → $0.1 \times \ldots$

(a) Show that $2r^2 - 60r - 45 = 0$

$\pi(r+1.5)^2 - \pi r^2 = 0.1\pi r^2$ (M1)

$\Rightarrow (r+1.5)^2 - r^2 = 0.1r^2$ (M1)

$\Rightarrow r^2 + 3r + 2.25 - r^2 = 0.1r^2$ (M1)

$\Rightarrow 3r + 2.25 = 0.1r^2$

→ $0.1r^2 - 3r - 2.25 = 0$

$\Rightarrow \underline{\underline{2r^2 - 60r - 45 = 0}}$

(b) Calculate the area of the pond.
Show your working clearly.
Give your answer correct to 3 significant figures.

Use $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$2r^2 - 60r - 45 = 0$

$a = 2$
$b = -60$
$c = -45$

$$x = \frac{-(-60) \pm \sqrt{(-60)^2 - 4(2)(-45)}}{2(2)} \quad \text{(M1)}$$

$$= \frac{60 \pm \sqrt{3600 + 360}}{4} \quad \text{(M1)}$$

30.732 (A1) $\qquad -0.732$

$\therefore$ AREA $= \pi \times (30.732)^2$ (M1) $= 2967.0\ldots$

$= \underline{\underline{2970}}\ \text{m}^2$ (A1)

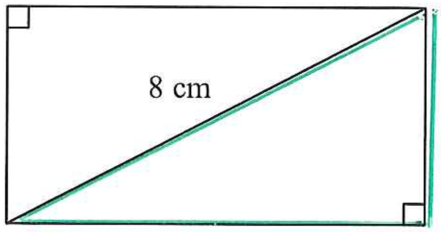


Diagram **NOT** accurately drawn

1ST

HEIGHT = $\sqrt{8^2 - x^2}$ B1

(USING PYTHAGORAS)

The diagram shows a rectangle.
The length of the rectangle is x cm.
The length of a diagonal of the rectangle is 8 cm.
The perimeter of the rectangle is 20 cm.

2ND

HEIGHT = $10 - x$

(a) Show that $x^2 - 10x + 18 = 0$

$10 - x = \sqrt{8^2 - x^2}$ M1

$(10 - x)^2 = 8^2 - x^2$ M1 [EITHER]

$100 - 20x + x^2 = 64 - x^2$

$36 - 20x + 2x^2 = 0$ M1

$2x^2 - 20x + 36 = 0$

$x^2 - 10x + 18 = 0$

(4)

(b) Solve $x^2 - 10x + 18 = 0$
Give your solutions correct to 3 significant figures.
Show your working clearly.

USE $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$, $b = -10$, $c = 18$

$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(18)}}{2(1)}$ M1

$= \frac{10 \pm \sqrt{100 - 72}}{2}$ M1

7.65

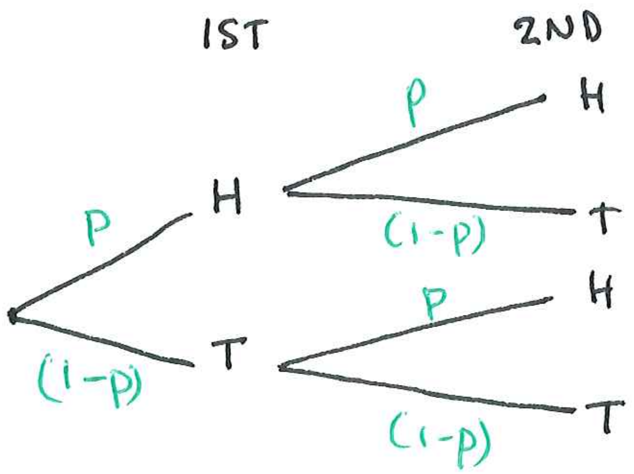
2.35 A1 [BOTH]

9. [Edexcel, 2007] Algebraic Proof (Quadratic Equations) **[3 Marks]**

A coin is biased so that the probability that it shows heads on any one throw is p.
The coin is thrown twice.

The probability that the coin shows heads exactly once is $\frac{8}{25}$

Show that $25p^2 - 25p + 4 = 0$



$$\left.\begin{aligned} P(HT) &= p(1-p) \\ P(TH) &= (1-p)p \end{aligned}\right\}$$ B1 [EITHER]

$\therefore$ P(EXACTLY ONE HEAD) $= p(1-p) + (1-p)p$

$$\Rightarrow \frac{8}{25} = p(1-p) + (1-p)p$$ M1 [EQUATION]

$$\Rightarrow \frac{8}{25} = p - p^2 + p - p^2$$

$$\Rightarrow \frac{8}{25} = 2p - 2p^2$$

$$\Rightarrow 8 = 50p - 50p^2$$ M1 [NO DENOMINATOR]

$$\Rightarrow 50p^2 - 50p + 8 = 0$$

$$\Rightarrow 25p^2 - 25p + 4 = 0$$

A bag contains x counters.
7 of the counters are blue. $P(B) = \frac{7}{x}$
Sam takes at random a counter from the bag and does not replace it. PROBABILITIES CHANGE
Jill then takes a counter from the bag.
The probability they both take a blue counter is 0.2

(a) Form an equation involving x.
Show that your equation can be expressed as $x^2 - x - 210 = 0$

$P(BB)$ $\frac{7}{x} \times \frac{6}{x-1} = 0.2$ B1

$\Rightarrow 42 = 0.2x(x-1)$ M1 [TWO+ STEPS OF WORKING]

$\Rightarrow 42 = 0.2x^2 - 0.2x \Rightarrow 210 = x^2 - x$

$\Rightarrow x^2 - x - 210 = 0$

(b) Solve $x^2 - x - 210 = 0$
Show clear algebraic working.

$x^2 - x - 210 = 0$

$(x+14)(x-15) = 0$ M1

$x = -14$ $\quad$ $x = 15$ A1 [FOR TWO SOLUTIONS]

[NOT POSSIBLE]

A1 [FOR SELECTING $x=15$ AS ACTUAL SOLUTION]

Clare buys some shares for $\$50x$.
Later, she sells the shares for $\$(600 + 5x)$.
She makes a profit of $x\%$

(a) Show that $x^2 + 90x - 1200 = 0$

ACTUAL PROFIT $= (600 + 5x) - 50x$

$= 600 - 45x$ B1

∴ PERCENTAGE PROFIT

$$x = \frac{600 - 45x}{50x} \times 100$$ M1

$$\Rightarrow x = \frac{600 - 45x}{x} \times 2$$

$$\Rightarrow x^2 = 1200 - 90x$$

M1 [EITHER]

$$\Rightarrow x^2 + 90x - 1200 = 0$$

(b) Solve $x^2 + 90x - 1200 = 0$
Find the value of x correct to 3 significant figures.

USE $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1,\ b = 90,\ c = -1200$

$$x = \frac{-(90) \pm \sqrt{90^2 - 4(1)(-1200)}}{2(1)}$$ M1 [CORRECT SUBSTITUTIONS]

$$= \frac{-90 \pm \sqrt{8100 + 4800}}{2}$$ M1 [SIMPLIFYING]

11.8 A1

-102 (NOT POSSIBLE)

(a) Show that

$$(a^2 + 1)(c^2 + 1) = (ac - 1)^2 + (a + c)^2$$

<u>LHS</u>

F O I L

$(a^2+1)(c^2+1) = a^2c^2 + a^2 + c^2 + 1$ M1 [EXPANSION OF LHS]

<u>RHS</u>

$\underbrace{(ac-1)^2} + \underbrace{(a+c)^2} = \underbrace{a^2c^2 - 2ac + 1} + \underbrace{a^2 + 2ac + c^2}$ M1

$= a^2c^2 + 1 + a^2 + c^2$

$= a^2c^2 + a^2 + c^2 + 1 = \underline{\underline{\text{LHS}}}$! M1

(b) By finding suitable values of a and c, use part (a) to write 650065 as the sum of two square numbers.

HINT: 65 MUST BE A FACTOR!

$650065 = 65 \times 10001$ B1

$= (64+1) \times (10000+1)$

$= (8^2+1) \times (100^2+1)$

$\therefore a = 8$, $c = 100$ B1 [BOTH]

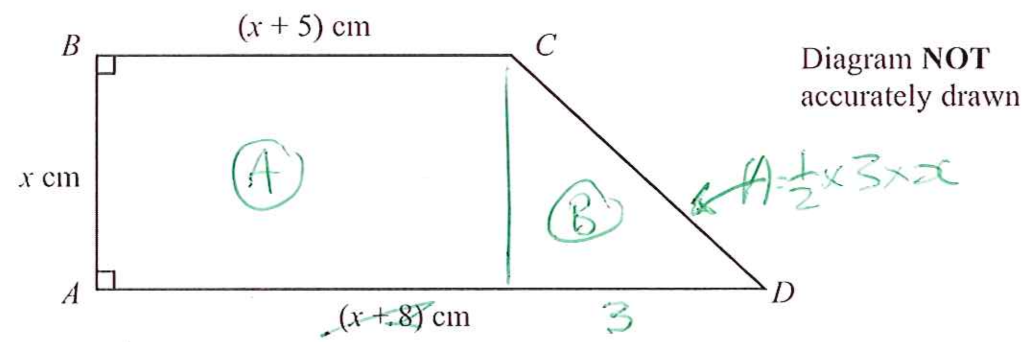
<u>TAKING RHS</u>

$(ac-1)^2 + (a+c)^2 = (8 \times 100 - 1)^2 + (8 + 100)^2$

$= \underline{\underline{799^2 + 108^2}}$ A1

[ACCEPT 638401 + 11664]

13. [Edexcel, 2012] Algebraic Proof (Quadratic Equations) **[7 Marks]**



The diagram shows a trapezium $ABCD$ with AD parallel to BC.
$AB = x$ cm, $BC = (x + 5)$ cm and $AD = (x + 8)$ cm.
The area of the trapezium is 42 cm^2.

(a) Show that $2x^2 + 13x - 84 = 0$

$A + B = 42$

$x(x+5) + 1{\cdot}5x = 42$ M1

$\Rightarrow x^2 + 5x + 1{\cdot}5x = 42$ M1

$\Rightarrow x^2 + 6{\cdot}5x = 42$

$\Rightarrow x^2 + 6{\cdot}5x - 42 = 0$

$\Rightarrow 2x^2 + 13x - 84 = 0$

(2)

(b) Calculate the perimeter of the trapezium.

$2x^2 + 13x - 84 = 0$

$(2x+21)(x-4) = 0$ B1

$x = -\frac{21}{2}$ $\qquad$ $x = \underline{\underline{4}}$ A1

$CD = \sqrt{3^2 + x^2} = \sqrt{9+16} = \underline{\underline{5}}$ A1

–VE NOT POSSIBLE

PERIMETER $= (x+8) + x + (x+5) + CD$ M1

$= (4+8) + 4 + (4+5) + 5$

$= \underline{\underline{30}}$

30 A1 cm

(5)

There are 10 beads in a box.
n of the beads are red. $P(R) = \frac{n}{10}$
Meg takes one bead at random from the box and does not replace it.
She takes a second bead at random from the box.
The probability that she takes 2 red beads is $\frac{1}{3}$.

PROBABILITIES CHANGE

Show that $n^2 - n - 30 = 0$

$$P(R) = \frac{n}{10}$$

$$P(R) = \frac{n-1}{9}$$

B1 [BOTH - ANYWHERE]

$$\therefore \quad P(RR) = \frac{n}{10} \times \frac{(n-1)}{9}$$

$$\Rightarrow \frac{n}{10} \times \frac{(n-1)}{9} = \frac{1}{3}$$

M1 [EQUATION]

$$\Rightarrow \frac{n \times (n-1)}{90} = \frac{1}{3}$$

$$\Rightarrow 3n(n-1) = 90$$

M1 [NO DENOMINATORS]

$$\Rightarrow 3n^2 - 3n = 90$$

M1 [FORMING A QUADRATIC]

$$\Rightarrow 3n^2 - 3n - 90 = 0$$

$$\Rightarrow \underline{\underline{n^2 - n - 30 = 0}}$$

15. [Edexcel, 2006] Algebraic Proof (Quadratic Equations) **[7 Marks]**

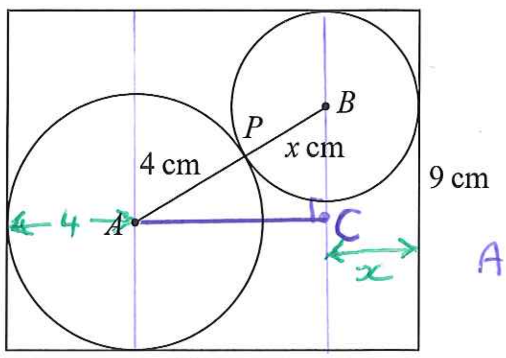


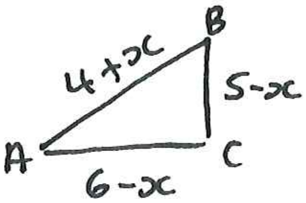
Diagram **NOT** accurately drawn

$AC = 10 - 4 - x$
$= 6 - x$ (B1)

$BC = 9 - 4 - x$
$= 5 - x$ (B1)

The diagram shows one disc with centre A and radius 4 cm and another disc with centre B and radius x cm.
The two discs fit exactly into a rectangular box 10 cm long and 9 cm wide.
The two discs touch at P.
APB is a straight line.

(a) Use Pythagoras' Theorem to show that $x^2 - 30x + 45 = 0$



$(6-x)^2 + (5-x)^2 = (4+x)^2$ (M1)

$36 - 12x + x^2 + 25 - 10x + x^2 = 16 + 8x + x^2$

$\Rightarrow 61 - 22x + 2x^2 = 16 + 8x + x^2$ (M1) [ANY]

$\Rightarrow 2x^2 - x^2 - 22x - 8x + 61 - 16 = 0$

$\Rightarrow x^2 - 30x + 45 = 0$

(b) Find the value of x.
Give your value correct to 3 significant figures.

USE $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1,\ b = -30,\ c = 45$

$$x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(1)(45)}}{2(1)}$$ (M1) [SUBSTITUTIONS]

$$= \frac{30 \pm \sqrt{900 - 180}}{2}$$ (M1) [SIMPLIFYING]

28.4 [TOO BIG!]

1.58 (A1)

Disclaimer

While reasonable endeavours have been used to verify the accuracy of these solutions, these solutions are provided on an "as is" basis and no warranties are made of any kind, whether express or implied, in relation to these solutions.

There is no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

Please report any errors or omissions that You find*. These solutions will be updated to correct errors that are discovered. It is recommended that You always check that You have the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or 'best' method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners' mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners' mark scheme.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site.

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words 'show that', 'show your working out', or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

* The best way to inform of errors or omissions is a direct Twitter message to @Maths4Everyone

Note: "You/Your" means the person who is using these solutions.