

CIRCLE THEOREMS

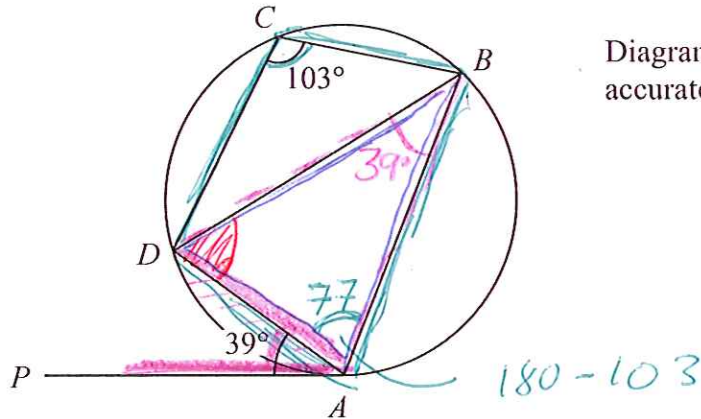
DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 69

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2013]

Angles in Circles (Inc Circle Theorems) [3 Marks]



A, B, C and D are points on a circle.

PA is a tangent to the circle.

Angle $PAD = 39^\circ$

Angle $BCD = 103^\circ$

Calculate the size of angle ADB .

$$\begin{aligned} ADB &= 180 - (39 + 77) \\ &= \underline{\underline{64^\circ}} \end{aligned}$$

A, B, C and D are points on a circle.
 Angle $BAC = 40^\circ$.
 Angle $DBC = 55^\circ$.

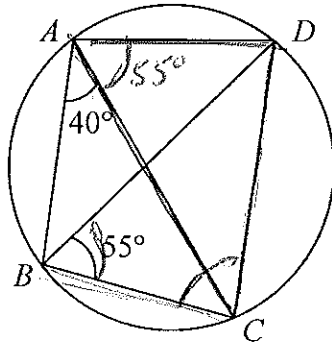


Diagram NOT accurately drawn

(a) (i) Find the size of angle DAC .

55° (A1)

(ii) Give a reason for your answer.

ANGLES IN THE SAME SEGMENT ARE EQUAL (A1)
 (2)

(b) (i) Calculate the size of angle DCB .

$$\begin{array}{r} 40 \\ + 55 \\ \hline 95 \end{array}$$
 (BT) '95' SEEN
 $180 - 95 = \underline{\underline{85}}$
 85° (A1)

(ii) Give reasons for your answer.

ABCD IS A CYCLIC QUADRILATERAL & OPPOSITE ANGLES IN A CYCLIC QUADRILATERAL ADD UP TO 180° (A1)
 (3)

(c) Is BD a diameter of the circle?

NO NO MARK!

Give a reason for your answer.

BECAUSE DCB WOULD BE 90° IF BD WAS A DIAMETER. (A1)

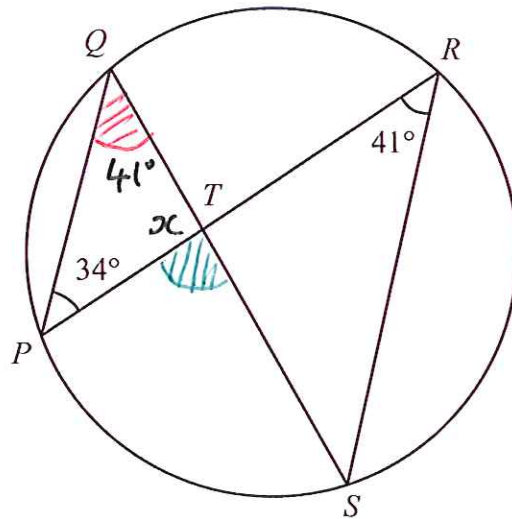


Diagram NOT
accurately drawn

P , Q , R and S are points on the circumference of a circle.
 PR and QS intersect at T .
Angle $QPR = 34^\circ$ and angle $PRS = 41^\circ$

(a) (i) Find the size of angle PQS .



41°

(ii) Give a reason for your answer.

ANGLES IN THE SAME SEGMENT ARE
EQUAL

(2)

(b) (i) Find the size of angle PTS .



$$x = 180 - (41 + 34) \\ = \underline{\underline{105}}$$

$$\hat{PTS} = 180 - 105 \\ = \underline{75}$$

75°

(ii) Explain why T cannot be the centre of the circle.

BECAUSE \hat{PTS} IS NOT TWICE THE ANGLE
AT THE CIRCUMFERENCE (41°)

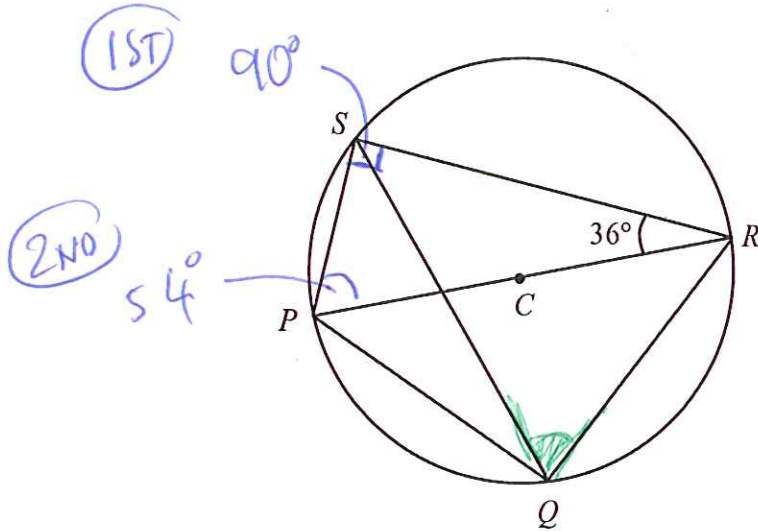


Diagram NOT
accurately drawn

P, Q, R and S are points on a circle, centre C .
 PCR is a straight line.
Angle $PRS = 36^\circ$.

Calculate the size of angle RQS .
Give a reason for each step in your working.

$$\hat{RSP} = 90^\circ \text{ (B1) (ANGLE FROM A DIAMETER)}$$

$$\hat{SPR} = 180 - (90 + 36) \text{ [ANGLES IN } \Delta \text{ ADD TO } 180^\circ \text{]} \text{ (M1)}$$

$$\hat{RQS} = \hat{RPS} \text{ BECAUSE THEY ARE FROM THE SAME CHORD. (A1)}$$

$$\text{So } \hat{RQS} = \underline{\underline{54^\circ}} \text{ (A0)}$$

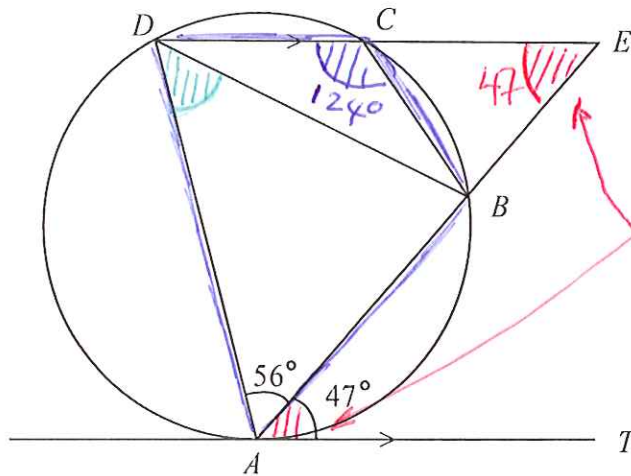



Diagram NOT
accurately drawn

ALTERNATE ANGLES


A, B, C and D are points on a circle.
 ABE and DCE are straight lines.
 AT is a tangent to the circle.
 DCE is parallel to AT .
 Angle $EAT = 47^\circ$. Angle $BAD = 56^\circ$.

(a) (i) Find the size of angle AED . 

$$\underline{\quad 47 \quad}^\circ \text{ (A1)}$$

(ii) Give a reason for your answer.


AED AND TAE ARE ALTERNATE ANGLES (A1)
 (2)

(b) Find the size of angle BCD . 

$$180 - 56$$

$$\underline{\quad 124 \quad}^\circ \text{ (A1)}$$

(1)

(c) (i) Find the size of angle ADB . 

$$\underline{\quad 47 \quad}^\circ \text{ (A1)}$$

(ii) Give a reason for your answer.

ANGLE BETWEEN A TANGENT AND A CHORD
IS EQUAL TO THE ANGLE IN THE ALTERNATE (A1)
SEGMENT
 (2)

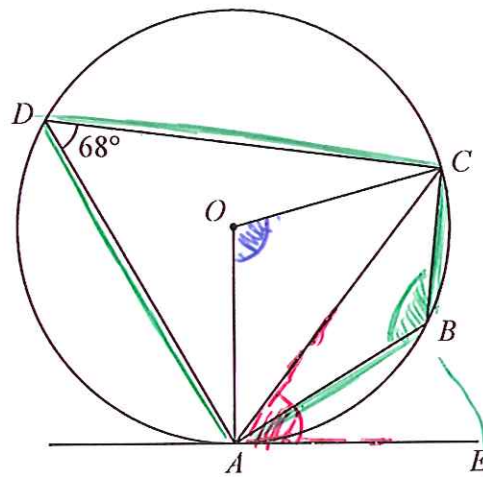


Diagram NOT
accurately drawn

A, B, C and D are points on a circle, centre O .
 AE is a tangent to the circle.
Angle $ADC = 68^\circ$

(a) (i) Find the size of angle ABC .

$$180 - 68$$

$$112^\circ$$

(AI)

(ii) Give a reason for your answer.

OPPOSITE ANGLES IN A CYCLIC QUADRILATERAL
ADD TO 180° (2)

(AI)

(b) (i) Find the size of angle AOC .

$$2 \times 68$$

$$136$$

(AI)

(ii) Give a reason for your answer.

ANGLE AT THE CENTRE IS TWICE ANGLE AT THE
CIRCUMFERENCE (2)

(AI)

(c) Find the size of angle CAE .

(ANGLE BETWEEN A TANGENT AND
CHORD IS EQUAL TO THE ANGLE IN
THE ALTERNATE SEGMENT)

$$68^\circ$$

(AI)

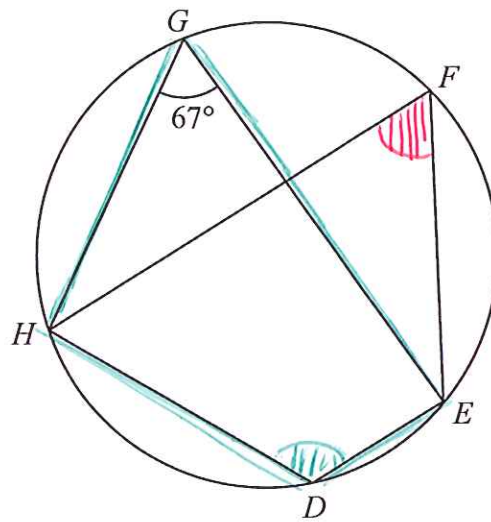


Diagram **NOT**
accurately drawn

CYCLIC
QUADRILATERAL

D, E, F, G and H are points on a circle.
Angle $EGH = 67^\circ$

(a) Find the size of angle EFH .



[ANGLES IN THE SAME SEGMENT ARE EQUAL!]

$$\frac{67}{(1)} \text{ } \circ \text{ (A)}$$

(b) (i) Find the size of angle EDH .



$$180 - 67$$

$$\frac{113}{(1)} \text{ } \circ \text{ (A)}$$

(ii) Give a reason for your answer.

OPPOSITE
ANGLES IN A CYCLIC QUADRILATERAL

ADD UP TO 180° .

(2)

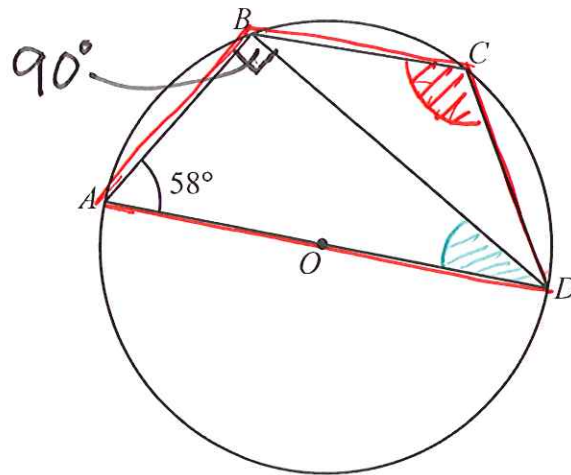


Diagram NOT
accurately drawn

A , B , C and D are four points on a circle, centre O .
 AD is a diameter of the circle.
Angle $BAD = 58^\circ$

(a) Calculate the size of angle ADB .

$$180 - (90 + 58)$$

\uparrow
(1)

32

(2)

(1)

(b) (i) Calculate the size of angle BCD .

$$180 - 58$$

122

(2)

(1)

(ii) Give a reason for your answer.

OPPOSITE ANGLES IN A CYCLIC QUADRILATERAL

ADD TO 180° (1)

(2)

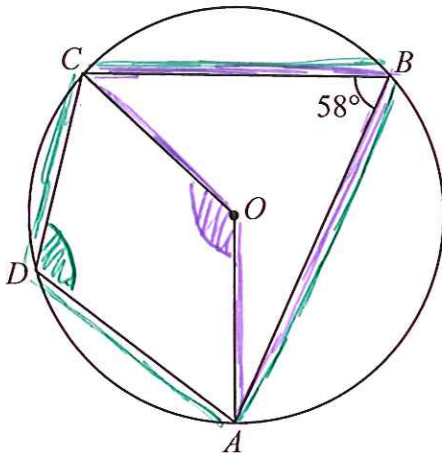


Diagram NOT
accurately drawn

A, B, C and D are points on a circle, centre O .
Angle $ABC = 58^\circ$.

- (a) (i) Calculate the size of angle AOC .

$$2 \times 58$$

$$\underline{\underline{116^\circ}} \quad (A1)$$

- (ii) Give a reason for your answer.

ANGLE AT THE CENTRE IS TWICE THE
ANGLE AT THE CIRCUMFERENCE (A1)
(2)

- (b) (i) Calculate the size of angle ADC .

$$180 - 58$$

$$\underline{\underline{122^\circ}} \quad (A1)$$

- (ii) Give a reason for your answer.

OPPOSITE ANGLES IN A CYCLIC
QUADRILATERAL ADD TO 180° (A1)
(2)

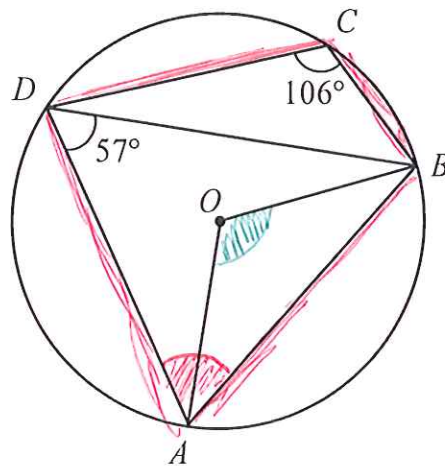


Diagram NOT
accurately drawn

A, B, C and D are points on a circle, centre O .
Angle $ADB = 57^\circ$.
Angle $BCD = 106^\circ$.

(a) (i) Calculate the size of angle AOB . 


$$2 \times 57$$

$$\underline{114}^\circ$$

(ii) Give a reason for your answer.

ANGLE AT THE CENTRE IS TWICE THE
ANGLE AT THE CIRCUMFERENCE

(2)

(b) Calculate the size of angle BAD . 

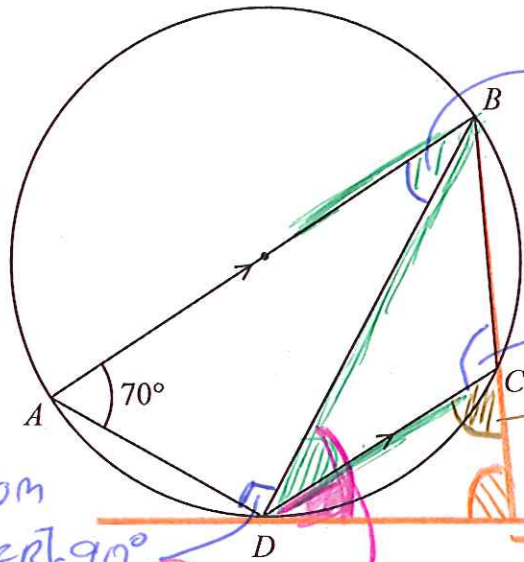
ABCD IS A CYCLIC QUADRILATERAL

$$180 - 106$$

$$\underline{74}^\circ$$

(1)

Diagram NOT accurately drawn



20° [ANGLES IN A TRIANGLE ADD TO 180°]

110° [OPPOSITE ANGLE IN CYCLIC QUAD ADD TO 180°]

[ANGLE FROM DIAMETER] = 90° (B1)

$70^\circ - 20^\circ = 50^\circ$ (B1) [FOR SHOWING 'T']

A, B, C and D are points on a circle.
 AB is a diameter of the circle.
 DC is parallel to AB.
 Angle $BAD = 70^\circ$

(a) Calculate the size of angle BDC .

$BDC = 20^\circ$ BECAUSE IT IS THE ALTERNATE ANGLE TO ABD

(A1)
 20°

 (2)

The tangent to the circle at D meets the line BC extended at T.

(b) Calculate the size of angle BTD .

$$180 - (70 + 50) = 60^\circ \text{ (A1)}$$

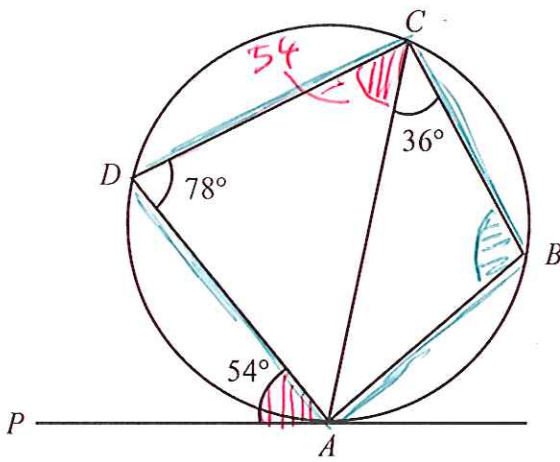


Diagram NOT
accurately drawn

A , B , C and D are points on a circle.
 PA is the tangent to the circle at A .
 Angle $PAD = 54^\circ$, angle $ACB = 36^\circ$ and angle $ADC = 78^\circ$.

(a) (i) Find the size of angle ACD .



54° (A1)

(ii) Give a reason for your answer.

ANGLES IN ALTERNATE SEGMENTS
ARE EQUAL

(A1)

(2)

(b) Explain why BD is a diameter of the circle.

(A1)

BECAUSE $\hat{BCD} = 36 + 54 = 90^\circ$ AND ANGLES
FROM DIAMETERS ARE 90°

(A1)

(2)

(c) (i) Work out the size of angle ABC .

$$180 - 78$$

102° (A1)

(ii) Give a reason for your answer.

OPPOSITE ANGLES IN A CYCLIC
QUADRILATERAL ADD UP TO 180°

(A1)

(2)

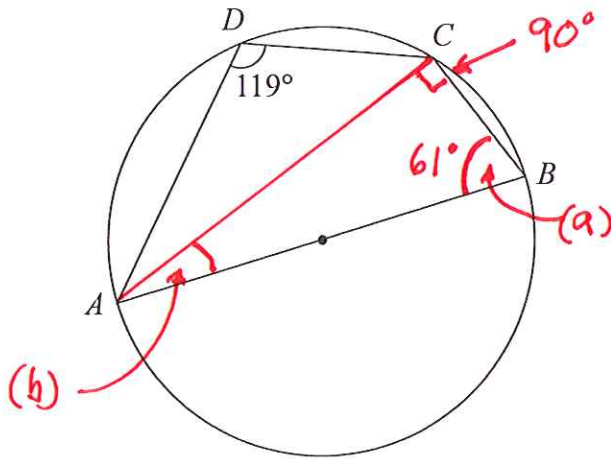


Diagram NOT
accurately drawn

A, B, C and D are points on the circumference of a circle.
 AB is a diameter of the circle.
Angle $ADC = 119^\circ$.

(a) (i) Work out the size of angle ABC .

$$180 - 119$$

$$\underline{\underline{61}} \text{ (AI)}^\circ$$

(ii) Give a reason for your answer.

OPPOSITE ANGLES IN A CYCLIC
QUADRILATERAL ADD UP TO 180° (AI)
(2)

(b) Work out the size of angle BAC .

$$\begin{array}{r} 90 \text{ (MI)} \\ + 61 \\ \hline 151 \end{array} \rightarrow 180 - 151 = \underline{\underline{29}}^\circ$$

$$\underline{\underline{29}} \text{ (AI)}^\circ$$

(2)

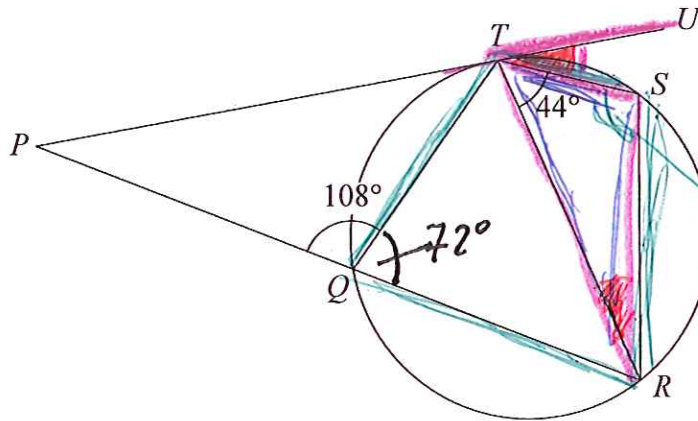


Diagram NOT
accurately drawn

$$180 - 72 = \underline{\underline{108^\circ}}$$

Q, R, S and T are points on the circumference of a circle.

PU is a tangent to the circle at T .

PQR is a straight line.

Angle $PQT = 108^\circ$.

Angle $STR = 44^\circ$.

Work out the size of angle STU .

You must give a reason for each step in your working.

$$\textcircled{1} \text{ ANGLE } TQR = 180 - \cancel{108} \\ = 72^\circ$$

BECAUSE PQR IS A STRAIGHT LINE

$$\textcircled{2} \text{ ANGLE } RST = 180 - 72 \\ = 108$$

BECAUSE $QTSR$ IS A CYCLIC
QUADRILATERAL

$$\textcircled{3} \text{ ANGLE } TRS = 180 - (44 + 108) \\ = 28^\circ$$

BECAUSE ANGLES IN A TRIANGLE ADD
TO 180°

$$\textcircled{4} \text{ ANGLE } STU = TRS = \underline{\underline{28^\circ}}$$

BECAUSE OF THE ALTERNATE SEGMENT
THEOREM!

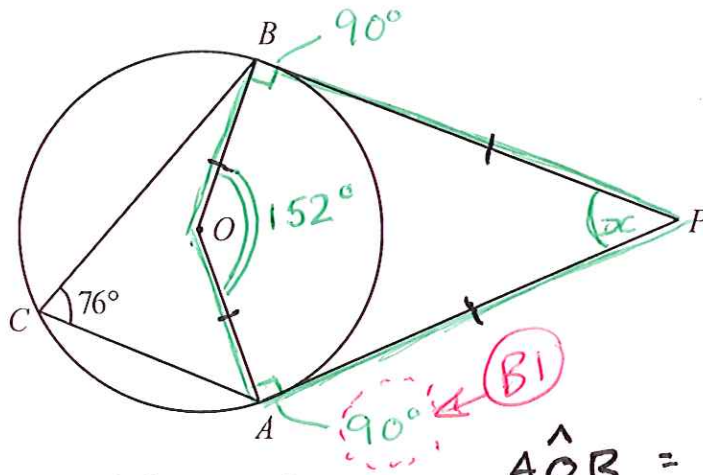


Diagram NOT
accurately drawn

A , B and C are points on a circle, centre O .

Angle $ACB = 76^\circ$

PA and PB are tangents to the circle.

Calculate the size of angle APB .

$$\hat{AOB} = 152$$

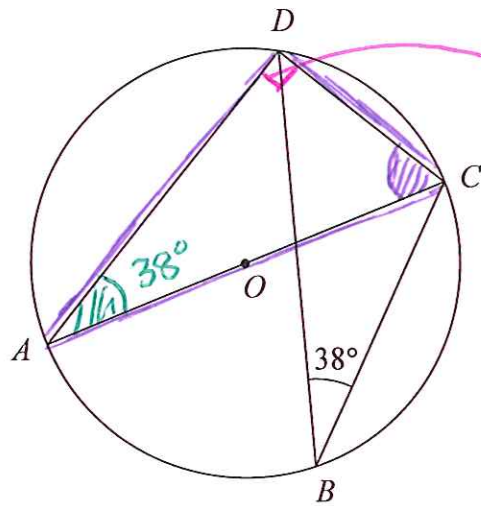
[ANGLE AT CENTRE =
 $2 \times$ ANGLE AT CIRC.]

$$360 - [152 + 90 + 90]$$

(m)

28

(A)



90° [ANGLE IN A SEMI-CIRCLE]

Diagram NOT accurately drawn

A, B, C and D are points on a circle, centre O .
 AC is a diameter of the circle.
 Angle $CBD = 38^\circ$.

(a) (i) Find the size of angle DAC .



(B1)
 38°

(ii) Give a reason for your answer.

..... ANGLES IN THE SAME SEGMENT
 ARE EQUAL (A1)
 (2)

(b) Find the size of angle ACD .



$$180 - (38 + 90)$$

(M1)

[FOR 90 IN WORKING OR ON DIAGRAM]

(A1)
 52°

 (2)

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There is no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

Please report any errors or omissions that You find*. These solutions will be updated to correct errors that are discovered. It is recommended that You always check that You have the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or ‘best’ method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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