

ANGLES IN POLYGONS

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 70

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2014]

Angles in Polygons (Interior and Exterior) [4 Marks]

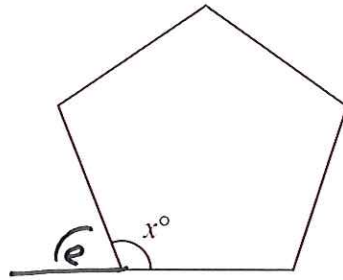


Diagram NOT
accurately drawn

The diagram shows a regular 5-sided polygon.

(a) Work out the value of x .

$$\begin{aligned} \text{EXTERIOR ANGLE} &= \frac{360}{5} \\ &= 72 \end{aligned} \quad \begin{array}{l} \text{INTERIOR ANGLE, } x \\ = 180 - 72 \\ = \underline{\underline{108}} \end{array}$$

(mi)

[LOTS OF OTHER
METHODS, TOO]

$$x = \frac{108}{(2)} \quad \text{(AL)}$$

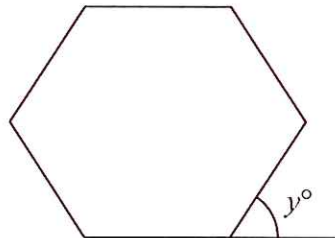


Diagram NOT
accurately drawn

The diagram shows a regular 6-sided polygon.

(b) Work out the value of y .

$$\begin{aligned} y = \text{EXTERIOR ANGLE} &= \frac{360}{6} \quad \text{(mi)} \\ &= \underline{\underline{60}} \quad \text{(AN)} \end{aligned}$$

The diagram shows a regular 5-sided polygon, with centre O .

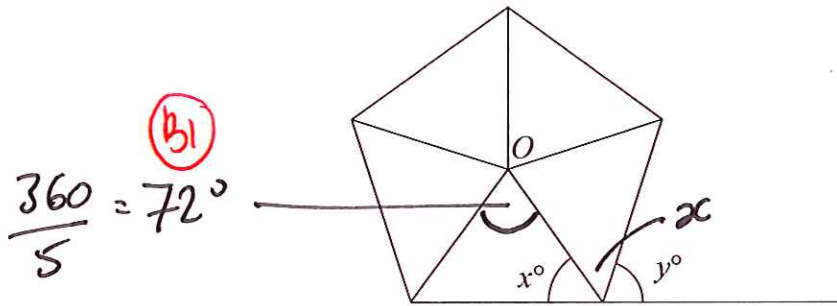


Diagram **NOT** accurately drawn

Work out the value of

(a) x ,

$$180 - 72 = 108$$

$$x = \frac{108}{2} \text{ (ml)}$$

$$x = \frac{54}{(3)} \text{ (Al)}$$

(b) y .

$$= 180 - 2x$$

$$= 180 - 2 \times 54 \text{ (ml)}$$

$$= \underline{\underline{72}}$$

$$y = \frac{72}{(2)} \text{ (Al)}$$

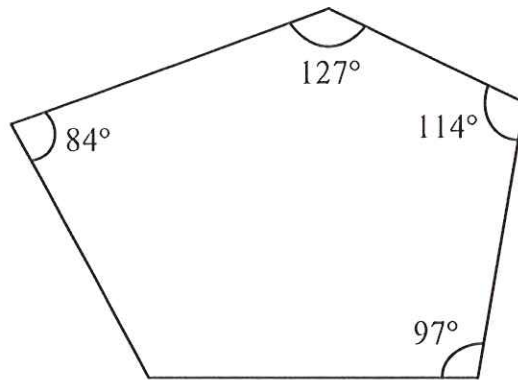


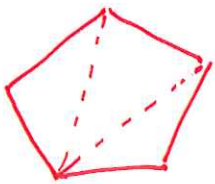
Diagram NOT
accurately drawn

Four of the angles of a pentagon are 97° , 114° , 127° and 84° .

Work out the size of the fifth angle.

$$84 + 127 + 114 + 97 = 422 \quad \text{(M1)}$$

$$\text{FOR } 540^\circ \quad \text{(B1)} \rightarrow 540 - 422 = \underline{\underline{118}} \quad \text{(M1)}$$



ANGLES IN A PENTAGON

$$3 \times 180 = \underline{\underline{540^\circ}}$$

$$\underline{\underline{118}} \quad \text{(A1)}^\circ$$

Work out the size of each exterior angle of a regular polygon with 15 sides.

$$\textcircled{M1} \left| \frac{360}{15} = \underline{\underline{24^\circ}} \textcircled{A1} \quad \left[\text{SUM OF EXTERIOR ANGLES} = 360^\circ \right]$$

Each exterior angle of a regular polygon is 15°

(a) How many sides has the regular polygon?

$$\textcircled{M1} \quad \left| \frac{360}{15} = 24 \right.$$

EXTERIOR ANGLES
ADD TO 360

$$\frac{24}{(2)} \quad \textcircled{A1}$$

The diagram shows 3 identical regular pentagons.

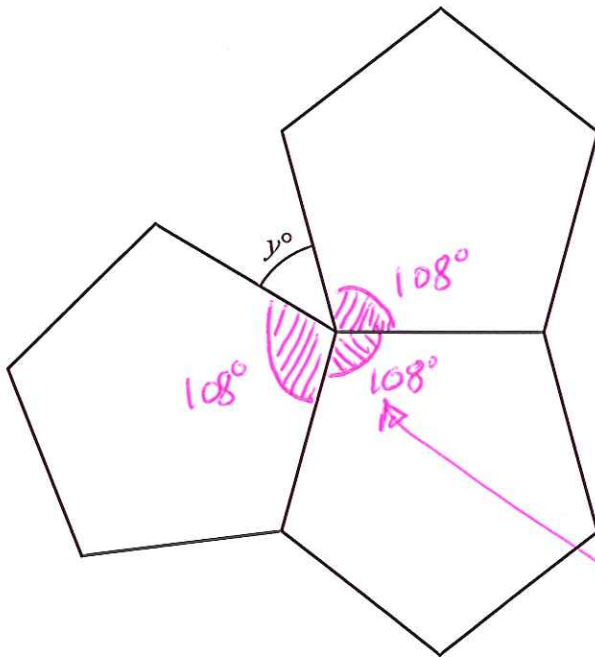


Diagram NOT
accurately drawn

1ST
EXTERIOR ANGLE
OF A PENTAGON
 $= \frac{360}{5} = \underline{\underline{72^\circ}}$

(b) Work out the value of y .

2ND INTERIOR ANGLE
OF A PENTAGON $= 180 - 72$
 $= \underline{\underline{108^\circ}}$ **B1**

3RD $y = \frac{360 - 3 \times 108}{(M1)}$

$$y = \frac{36}{(3)} \quad \textcircled{A1}$$

- (a) The diagram shows a **regular octagon**, with centre O .

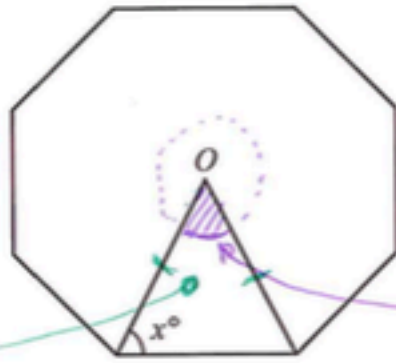


Diagram NOT accurately drawn

$$\text{IST } \frac{360}{8} = \underline{45^\circ} \text{ (B)}$$

ISOSCELES TRIANGLE

Work out the value of x .

$$x = \frac{180 - 45}{2} \text{ (M)}$$

$$x = \frac{67.5}{(3)} \text{ (A)}$$

- (b) A regular polygon has an exterior angle of 30° .
Work out the number of sides of the polygon.

$$n = \frac{360}{\text{EXTERIOR}}$$

$$\begin{aligned} n &= \frac{360}{30} \text{ (M)} \\ &= \underline{\underline{12}} \text{ (A)} \end{aligned}$$

(a) Find the sum of the interior angles of a polygon with 7 sides.

$$\text{EXTERIOR} = \frac{360}{7} = 51.43$$

$$\text{INTERIOR} = 180 - 51.43 = 128.57 \quad (\text{BI})$$

$$\begin{aligned} \text{SUM OF INTERIOR} &= 128.57 \times 7 \\ &= \underline{\underline{900}} \quad (\text{AI}) \end{aligned}$$

ALTERNATE

$$3 - 180$$

$$4 - 360$$

$$5 - 540$$

$$6 - 720$$

$$7 - 900$$

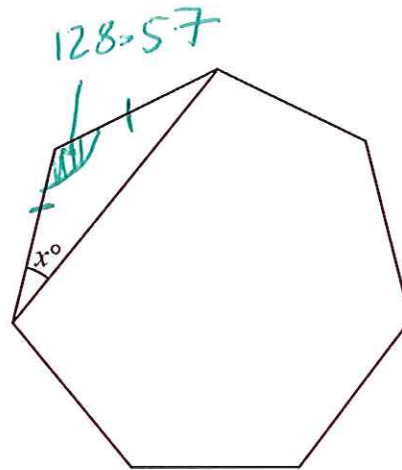


Diagram NOT
accurately drawn

The diagram shows a regular polygon with 7 sides.

(b) Work out the value of x .

Give your answer correct to 1 decimal place.

$$x = \frac{180 - 128.57}{2} \quad (\text{mi})$$

$$= 25.7142 \dots$$

$$= \underline{\underline{25.7}} \quad (\text{AI})$$

The size of each exterior angle of a regular polygon is 18° .

(a) Work out how many sides the polygon has.

$$\frac{360}{18} \text{ (m)}$$

SUM OF EXTERIOR = 360°

$$\frac{20}{(2)} \text{ (A)}$$

(b) Work out the sum of the interior angles of the polygon.

SUM = $(n - 2) \times 180$
INTERIOR

$$= (20 - 2) \times 180 \text{ (m)}$$

$$= 18 \times 180$$

$$= \underline{\underline{3240}}$$

$$\frac{3240}{(2)} \text{ (A)}$$

[OTHER METHODS
ARE ACCEPTABLE]

The diagram shows an incomplete regular polygon.

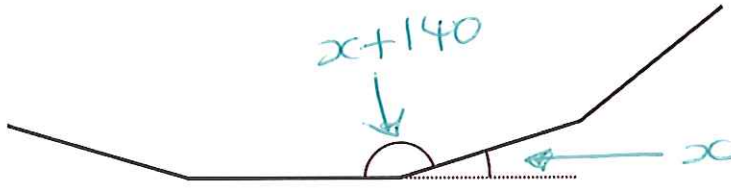


Diagram NOT accurately drawn

The size of each interior angle is 140 degrees greater than the size of each exterior angle.

Work out the number of sides the regular polygon has.

$$\begin{aligned}
 & \text{(BI)} \\
 & (x + 140) + x = 180 \quad \text{(MI)} \\
 & \Rightarrow 2x + 140 = 180 \\
 & \quad 2x = 40 \\
 & \quad x = \underline{\underline{20^\circ}} \quad \text{(AI)} \\
 & \text{EXTERIOR,}
 \end{aligned}
 \quad \rightarrow \quad
 \begin{aligned}
 \text{SIDES} &= \frac{360}{20} \\
 &= \underline{\underline{18}} \quad \text{(AI)}
 \end{aligned}$$

Here is a regular 10-sided polygon.

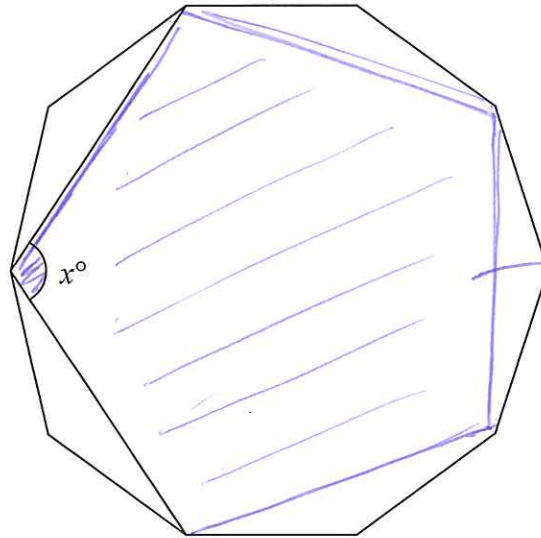


Diagram NOT
accurately drawn

REGULAR
PENTAGON

Work out the value of x .
Show your working clearly.

x IS THE INTERIOR ANGLE OF A
REGULAR PENTAGON. (M1)

$$\begin{aligned} \text{SUM} &= (n-2) \times 180 && \text{(M1)} \\ &= 3 \times 180 \\ &= 540 \end{aligned}$$

$$\begin{aligned} \therefore \text{EACH ANGLE} &= \frac{540}{5} && \text{(M1)} \\ &= \underline{\underline{108^\circ}} && \text{(A1)} \end{aligned}$$

ALTERNATIVE
METHODS ARE
FINE

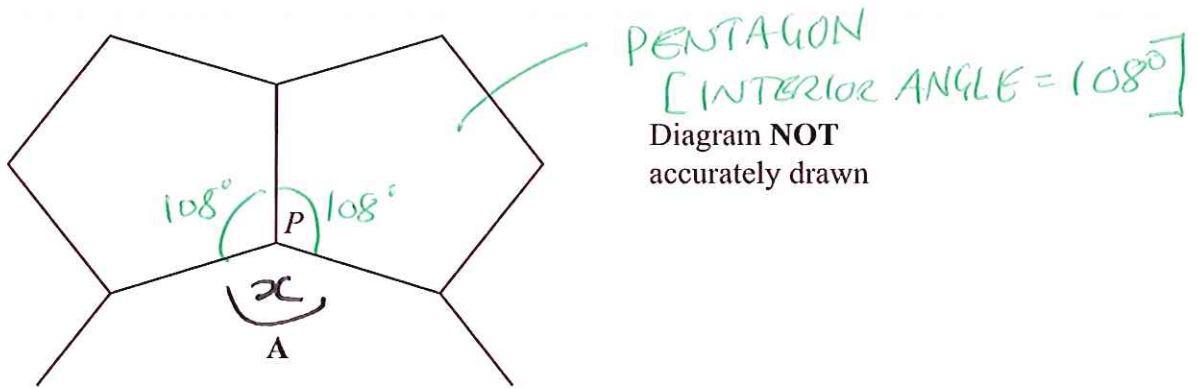
$x = \dots\dots\dots$

Work out the size of an exterior angle of a regular polygon with 8 sides.

$$\text{EXTERIOR} = \frac{360}{\text{SIDES}}$$

$$\frac{360}{8} \text{ (ml)}$$

$$\underline{\quad\quad\quad} 45 \text{ (AI)}^\circ$$



The diagram shows two congruent regular pentagons and part of a regular n -sided polygon A.

Two sides of each of the regular pentagons and two sides of A meet at the point P.

Calculate the value of n .

Show your working clearly.

$$\begin{aligned} \text{ANGLE } \alpha &= 360 - 2 \times 108 \quad \leftarrow \text{(B1)} \\ &= 144^\circ \quad \text{(M1)} \end{aligned}$$

EXTERIOR

$$\begin{aligned} \text{ANGLE OF } A &= 180 - 144 \\ &= 36^\circ \quad \text{(M1)} \end{aligned}$$

NUMBER OF SIDES

$$\begin{aligned} &= \frac{360}{36} \quad \text{(M1)} \\ &= \underline{\underline{10}} \end{aligned}$$

$$n = \underline{\underline{10}} \quad \text{(A1)}$$

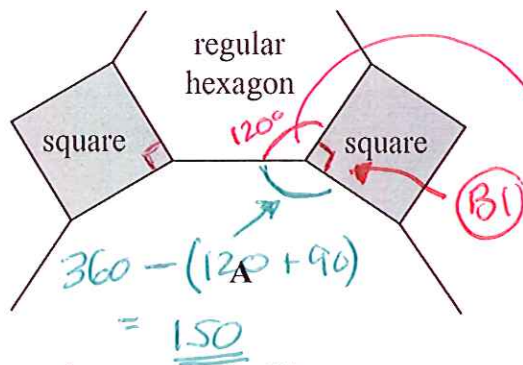


Diagram NOT accurately drawn

HEXAGON EXTERNAL

$$\frac{360}{6} = \underline{\underline{60}}$$

\therefore HEXAGON INTERNAL

$$180 - 60 = \underline{\underline{120}} \quad \text{(BI)}$$

The diagram shows part of a tiling pattern.
The tiling pattern is made from three shapes.
Two of the shapes are squares and regular hexagons.
The third shape is a regular n -sided polygon A.

Work out the value of n .

$$\text{INTERNAL ANGLE} = 150 \quad \text{(BI)}$$

$$\therefore \text{EXTERNAL ANGLE} = 180 - 150 = 30$$

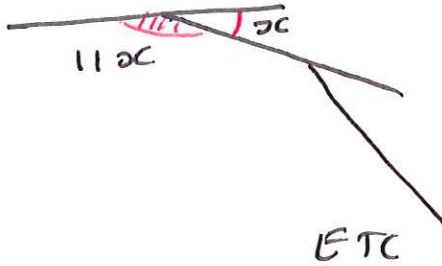
$$\therefore \text{NUMBER OF SIDES} = \frac{360}{30} \quad \text{(mi)}$$

$$= \underline{\underline{12}}$$

$$n = \dots\dots\dots 12 \quad \text{(AI)}$$

The size of each interior angle of a regular polygon is 11 times the size of each exterior angle.

Work out the number of sides the polygon has.



$$12x = 180 \quad (B1)$$

$$x = 15 \quad (B1)$$

$$\text{SIDES} = \frac{360}{15} \quad \left. \vphantom{\frac{360}{15}} \right\} (M1)$$

$$= \underline{\underline{24}} \quad (A1)$$

The size of each interior angle of a regular polygon with n sides is 140°

Work out the size of each interior angle of a regular polygon with $2n$ sides.

$$\text{INTERIOR ANGLE} = 140$$

$$\begin{aligned} \text{EXTERIOR} &= 180 - 140 \\ &= \underline{\underline{40}} \quad (\text{m1}) \end{aligned}$$

$$\begin{aligned} \text{NUMBER OF SIDES} &= \frac{360}{40} \quad (\text{m1}) \\ &= \underline{\underline{9}} \quad (\text{m1}) \end{aligned}$$

$$2n = 18 \text{ SIDES}$$

$$\begin{aligned} \text{EXTERIOR ANGLE} &= \frac{360}{18} \quad (\text{m1}) \\ &= 20 \end{aligned}$$

$$\begin{aligned} \therefore \text{INTERIOR ANGLE} &= 180 - 20 \\ &= \underline{\underline{160^\circ}} \quad (\text{ft}) \end{aligned}$$

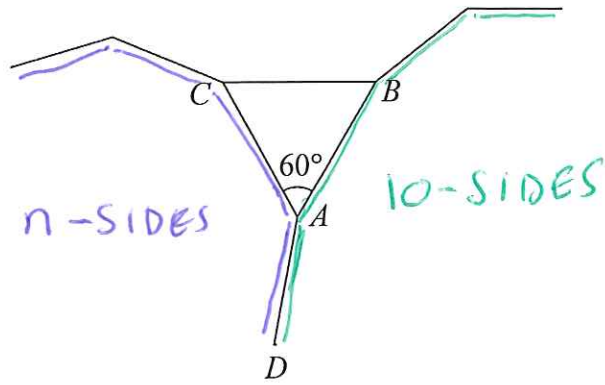


Diagram NOT
accurately drawn

The sides of an equilateral triangle ABC and two **regular** polygons meet at the point A .
 AB and AD are adjacent sides of a regular 10-sided polygon.
 AC and AD are adjacent sides of a regular n -sided polygon.

Work out the value of n .

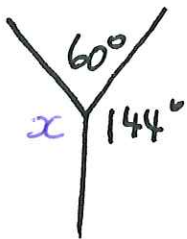
10-SIDED SHAPE

$$\begin{aligned} \text{EXTERIOR ANGLE} &= \frac{360}{10} \\ &= \underline{36^\circ} \end{aligned}$$

$$\begin{aligned} \text{INTERIOR ANGLE} &= 180 - 36 \\ &= \underline{144^\circ} \end{aligned}$$

(m)

(B)



$$\begin{aligned} x &= 360 - (60 + 144) \\ &= \underline{156^\circ} \end{aligned}$$

(m)

n-SIDED SHAPE

$$\begin{aligned} \text{EXTERIOR ANGLE} &= 180 - 156 \\ &= \underline{24^\circ} \end{aligned}$$

(m)

$$\begin{aligned} \therefore n &= \frac{360}{24} \\ &= \underline{15} \end{aligned}$$

(A)



Diagram NOT
accurately drawn

The diagram shows part of a regular polygon.

The interior angle and the exterior angle at a vertex are marked.

The size of the interior angle is 7 times the size of the exterior angle.

Work out the number of sides of the polygon.

$$7x + x = 180 \quad (\text{m})$$

$$\Rightarrow 8x = 180$$

$$x = \frac{180}{8}$$

$$= \underline{\underline{22.5^\circ}} \quad (\text{A1})$$

$$\frac{360}{22.5} = \underline{\underline{16 \text{ sides}}} \quad (\text{A1})$$

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There is no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

Please report any errors or omissions that You find*. These solutions will be updated to correct errors that are discovered. It is recommended that You always check that You have the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or ‘best’ method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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