

AREA OF TRIANGLE

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 40

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2015]

Area of Triangle (From Sine of the Angle) [3 Marks]

$ABCD$ is a kite.

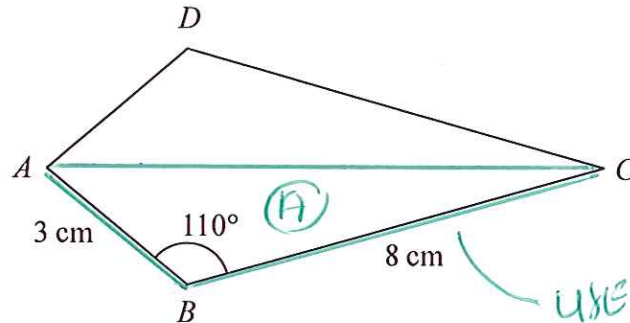


Diagram NOT
accurately drawn

$AB = 3 \text{ cm}$
 $BC = 8 \text{ cm}$
Angle $ABC = 110^\circ$

Calculate the area of the kite $ABCD$.
Give your answer correct to 3 significant figures.

$$\begin{aligned} \text{AREA OF } \textcircled{A} &= \frac{1}{2} \times 3 \times 8 \times \sin 110 \quad \textcircled{m1} \\ &= 11.276 \dots \quad \textcircled{A1} \end{aligned}$$

$$\begin{aligned} \therefore \text{TOTAL AREA} &= 11.276 \times 2 \\ &= 22.552 \dots \\ &= \underline{\underline{22.6 \text{ cm}^2}} \quad \textcircled{A1} \end{aligned}$$

USE $A = \frac{1}{2} ab \sin C$

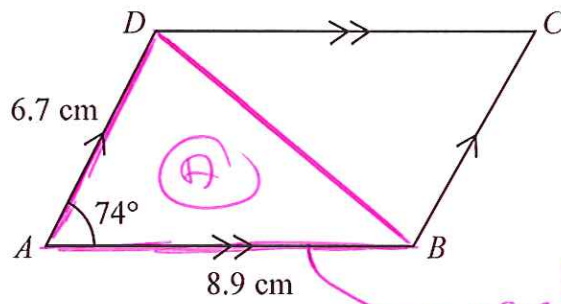


Diagram **NOT**
accurately drawn

$ABCD$ is a parallelogram.
 $AB = 8.9$ cm.
 $AD = 6.7$ cm.
 Angle $BAD = 74^\circ$

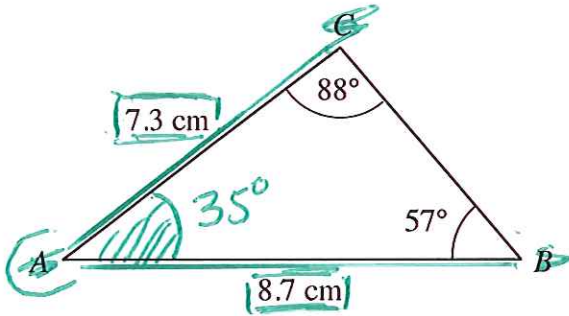
Calculate the area of parallelogram $ABCD$.
 Give your answer correct to 3 significant figures.

USE $A = \frac{1}{2} ab \sin C$

$$\begin{aligned} \textcircled{A} &= \frac{1}{2} \times 8.9 \times 6.7 \times \sin 74 \quad \textcircled{m1} \\ &= \underline{\underline{28.66}} \quad \textcircled{A1} \end{aligned}$$

$$\begin{aligned} \therefore \text{WHOLE PARALLELOGRAM} &= 28.66 \times 2 \\ &= 57.32 \\ &= \underline{\underline{57.3}} \text{ cm}^2 \quad \textcircled{A1} \end{aligned}$$

[3 marks]

Diagram NOT
accurately drawn

USE

$$A = \frac{1}{2} ab \sin C$$

Calculate the area of triangle ABC .
Give your answer correct to 3 significant figures.

$$\begin{aligned} A &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times 7.3 \times 8.7 \times \sin 35 \\ &= 18.2139 \dots \end{aligned}$$

(B1) [FOR '35' ANYWHERE
IN WORKING]

(M1) [CORRECT USE OF
FORMULA]

$$\begin{aligned} &18.2 \text{ cm}^2 \\ &\dots\dots\dots \text{cm}^2 \\ &\quad \quad \quad \text{(A1)} \\ &\quad \quad \quad \text{(ft)} \quad \text{(3)} \end{aligned}$$

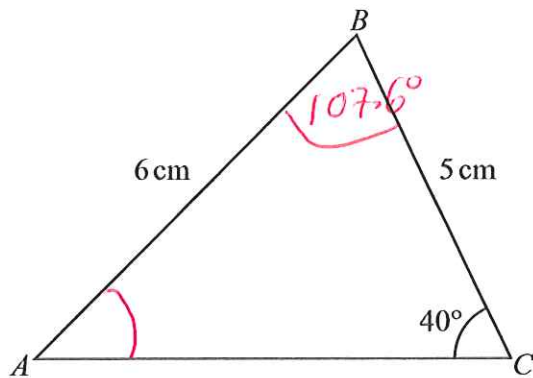


Diagram NOT
accurately drawn

LOTS OF DIFFERENT
METHODS.
YOU NEED TO USE SINE
OR COSINE RULE FIRST

Calculate the area of triangle ABC.
Give your answer correct to 3 significant figures.

$$\frac{\sin A}{5} = \frac{\sin 40}{6} \Rightarrow \sin A = \frac{5 \times \sin 40}{6} \quad (M1)$$

$$A = \sin^{-1} \left(\frac{5 \times \sin 40}{6} \right) \quad (M1)$$

$$= \underline{32.388^\circ} \quad (A1)$$

$$\therefore B = 180 - (40 + 32.388)$$

$$= \underline{107.612} \quad (B1)$$

$$\therefore \text{AREA} = \frac{1}{2} \times 5 \times 6 \times \sin 107.612 \quad (M1)$$

$$= 14.2969 \dots$$

$$\underline{14.3} \text{ cm}^2 \quad (A1)$$

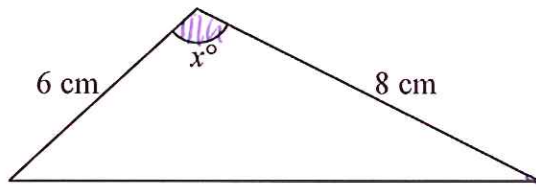


Diagram NOT
accurately drawn

The area of the triangle is 12 cm^2 .
The angle x° is obtuse. $\overline{\overline{\longrightarrow}}$
Calculate the value of x .

USE $A = \frac{1}{2}ab\sin C$

$$\frac{1}{2} \times 6 \times 8 \times \sin x = 12 \quad \text{(M1) [EQUATION]}$$

$$\Rightarrow 24 \times \sin x = 12$$

$$\sin x = 0.5$$

$$x = \sin^{-1}(0.5) \quad \text{(M1) [EITHER]}$$

$$= \underline{\underline{30^\circ}} \quad \text{(A1)}$$

BUT x IS OBTUSE SO

$$x = 180 - 30$$

$$= \underline{\underline{150^\circ}} \quad \text{(A1)}$$

ABC is a triangle.

$AB = 12$ cm

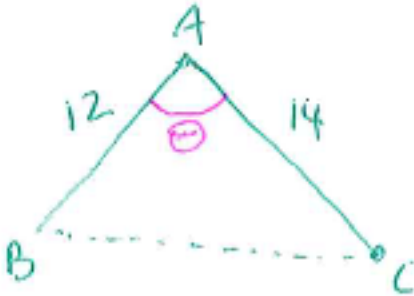
$AC = 14$ cm

The area of triangle ABC is 72 cm²

USE $A = \frac{1}{2}ab \sin C$

Find, in degrees, the two possible sizes of angle BAC .

Give your answers correct to the nearest degree.



$$\frac{1}{2} \times 12 \times 14 \sin \theta = 72 \quad (M1)$$

$$\Rightarrow \sin \theta = \frac{72 \times 2}{12 \times 14}$$

$$\Rightarrow \sin \theta = \frac{6}{7} \quad (M1)$$

$$\begin{aligned} \therefore \theta &= \sin^{-1}\left(\frac{6}{7}\right) \\ &= 58.997\dots \\ &= \underline{\underline{59^\circ}} \quad (A1) \end{aligned}$$

BUT θ COULD ALSO BE OBTUSE $\Rightarrow 180 - 59 = \underline{\underline{121^\circ}} \quad (A1)$

Here is triangle LMN , where angle LMN is an obtuse angle.

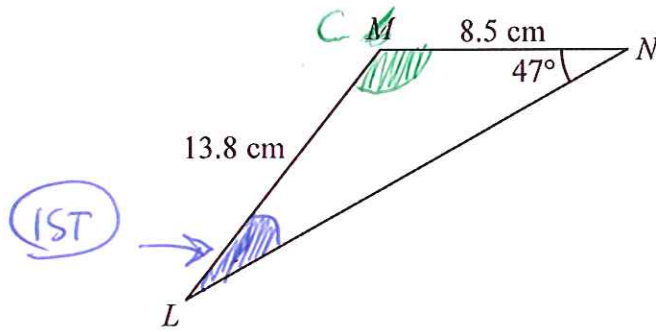


Diagram NOT accurately drawn

$$ML = 13.8 \text{ cm}$$

$$MN = 8.5 \text{ cm}$$

$$\text{Angle } MNL = 47^\circ$$

Work out the area of triangle LMN .

Give your answer correct to 3 significant figures.

$$A = \frac{1}{2} ab \sin C$$

1ST FIND L:

$$\frac{\sin L}{8.5} = \frac{\sin 47}{13.8}$$

$$\Rightarrow \sin L = \frac{8.5 \times \sin 47}{13.8}$$

$$= 0.45047\dots$$

$$L = \underline{\underline{26.77^\circ}}$$

$$\therefore M = 180 - (26.77 + 47)$$

WE LABELLED IT AS 'C'

$$= \underline{\underline{106.23}}$$

$$A = \frac{1}{2} \times 8.5 \times 13.8 \times \sin 106.23$$

$$= 56.312\dots$$

$$= \underline{\underline{56.3 \text{ cm}^2}}$$

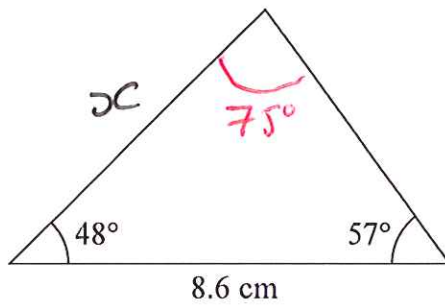


Diagram NOT
accurately drawn

Calculate the area of the triangle.

Give your answer correct to 3 significant figures.

$$\frac{x}{\sin 57} = \frac{8.6}{\sin 75} \quad (M1)$$

$$\Rightarrow x = \frac{8.6 \times \sin 57}{\sin 75}$$

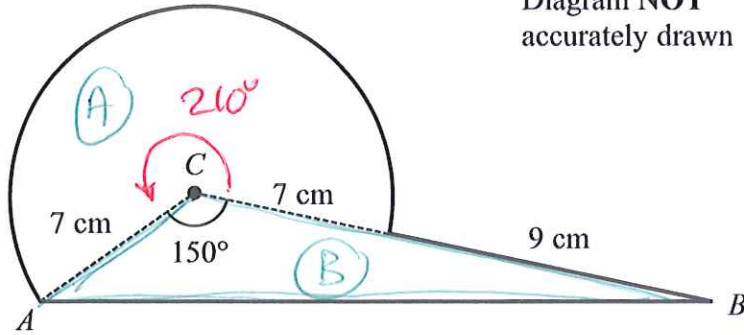
$$= \underline{7.4669\dots} \quad (A1)$$

$$\text{AREA} = \frac{1}{2} \times 7.4669 \times 8.6 \times \sin 48 \quad (M1)$$

$$= \underline{\underline{23.86\dots}}$$

$$\dots\dots\dots 23.9 \text{ cm}^2 \quad (A1)$$

Here is a shape.



The shape is made from triangle ABC and a sector of a circle, centre C and radius CA .

$CA = 7$ cm.

$CB = 16$ cm.

Angle $ACB = 150^\circ$

Calculate the area of the shape.

Give your answer correct to 3 significant figures.

$$\begin{aligned} \text{AREA OF SECTOR} &= \frac{210}{360} \times \pi \times 7^2 \\ &= \underline{89.797\dots} \end{aligned}$$

$$\begin{aligned} \text{AREA OF TRIANGLE} &= \frac{1}{2} \times 7 \times 16 \times \sin 150^\circ \\ &= \underline{28} \end{aligned}$$

$$\begin{aligned} \text{TOTAL} &= 89.797\dots + 28 \\ &= 117.797 \\ &= \underline{\underline{118 \text{ cm}^2}} \end{aligned}$$

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There is no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

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The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or ‘best’ method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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