# COMPLETING THE SQUARE

DATE OF SOLUTIONS: 11/06/2018
MAXIMUM MARK: 59

# SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

**1.** Completing the square **[2 marks]**

Write $x^2 + 4x + 5$ in the form $(x + a)^2 + b$ where $a$ and $b$ are integers.

$$x^2 + 4x + 5 = (x+2)^2 - 2^2 + 5$$

$$= (x+2)^2 - 4 + 5$$

B1 B1

$(x+2)^2 + 1$

**2.** Completing the square **[2 marks]**

For all values of $x$, $x^2 - 10x + 7 = (x - p)^2 - q$

Find the value of the constants $p$ and $q$.

$$x^2 - 10x + 7 = (x-5)^2 - 5^2 + 7$$
$$= (x-5)^2 - 25 + 7$$
$$= (x-5)^2 - 18$$

$p =$ 5 B1

$q =$ 18 B1

**3.** Completing the square **[2 marks]**

Solve the equation $(x-7)^2 - 5 = 0$

Write your answer in the form $a \pm \sqrt{b}$ where $a$ and $b$ are integers.

$(x-7)^2 - 5 = 0$

$\Rightarrow (x-7)^2 = 5$ B1 [FOR '+' AND '−']

$\Rightarrow x - 7 = \pm\sqrt{5}$

A1

$x = 7 \pm \sqrt{5}$

| **4.** | Completing the square **[4 marks]** |
|---|---|

By completing the square, solve the equation $x^2 + 10x - 3 = 0$

Give your answer in the form $p \pm \sqrt{q}$ where $p$ and $q$ are integers.

$$x^2 + 10x - 3 = 0$$

$$(x+5)^2 - 5^2 - 3 = 0 \quad \text{M1}$$

$$(x+5)^2 - 25 - 3 = 0$$

$$\Rightarrow (x+5)^2 = 28 \quad \text{M1}$$

$$\Rightarrow x+5 = \pm\sqrt{28}$$

B1

[FOR '+' AND '-']

ACCEPT $-5 \pm 2\sqrt{7}$

A1

$x = -5 \pm \sqrt{28}$

| | |
|---|---|
| **5.** | Completing the square **[3 marks]** |

(a) Write $x^2+14x-9$ in the form $(x+a)^2+b$ where $a$ and $b$ are integers.

$$x^2+14-9 = (x+7)^2-7^2-9$$
$$= (x+7)^2-49-9$$

B1 B1

$(x+7)^2-58$

**(2)**

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph $y=x^2+14x-9$

$(x+7)^2-58$

'$x$' IS THE NEGATIVE OF THIS

THIS IS '$y$'

A1

( $-7$ , $-58$ )

**(1)**

**6.** Completing the square **[3 marks]**

$6x-7-x^2 = p-(x+q)^2$ where $p$ and $q$ are integers.

Find the value of $p$ and the value of $q$.

IF $6x-7-x^2 = p-(x+q)^2$

[MULTIPLY EACH TERM BY "-1"]

THEN $x^2-6x+7 = (x+q)^2-p$ (M1)

$\Rightarrow (x-3)^2-3^2+7 = (x+q)-p$

$\Rightarrow (x-3)-2 = (x+q)-p$

[$+q = -3$]

[$-p = -2$]

$p =$ 2 (A1)

$q =$ -3 (A1)

Write $x^2 - 6x - 3$ in the form $(x+a)^2 + b$ where $a$ and $b$ are integers.

$$x^2 - 6x - 3 = (x-3)^2 - 3^2 - 3$$
$$= (x-3)^2 - 9 - 3$$

B1 B1

$(x-3)^2 - 12$

**(2)**

(b) Hence, or otherwise, solve the equation $x^2 - 6x - 3 = 0$

Write your answers in the form $p \pm \sqrt{q}$ where $p$ and $q$ are integers.

$$(x-3)^2 - 12 = 0$$
$$(x-3)^2 = 12$$

B1 [FOR '+' AND '−']

$$x - 3 = \pm\sqrt{12}$$

A1

$x = 3 \pm \sqrt{12}$ (OR $3 \pm 2\sqrt{3}$)

**(2)**

| **8.** | Completing the square **[3 marks]** |
|---|---|

Write $2x^2 + 8x + 13$ in the form $a(x+b)^2 + c$ where $a$, $b$ and $c$ are integers.

$$2x^2 + 8x + 13 = 2[x^2 + 4x] + 13$$
$$= 2[(x+2)^2 - 2^2] + 13$$
$$= 2(x+2)^2 - 2 \times 2^2 + 13$$
$$= \underline{\underline{2(x+2)^2 + 5}}$$

B1

B1

B1

| **9.** | Completing the square **[4 marks]** |
|---|---|

(a) Write $2x^2 - 12x + 17$ in the form $a(x-b)^2 + c$ where $a$, $b$ and $c$ are integers.

$$2x^2 - 12x + 17 = 2[x^2 - 6x] + 17$$
$$= 2[(x-3)^2 - 3^2] + 17$$
$$= 2(x-3)^2 - 2 \times 3^2 + 17$$

B1 B1 B1

$2(x-3)^2 - 1$

**(3)**

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph $y = 2x^2 - 12x + 17$

$2(x\underbrace{-3}_{}) ^2 \underbrace{-1}_{}$

'x' IS THE NEGATIVE OF THIS

THIS IS 'y'

A1

(3 , -1)

**(1)**

**10.** Completing the square **[3 marks]**

$x^2 + hx + 15 = (x+3)^2 + k$

Find the value of the constants $h$ and $k$.

$\boxed{1ST}$ $h = \underline{\underline{6}}$ [THE 'x' TERM IS HALVED WHEN COMPLETING THE SQUARE]

$\boxed{2ND}$ $x^2 + 6x + 15 = (x+3)^2 - 3^2 + 15$ (M1)

$= (x+3)^2 - 9 + 15$

$= (x+3)^2 + 6$ $[+k = +6]$

$h =$ ......6...... (B1)

$k =$ ......6...... (A1)

**11.** Completing the square **[3 marks]**

$2x^2 - 20x + 10 = p(x-q)^2 - r$

Find the value of the constants $p$, $q$ and $r$.

$$2x^2 - 20x + 10 = 2\left[x^2 - 10x\right] + 10$$
$$= 2\left[(x-5)^2 - 5^2\right] + 10$$
$$= 2(x-5)^2 - 2 \times 5^2 + 10$$
$$= 2(x-5)^2 - 50 + 10$$
$$= 2(x-5)^2 - 40$$

$p =$ 2 B1

$q =$ 5 B1

$r =$ 40 B1

**12.** Completing the square **[3 marks]**

The expression $x^2 - 8x + 21$ can be written in the form $(x - a)^2 + b$ for all values of $x$.

(a) Find the value of $a$ and the value of $b$.

$$x^2 - 8x + 21 = (x-4)^2 - 4^2 + 21$$
$$= (x-4)^2 - 16 + 21$$
$$= (x-4)^2 + 5$$

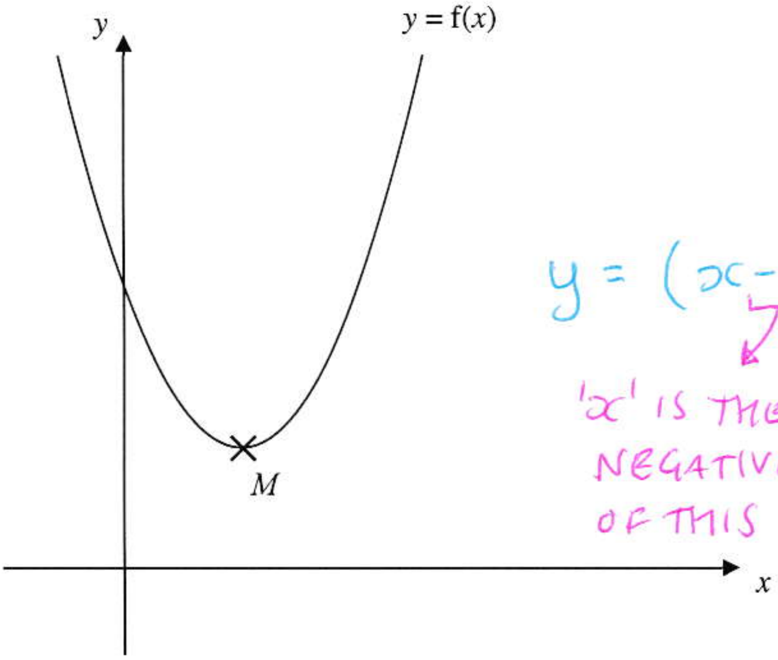
$a =$ 4 B1

$b =$ 5 B1

**(2)**

The equation of a curve is $y = f(x)$ where $f(x) = x^2 - 8x + 21$

The diagram shows part of a sketch of the graph of $y = f(x)$.




$y = \underbrace{(x-4)}^2 + \underbrace{5}$

'x' IS THE NEGATIVE OF THIS

THIS IS 'y'

The minimum point of the curve is $M$.

(b) Write down the coordinates of $M$.

(4 , 5) A1

**(1)**

(a) Write $x^2+6x+13$ in the form $(x+a)^2+b$ where $a$ and $b$ are integers.

$$x^2+6x+13 = (x+3)^2-3^2+13$$
$$= (x+3)^2-9+13$$

B1 B1

$(x+3)^2 + 4$

**(2)**

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph $y=x^2+6x+13$

$y = (x+3)^2 + 4$

'x' IS NEGATIVE OF THIS

THIS IS 'y'

A1

( -3 , 4 )

**(1)**

(c) Using your answer to part (b) state the number of solutions to the equation $x^2+6x+13=0$
Give a reason for your answer.

Number of solutions NONE

Reason: THE y-COORDINATE OF THE TURNING POINT IS POSITIVE. THIS MEANS THE CURVE $y=x^2+6x+13$ NEVER CROSSES THE x-ASIS MEANING THERE ARE NO SOLUTIONS A1

**(1)**

**14.** Completing the square **[5 marks]**

(a) Write $3x^2 - 12x - 1$ in the form $a(x-b)^2 + c$ where $a$, $b$ and $c$ are integers.

$$3x^2 - 12x - 1 = 3\left[x^2 - 4x\right] - 1$$
$$= 3\left[(x-2)^2 - 2^2\right] - 1$$
$$= 3(x-2)^2 - 3 \times 2^2 - 1$$
$$= 3(x-2)^2 - 12 - 1$$

B1 B1 B1

$3(x-2)^2 - 13$

**(3)**

(b) Hence, or otherwise, find the $\boxed{\text{minimum value}}$ of $y = 3x^2 - 12x - 1$

* QUESTION DOESN'T ASK FOR COORDINATES – JUST THE MINIMUM!

$y = 3(x\underbrace{-2}_{}) ^2 \underbrace{-13}_{}$

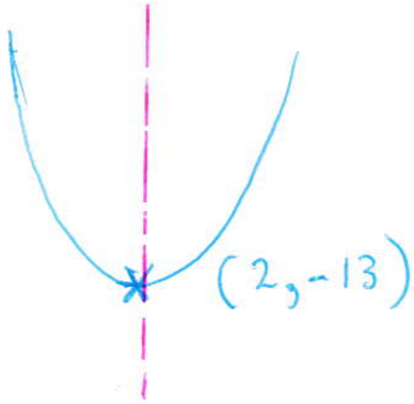
'x' IS THE NEGATIVE OF THIS

THIS IS 'y'

$-13$ A1

**(1)**

(c) Using your answer to part (b) find the equation of the line of symmetry of the curve $y = 3x^2 - 12x - 1$




$x = 2$ A1

**(1)**

LINE OF SYMMETRY PASSES THROUGH TURNING POINT!

**15.** Completing the square **[2 marks]**

Write $x^2+5x+9$ in the form $(x+a)^2+b$

$$x^2+5x+9 = (x+2.5)^2-2.5^2+9$$
$$= (x+2.5)^2-6.25+9$$
$$= \underline{\underline{(x+2.5)^2+2.75}}$$

B1 B1

**16.** Completing the square **[3 marks]**

$x^2 + px + 4 = (x+q)^2 \boxed{-5}$ THESE ARE EQUAL!

Find the value of the constants $p$ and $q$.

$\boxed{\text{1ST}}$ $x^2 + px + 4 = (x + \frac{p}{2})^2 \boxed{- \left(\frac{p}{2}\right)^2 + 4}$

'q'

$\boxed{\text{2ND}}$

$-\left(\frac{p}{2}\right)^2 + 4 = -5$

$-\left(\frac{p}{2}\right)^2 = -9$

$\Rightarrow \left(\frac{p}{2}\right)^2 = 9$

$\Rightarrow \frac{p}{2} = \pm 3$

$\Rightarrow p = \pm 6$

$\boxed{\text{3RD}}$ $q = \frac{p}{2}$

$= \frac{\pm 6}{2}$

$p =$ ....$\pm 6$....

$q =$ ....$\pm 3$....

**17.** Completing the square **[3 marks]**

$x^2 + 4x + p = (x+2)^2 + 3p$

Find the value of $p$.

THESE ARE EQUAL!

$x^2 + 4x + p = (x+2)^2 - 2^2 + p$ M1

$\Rightarrow 3p = -2^2 + p$ M1

$3p = -4$

$2p = -4$

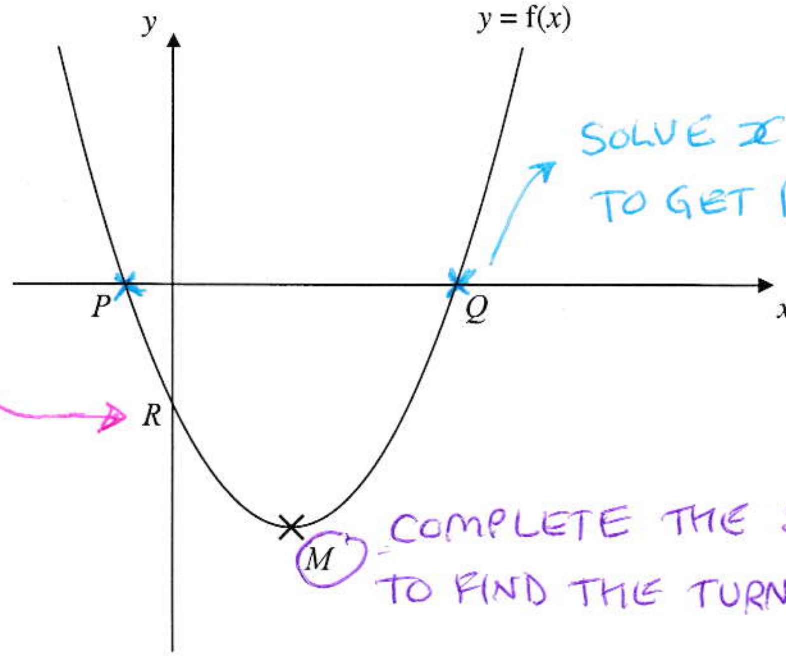
$p = -2$ A1

**18.** Completing the square **[6 marks]**

$f(x) = x^2 - 4x - 5$

The diagram show a sketch of the graph of $y = f(x)$

y-AXIS INTERCEPT




SOLVE $x^2 - 4x - 5 = 0$ TO GET P AND Q

COMPLETE THE SQUARE TO FIND THE TURNING POINT

The minimum point of the curve is $M$.

The points at which the curve crosses the $x$-axis and the $y$-axis are $P$, $Q$ and $R$.

Find the coordinates of the points $M$, $P$, $Q$ and $R$.

$\boxed{\text{1ST}}$

$$x^2 - 4x - 5 = (x-2)^2 - 2^2 - 5 \quad \text{M1}$$
$$= (x-2)^2 - 9$$

$\boxed{\text{2ND}}$

$$x^2 - 4x - 5 = 0$$
$$(x-5)(x+1) = 0 \quad \text{M1}$$

$x = 5$ $\qquad$ $x = -1$

$M$(.....2..... , .....$-9$.....) B1

$P$(.....$-1$..... , .....0.....) A1

$Q$(.....5..... , .....0.....) A1

$R$(.....0..... , .....$-5$.....) B1

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