AVERAGES AND THE RANGE

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 75

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2017] Averages and the Range **[2 Marks]**

Here is a list of numbers

12 19 12 15 11 15 12 13 17

Find the median.

11 12 12 12 13 15 15 17 19 (M1)

[ARRANGE IN ORDER]

9 VALUES

MIDDLE = $\frac{9+1}{2}$

= 5TH VALUE

MEDIAN = 13 (A1)

2. [Edexcel, 2017] Averages and the Range **[5 Marks]**

Here is a list of numbers.

12 15 14 17 22 19 13

Bridgit says,

"To work out the median you find the middle number, so the median of these numbers is 17"

Bridgit's answer is **not** correct.

(a) What is wrong with Bridgit's method?

SHE DOES NOT ARRANGE THEM IN ORDER OF SIZE (A1)

(1)

(b) Work out the range of the numbers in the list.

HIGHEST - LOWEST = 22 - 12 (M1)

10 (A1)

(2)

(c) Work out the mean of the numbers in the list.

(M1) $\frac{12+15+14+17+22+19+13}{7}$

16 (A1)

(2)

3. [Edexcel, 2014] Averages and the Range **[3 Marks]**

Here are Ryan's scores in nine French tests.

4	6	4	7	8	a	6	7	7

The mean of Ryan's nine scores is 6

Work out the value of a.

TOTAL $= 9 \times 6$

$= 54$. (B1)

$4 + 6 + 4 + \ldots + 7 = 49$ (B1)

$a = 54 - 49$

$= 5$

$a =$ 5 (A1)

4. [Edexcel, 2017] Averages and the Range **[2 Marks]**

The manager of a clothes shop recorded the size of each dress sold one morning.

10	10				
12	12				
14	14	14	14	14	14
16	16	16	16		
18	18	18			
20	20	20			

The sizes of dresses are always even numbers.
The mean size of the dresses sold that morning is 15.3

The manager says,

"The mean size of the dresses is **not** a very useful average."

(i) Explain why the manager is right.

MANAGER WANTS TO KNOW SIZES OF DRESSES TO ORDER, BUT THERE ARE NO DRESSES OF SIZE 15.3 (C1)

(ii) Which is the more useful average for the manager to know, the median or the mode?
You must give a reason for your answer.

THE MODE IS THE BEST AVERAGE IN THIS SITUATION BECAUSE IT GIVES THE MOST POPULAR SIZE, WHICH IS EXACTLY WHAT THE MANAGER WANTS TO KNOW. (C1)

[NOTE THAT 'C1' IS A 'COMMUNICATION' MARK AND SO THERE MUST BE SUFFICIENT EXPLANATION OR A CLEARLY STATED REASON]

5. [Edexcel, 2011] Averages and the Range **[3 Marks]**

Six numbers have a mean of 5 — TOTAL = 30 (B1)

Five of the numbers are

3 2 7 6 2

The other number is x.

Work out the value of x.

3 + 2 + 7 + 6 + 2 = 20 (M1)

30 − 20 = 10

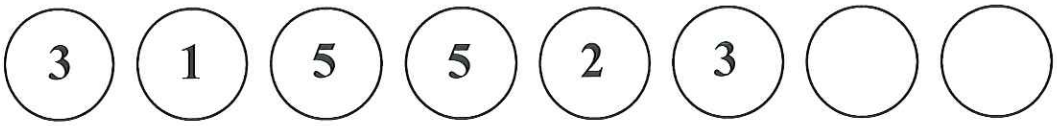
(A1)

$x =$ 10

6. [Edexcel, 2016] Averages and the Range **[3 Marks]**

Here are 8 cards.

There is a number on each of six cards.
Two cards are blank.



Uzma wants the mean of the numbers on the 8 cards to be 4
She wants the range of the numbers on the 8 cards to be 9

Find the numbers that she should write on the two blank cards.

TOTAL NEEDS TO BE $8 \times 4 = 32$ (B1)

CURRENT TOTAL IS $3+1+5+5+2+3 = 19$ (B1)

$\therefore$ LAST TWO MUST ADD TO $32 - 19 = 13$

RANGE MUST BE 9.

CURRENT LOWEST IS 1

$\therefore$ HIGHEST MUST BE 10 $\longrightarrow$ 10 and 3 (A1 [BOTH])

7. [Edexcel, 2016] Averages and the Range **[3 Marks]**

Here is a list of numbers written in order of size.

3 6 x y

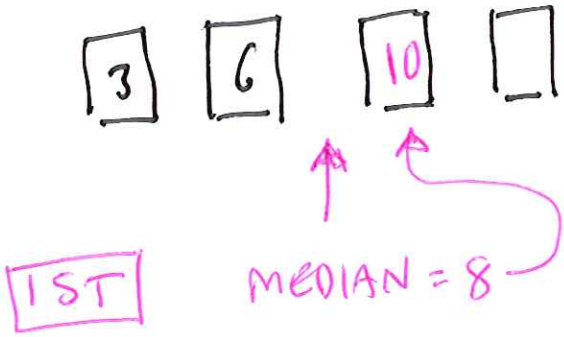
The numbers

have a median of 8

have a mean of 11 → TOTAL = 11 × 4

= 44 (B1)

Find the value of x and the value of y.



1ST MEDIAN = 8

2ND $y = 44 - (3 + 6 + 10)$

$= 25$

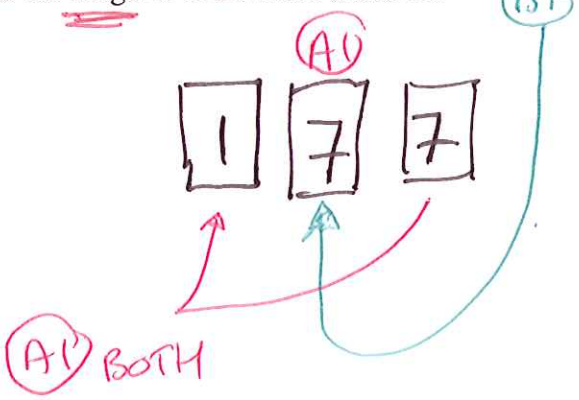
$x =$ 10 (A1)

$y =$ 25 (A1)

8. [Edexcel, 2012] Averages and the Range **[3 Marks]**

Three positive whole numbers have a median of 7 and a mean of 5 — TOTAL = 15

Find the range of these three numbers.



15 − 7 = 8 (SO FIRST AND THIRD ADD TO 8)

RANGE = 7 − 1 = 6 A1

9. [Edexcel, 2006] Averages and the Range **[6 Marks]**

(a) Four numbers have a mean of 6
Three of the numbers are 3, 7 and 10
Find the other number.

TOTAL = 4 × 6
= 24 (B1)

TOTAL = 20

24 − 20 = 4

4 (A1)

(2)

(b) Three numbers have a mode of 5 and a mean of 6
Find the three numbers.

TOTAL = 3 × 6
= 18 (B1)

18 − 10 = 8

MODE (TOTAL = 10)

8 (A1)

(2)

(c) Find four numbers which have a mode of 7 and a median of 6

MODE

MEDIAN = 6

ANY NUMBER LESS THAN 5

(A1) (A1)

4, 5, 7, 7

(2)

OR ANY NUMBER LESS THAN 4.

10. [Edexcel, 2004] Averages and the Range **[4 Marks]**

The mean height of a group of 4 girls is 158 cm.

(a) Work out the total height of the 4 girls.

4×158

632 (A1)

..... cm

(1)

Sarah joins the group and the mean height of the 5 girls is 156 cm.

(b) Work out Sarah's height.

$5 \times 156 = 780$ (A1)

$780 - 632 = 148$ (M1)

148 (A1)

..... cm

(3)

11. [Edexcel, 2015] Averages and the Range **[4 Marks]**

w, x, y and z are 4 integers written in order of size, starting with the smallest.

The mean of w, x, y and z is 13
The sum of w, x and y is 33

(a) Find the value of z.

TOTAL $= 4 \times 13$
$= 52$ (B1)

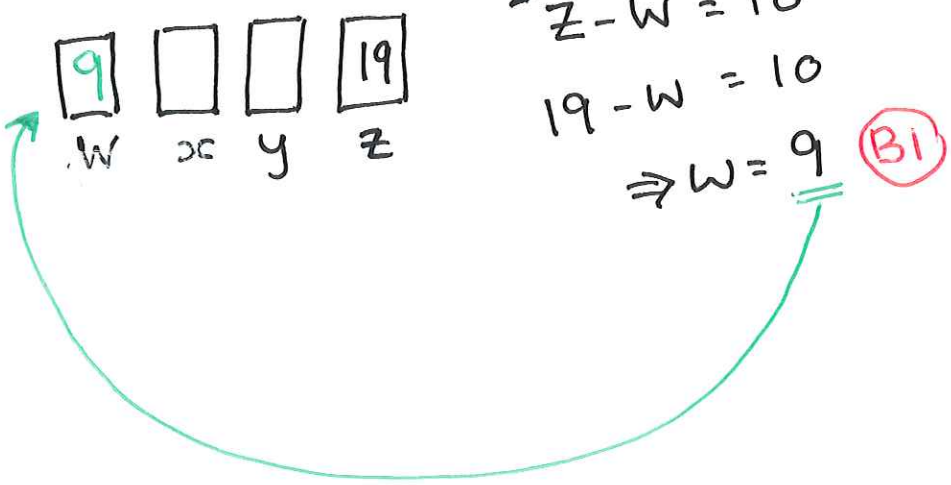
$z = 52 - 33$

$z =$ 19 (A1)

(2)

Given also that the range of w, x, y and z is 10,

(b) work out the median of w, x, y and z.



(1ST)
$z - w = 10$
$19 - w = 10$
$\Rightarrow w = 9$ (B1)

$x + y = 52 - (9 + 19)$
$= 24$

$\therefore \frac{x+y}{2} = 12$ [MEDIAN] (A1)

Three positive whole numbers have a mean of 4 and a range of 7

Find the three positive whole numbers.

TOTAL = 3 × 4 = 12

POSSIBILITIES:

1 AND 8

2 AND 9

3 AND 10

ETC

SUM IS TOO BIG!

IF 1 AND 8, THIRD NUMBER = 3

IF 2 AND 9, THIRD NUMBER = 1

RANGE OF 8! (NOT POSSIBLE

1 3 8

A1

1 3 8

B1 FOR SEEING TOTAL = 12

The mean height of a group of 6 children is 165 cm.
One child, whose height is 155 cm, leaves the group.

Find the mean height of the remaining 5 children.

TOTAL HEIGHT TO START = 6×165

$= 990$ (B1)

TOTAL HEIGHT AFTER CHILD LEAVES $= 990 - 155$

$= 835$ (B1)

MEAN HEIGHT OF REMAINING FIVE $= \frac{835}{5}$

$= 167$ cm (A1)

There are 10 boys and 20 girls in a class.
The class has a test.

The mean mark for all the class is 60
The mean mark for the girls is 54

Work out the mean mark for the boys.

CLASS SIZE = 30

TOTAL MARK FOR WHOLE CLASS = 60×30

$= 1800$

TOTAL FOR THE GIRLS = $54 \times 20 = 1080$

M1 [EITHER]

$\therefore$ BOYS TOTAL = $1800 - 1080$

$= 720$ M1

MEAN OF BOYS = $\frac{720}{10}$

$= 72$ A1

15. [Edexcel, 2012] Averages and the Range **[4 Marks]**

A group of students take a test.
The group consists of 12 boys and 8 girls.
The mean mark for the boys is 18
The mean mark for the girls is 16.5

Calculate the mean mark for the whole group.

TOTAL $= 12 \times 18$
$= 216$

TOTAL $= 8 \times 16.5$
$= 132$

TOTAL FOR BOTH BOYS AND GIRLS

$216 + 132 = 348$

MEAN MARK $= \frac{348}{20}$

$= 17.4$

17.4

Three positive whole numbers are all different.
The numbers have a median of 8 and a mean of 6
Find the three numbers.

TOTAL = 6 × 3
= 18

THESE TWO ADD TO 10

POSSIBLES:

0 8 10 ✗ ZERO IS NOT POSITIVE

1 8 9

2 8 8 ✗ NOT ALL DIFFERENT

(B1) 8 IS MEDIAN

(B1) ALL POSITIVE AND ADD TO 18

1, 8, 9

There are 30 apples in a box.
The mean weight of these 30 apples is 120 grams. → TOTAL WEIGHT = 3600 g

There are 10 apples in a bag.
The mean weight of these 10 apples is 95 grams. → TOTAL WEIGHT = 950 g

Work out the mean weight of the 40 apples.

M1 [EITHER]

$$\text{MEAN} = \frac{3600 + 950}{40}$$ M1

113.75 A1 grams

18. [Edexcel, 2017] Averages and the Range **[3 Marks]**

Walkden Reds is a basketball team.

At the end of 11 games, their mean score was 33 points per game.
At the end of 10 games, their mean score was 2 points higher.

Jordan says,

"Walkden Reds must have scored 13 points in their 11th game."

Is Jordan right?
You must show how you get your answer.

TOTAL AFTER 11 GAMES = $11 \times 33 = \underline{363}$ (M1)

TOTAL AFTER 10 GAMES = $10 \times 35 = \underline{350}$ (M1)

POINTS SCORED IN 11TH GAME = $363 - 350$

$= \underline{\underline{13}}$

[SO JORDAN IS CORRECT] (A1) [WORKING MUST BE SHOWN]

Zara must take 5 tests.
Each test is out of 100
After 4 tests, her mean score is 64%.

TOTAL = 4 × 64 = 256 (B1)

What score must Zara get in her 5th test to increase her mean score in all 5 tests to 70%?

TOTAL NEEDS TO BE 5 × 70 = 350 (B1)

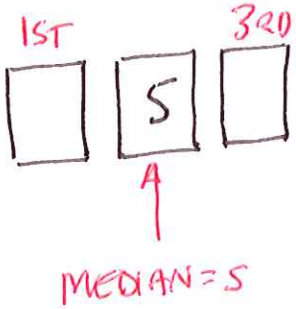
SO SHE NEEDS 350 − 256 = 94 (A1)

(M1)

20. [Edexcel, 2010] Averages and the Range **[4 Marks]**

(a) Three positive whole numbers are all different.
They have a median of 5 and a mean of 4
Find the three numbers.

TOTAL = 12



$1^{ST} + 3^{RD} = 12 - 5$
$= 7$

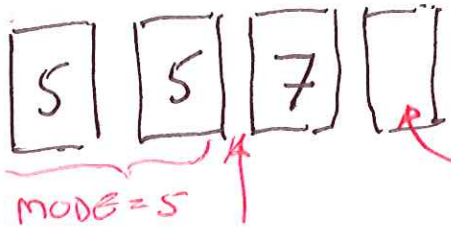
1 + 6
2 + 5
3 + 4 NOT POSSIBLE

1, 5, 6 OR 2, 5, 5

(2)

(b) Find four whole numbers which have a mode of 5 and a median of 6

AT LEAST TWO 5s



CAN BE 9, 10, 11, ETC

5, 5, 7, 8

(2)

21. [Edexcel, 2014] Averages and the Range **[3 Marks]**

The mean of four numbers is 2.6
One of the four numbers is 5

Find the mean of the other three numbers.

→ TOTAL = 4×2.6

$= 10.4$ (B1)

$10.4 - 5 = 5.4$ (M1)

$\frac{5.4}{3} = 1.8$ (A1)

22. [Edexcel, 2010] Averages and the Range **[2 Marks]**

Three numbers a, b and c have a median of 4 and a range of 7

(a) Find the median of the three numbers $a + 2$, $b + 2$ and $c + 2$

NEW MEDIAN = MEDIAN + 2

= 4 + 2

A1

6

(1)

(b) Find the range of the three numbers $a + 2$, $b + 2$ and $c + 2$

R RANGE IS UNCHANGED

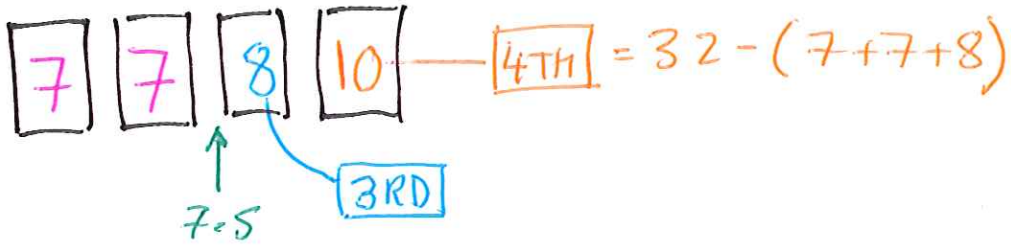
A1

7

(1)

a, b, c and d are four integers.
Their mean is 8 → TOTAL = 4 × 8 = 32
Their mode is 7 (2ND)
Their median is 7.5 (1ST)

(a) Find the value of the largest of the four integers.



10

(2)

(b) Find the mean value of the numbers $(2a - 3)$, $(2b - 3)$, $(2c - 3)$ and $(2d - 3)$.

ALL VALUES ARE DOUBLED
THEN 3 IS SUBTRACTED
[SAME HAPPENS TO THE MEAN]

So

NEW MEAN = 2 × 8 − 3
= 13

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B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

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Note that some questions contain the words 'show that', 'show your working out', or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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