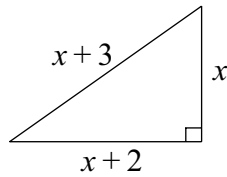# ALGEBRAIC PROOF

1. [**6 marks**]

A right-angled triangle has sides of length x cm, $(x + 2)$ cm and $(x + 3)$ cm.

(a) Use Pythagoras' theorem to write down an equation in x.

....

(1)

(b) Show that your equation simplifies to $x^2 - 2x - 5 = 0$

(2)

(c) By solving the equation $x^2 - 2x - 5 = 0$, find the length of each side of the triangle.
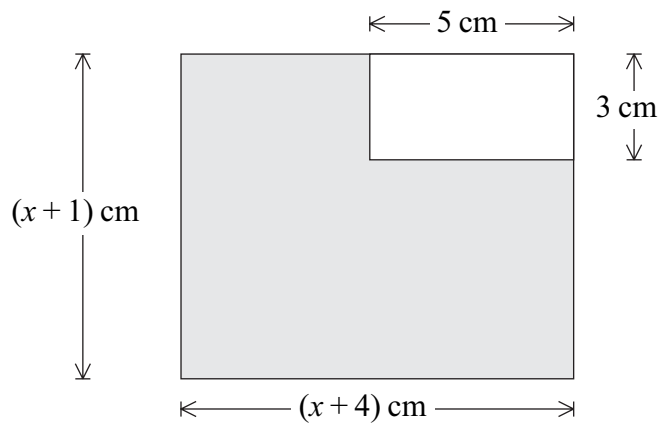Give your answers correct to one decimal place.

.... cm, ... cm, ... cm

(3)

A rectangular piece of card has length $(x + 4)$ cm and width $(x + 1)$ cm.
A rectangle 5 cm by 3 cm is cut from the corner of the piece of card.
The remaining piece of card, shown shaded in the diagram, has an area of 35 cm^2.

(a) Show that $x^2 + 5x - 46 = 0$

(3)

(b) Solve $x^2 + 5x - 46 = 0$ to find the value of x.
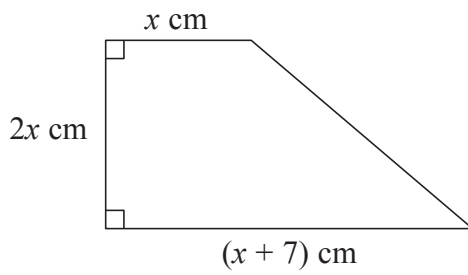   Give your answer correct to 3 significant figures.

$x = $ ....
(3)

3.   **[6 marks]**

x cm

Diagram **NOT** accurately drawn

$2x$ cm

$(x + 7)$ cm

The diagram shows a trapezium.
The trapezium has an area of 17 cm^2

(a) Show that $2x^2 + 7x - 17 = 0$

(3)

(b) Work out the value of x.
    Give your answer correct to 3 significant figures.
    Show your working clearly.

$x = $ ....
(3)

4. [6 marks]

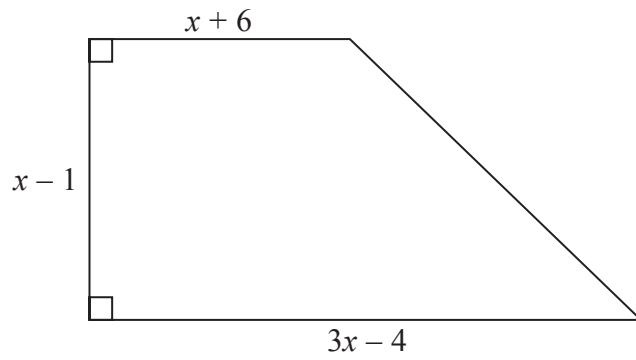The diagram shows a trapezium.

Diagram **NOT**
accurately drawn

All measurements on the diagram are in centimetres.

The area of the trapezium is 119 cm²

(i) Show that $\quad 2x^2 - x - 120 = 0$

(ii) Find the value of x.
Show your working clearly.

$x =$ ....

A rectangular lawn has a length of $3x$ metres and a width of $2x$ metres.
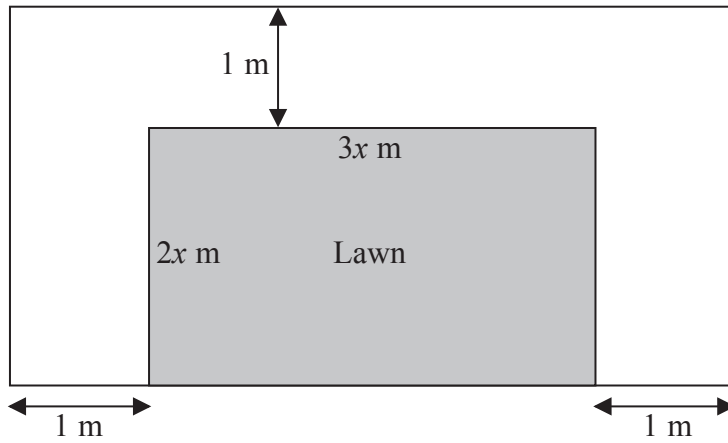The lawn has a path of width 1 metre on three of its sides.

Diagram **NOT**
accurately drawn

The total area of the lawn and the path is 100 m²

(a) Show that $\quad 6x^2 + 7x - 98 = 0$

(2)

(b) Calculate the area of the lawn.
Show clear algebraic working.

.... m²

(5)

6.   **[8 marks]**

The diagram shows a rectangular playground of width x metres and length $3x$ metres.
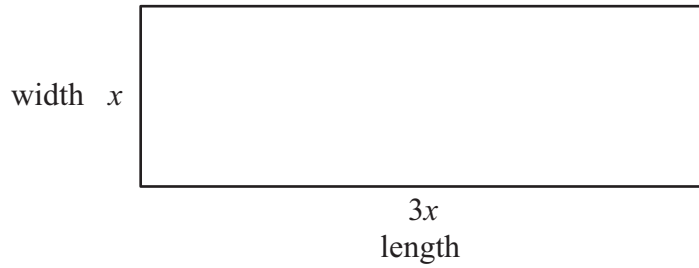
width x

Diagram **NOT**
accurately drawn

$3x$
length

The playground is extended, by adding 10 metres to its width and 20 metres to its length, to form a larger rectangular playground.

The area of the larger rectangular playground is double the area of the original playground.

(a) Show that $3x^2 - 50x - 200 = 0$

(3)

(b) Calculate the area of the original playground.

.... m^2

(5)

The diagram shows a circular pond, of radius r metres, surrounded by a circular path.
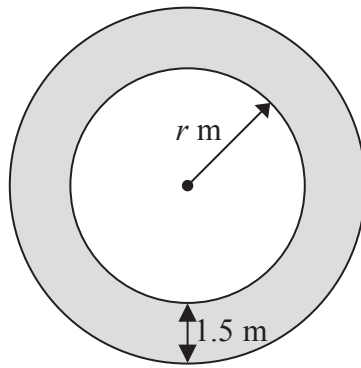The circular path has a constant width of 1.5 metres.

Diagram **NOT**
accurately drawn

r m

1.5 m

The area of the path is $\dfrac{1}{10}$ the area of the pond.

(a) Show that $\quad 2r^2 - 60r - 45 = 0$

(3)

(b) Calculate the area of the pond.
  Show your working clearly.
  Give your answer correct to 3 significant figures.

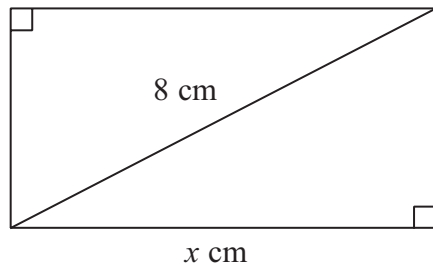.. m^2

(5)

8 cm

x cm

Diagram **NOT** accurately drawn

The diagram shows a rectangle.
The length of the rectangle is x cm.
The length of a diagonal of the rectangle is 8 cm.
The perimeter of the rectangle is 20 cm.

(a) Show that $x^2 - 10x + 18 = 0$

(4)

(b) Solve $x^2 - 10x + 18 = 0$
Give your solutions correct to 3 significant figures.
Show your working clearly.

....

(3)

9.

A coin is biased so that the probability that it shows heads on any one throw is p.
The coin is thrown twice.

The probability that the coin shows heads exactly once is $\dfrac{8}{25}$

Show that $25p^2 - 25p + 4 = 0$

A bag contains x counters.
7 of the counters are blue.
Sam takes at random a counter from the bag and does not replace it.
Jill then takes a counter from the bag.
The probability they both take a blue counter is 0.2

(a) Form an equation involving x.
Show that your equation can be expressed as $x^2 - x - 210 = 0$

(2)

(b) Solve $x^2 - x - 210 = 0$
Show clear algebraic working.

....
(3)

11. **[6 marks]**

Clare buys some shares for $50x$.
Later, she sells the shares for $\$(600 + 5x)$.
She makes a profit of $x\%$

(a) Show that $x^2 + 90x - 1200 = 0$

(3)

(b) Solve $x^2 + 90x - 1200 = 0$
Find the value of x correct to 3 significant figures.

$x =$ ....

(3)

(a) Show that

$$(a^2 + 1)(c^2 + 1) = (ac - 1)^2 + (a + c)^2$$

(3)

(b) By finding suitable values of a and c, use part (a) to write 650065 as the sum of two square numbers.

$$650065 = \text{.............................} + \text{.............................}$$

(3)

B $(x + 5)$ cm C Diagram **NOT** accurately drawn

x cm

A $(x + 8)$ cm D
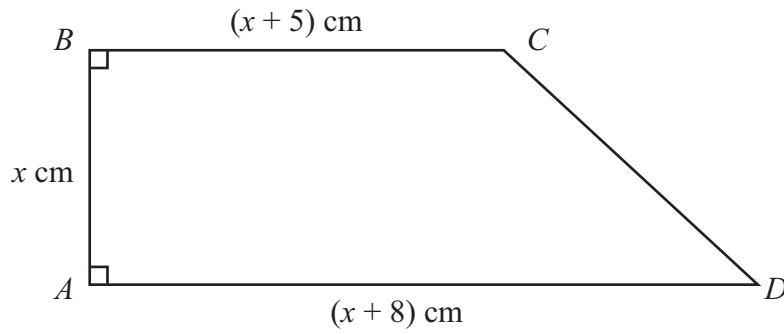
The diagram shows a trapezium $ABCD$ with AD parallel to BC.
$AB = x$ cm, $BC = (x + 5)$ cm and $AD = (x + 8)$ cm.
The area of the trapezium is 42 cm².

(a) Show that $2x^2 + 13x - 84 = 0$

(2)

(b) Calculate the perimeter of the trapezium.

.... cm

(5)

There are 10 beads in a box.

n of the beads are red.

Meg takes one bead at random from the box and does not replace it.

She takes a second bead at random from the box.

The probability that she takes 2 red beads is $\frac{1}{3}$.
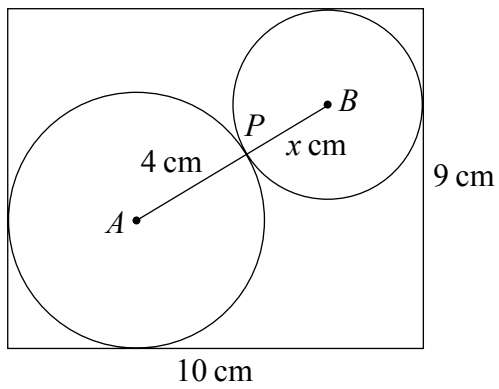
Show that $n^2 - n - 30 = 0$

Diagram **NOT** accurately drawn

The diagram shows one disc with centre A and radius 4 cm and another disc with centre B and radius x cm.
The two discs fit exactly into a rectangular box 10 cm long and 9 cm wide.
The two discs touch at P.
APB is a straight line.

(a) Use Pythagoras' Theorem to show that $x^2 - 30x + 45 = 0$

(4)

(b) Find the value of x.
      Give your value correct to 3 significant figures.

$x = $ ....
(3)