

COMPOUND EVENTS

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 72

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2009]

Probability (Compound Events) [5 Marks]

Each time Jeni plays a computer game the probability that she will win is $\frac{2}{3}$

Jeni plays the computer game 3 times.

Calculate the probability that Jeni will win

(a) all 3 games,

$$P(WWW) = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \quad (m)$$
$$= \frac{8}{27} \quad (A1)$$

(b) exactly 2 out of the 3 games.

$$P(WWL) = \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}$$
$$P(WLW) = \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{27}$$
$$P(LWW) = \frac{1}{3} \times \dots = \frac{4}{27}$$

TOTAL
= $\frac{12}{27}$ (A1)

(m)
↑
THREE
OUTCOMES

(m)
MULTIPLYING

There are 9 counters in a bag.
7 of the counters are red and 2 of the counters are white.

Ajit takes at random two counters from the bag without replacement.

PROBABILITIES
CHANGE

(a) Calculate the probability that the two counters are red.

$$P(R,R) = \frac{7}{9} \times \frac{6}{8} = \frac{42}{72}$$

(mu)

$$\frac{7}{12} \quad (AI)$$

(2)

(b) Calculate the probability that the two counters have different colours.

$$P(R,W) = \frac{7}{9} \times \frac{2}{8} = \frac{14}{72}$$

$$P(W,R) = \frac{2}{9} \times \frac{7}{8} = \frac{14}{72}$$

(mu)

} $\frac{28}{72}$

$$\frac{7}{18} \quad (AI)$$

(3)

Naveed has two bags of tiles, bag A and bag B.

There are 10 tiles in bag A.
7 of these tiles are red.
The other 3 tiles are white.

There are 8 tiles in bag B.
5 of these tiles are red.
The other 3 tiles are white.

Naveed takes at random one tile from each bag.

(a) Work out the probability that the tiles are the same colour.

$$\begin{aligned}
 P(R,R) &= \frac{7}{10} \times \frac{5}{8} = \frac{35}{80} \quad \text{(BI)} \\
 P(W,W) &= \frac{3}{10} \times \frac{3}{8} = \frac{9}{80} \quad \text{(BI)}
 \end{aligned}
 \left. \vphantom{\begin{aligned} P(R,R) \\ P(W,W) \end{aligned}} \right\} \text{TOTAL} = \frac{44}{80} \quad \text{(AI)}$$

$$\frac{11}{20}$$

(3)

All 18 tiles are put in a box.

Naveed takes at random one tile from the box.

He does not replace the tile.

Naveed then takes at random a second tile from the box.

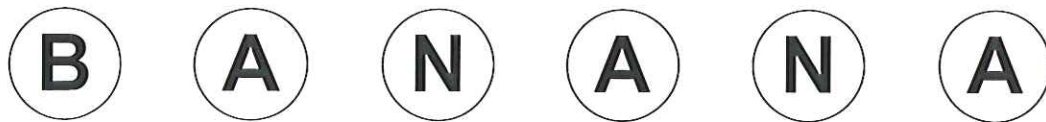
PROBABILITIES CHANGE!

(b) Work out the probability that both tiles are red.

$$P(R,R) = \frac{12}{18} \times \frac{11}{17} \quad \text{(MI)}$$

$$= \frac{22}{51} \quad \text{(AI)} \quad \text{[ACCEPT EQUIVALENTS!]}$$

The diagram shows six counters.



Each counter has a letter on it.

Bishen puts the six counters into a bag.

He takes a counter at random from the bag.

He records the letter which is on the counter and replaces the counter in the bag.

He then takes a second counter at random and records the letter which is on the counter.

(a) Calculate the probability that the first letter will be A and the second letter will be N.

$$P(A, N) = \frac{3}{6} \times \frac{2}{6} = \frac{6}{36}$$

(m)

$$\frac{1}{6} \frac{(A)}{(2)}$$

(b) Calculate the probability that both letters will be the same.

$$P(B, B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

$$P(A, A) = \frac{3}{6} \times \frac{3}{6} = \frac{9}{36}$$

$$P(N, N) = \frac{2}{6} \times \frac{2}{6} = \frac{4}{36}$$

$$\left. \begin{array}{l} P(B, B) \\ P(A, A) \\ P(N, N) \end{array} \right\} \text{TOTAL} = \frac{14}{36}$$

$$\frac{7}{18} \frac{(A)}{(4)}$$

PROBABILITIES
STAY THE SAME

A box contains 7 good apples and 3 bad apples. $P(G) = \frac{7}{10}$ $P(B) = \frac{3}{10}$

Nick takes two apples at random from the box, without replacement.

(a) (i) Calculate the probability that both of Nick's apples are bad.

$$P(B, B) = \frac{3}{10} \times \frac{2}{9} = \frac{6}{90}$$

(m)

$$\frac{1}{15} \quad (A1)$$

PROBABILITIES CHANGE

(ii) Calculate the probability that at least one of Nick's apples is good.

$$P(G, B) = \dots$$

$$P(B, G) = \dots$$

$$P(G, G) = \dots$$

* SHORTCUT *
 $1 - P(B, B)$
 (m)

$$\frac{14}{15} \quad (A1)$$

(4)

Another box contains 8 good oranges and 4 bad oranges.

$$P(G) = \frac{8}{12}, \quad P(B) = \frac{4}{12}$$

Crystal keeps taking oranges at random from the box one at a time, without replacement, until she gets a good orange.

(b) Calculate the probability that she takes exactly three oranges.

$$P(B, B, G) = \frac{4}{12} \times \frac{3}{11} \times \frac{8}{10} \quad (m)$$

$$= \frac{96}{1320}$$

$$\frac{4}{55} \quad (A1)$$

(2)

The sides of a fair six-sided dice are numbered from 1 to 6

The dice is thrown three times.

Find the probability that it shows a 1 at least twice.

$$\begin{aligned}
 P(1, 1, \bar{1}) &= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{216} \\
 P(1, \bar{1}, 1) &= \frac{1}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{5}{216} \\
 P(\bar{1}, 1, 1) &= \frac{5}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{5}{216} \\
 P(1, 1, 1) &= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}
 \end{aligned}$$

$\left. \begin{array}{l} \frac{5}{216} \\ \frac{5}{216} \\ \frac{5}{216} \end{array} \right\} \frac{16}{216} \text{ (A1)}$

$\frac{2}{27} \text{ (A1)}$

$\underbrace{\hspace{10em}}_{\text{(M2)}}$

Younis spins a biased coin twice.

The probability that it will come down heads both times is 0.36

Calculate the probability that it will come down tails both times.

$$\text{LET } P(H) = x$$

$$\therefore P(H, H) = x \times x = x^2$$

$$\text{AND } x^2 = 0.36$$

$$\Rightarrow x = 0.6 \quad (\text{BI})$$

$$\therefore P(T) = 1 - 0.6$$

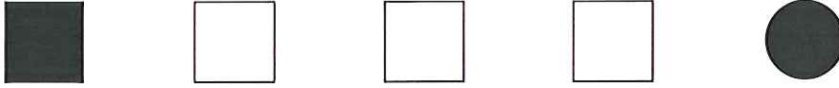
$$= 0.4 \quad (\text{MI})$$

AND

$$P(T, T) = 0.4 \times 0.4$$

$$= \underline{\underline{0.16}} \quad (\text{AI})$$

Here are five shapes.



Four of the shapes are squares and one of the shapes is a circle.

One square is black.

Three squares are white.

The circle is black.

The five shapes are put in a bag.

- (a) Jasmine takes a shape at random from the bag 150 times. She replaces the shape each time.

Work out an estimate for the number of times she will take a white square.

$$P(\text{WHITE SQUARE}) = \frac{3}{5} \text{ (m1)}$$

$$E(\text{WHITE SQUARE}) = \frac{3}{5} \times 150 \text{ (m1)} \quad \dots \quad 90 \text{ (A1)}$$

(3)

- (b) Alec takes a shape at random from the bag and does **not** replace it. Bashir then takes a shape at random from the bag.

Work out the probability that

PROBABILITY CHANGE

- (i) they both take a square,

$$P(S, S) = \frac{4}{5} \times \frac{3}{4} = \frac{12}{20} \text{ (m1)}$$

$$\dots \quad 0.6 \text{ (A1)}$$

- (ii) they take shapes of the same colour.

$$\left. \begin{aligned} P(B, B) &= \frac{2}{5} \times \frac{1}{4} = \frac{2}{20} \\ P(W, W) &= \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} \end{aligned} \right\} \text{TOTAL} = \frac{8}{20}$$

(m1)

(m1)

$$\dots \quad 0.4 \text{ (A1)}$$

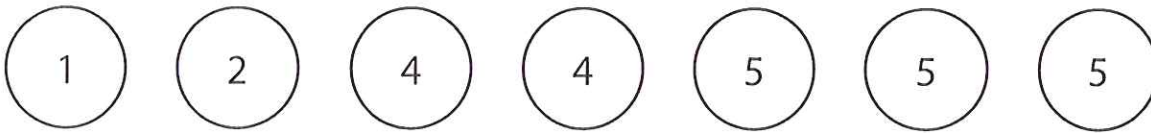
(5)

BOTH
OUTCOMES

MULTIPLYING

Here are seven counters.
Each counter has a number on it.

7 COUNTERS



Ali puts the seven counters in a bag.

He takes, at random, a counter from the bag and does not replace the counter.

He then takes, at random, a second counter from the bag.

ONLY 6 COUNTERS FOR SECOND PICK

Calculate the probability that

(i) the number on the second counter is 2 more than the number on the first counter,

$$P(2,4) = \frac{1}{7} \times \frac{2}{6} = \frac{2}{42}$$

(mi)

$$= \frac{1}{21}$$

$\frac{1}{21}$ (AI)

(ii) the number on the second counter is 1 more than the number on the first counter.

$$P(1,2) = \frac{1}{7} \times \frac{1}{6} = \frac{1}{42}$$

(mi)

$$P(4,5) = \frac{2}{7} \times \frac{3}{6} = \frac{6}{42}$$

(mi)

TOTAL = $\frac{7}{42}$

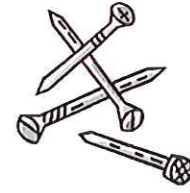
$$= \frac{1}{6}$$

$\frac{1}{6}$ (AI)

A box contains 20 nails.

The table shows information about the length of each nail.

Length of nail (mm)	25	30	40	50	60
Number of nails	1	8	4	5	2



(a) Viraj takes at random one nail from the box.

Find the probability that the length of the nail he takes is

(i) 50 mm or 60 mm,

$$\text{(M1)} \quad \frac{5+2}{20} = \frac{7}{20}$$

$$\text{(A1)} \quad \underline{0.35}$$

(ii) less than 35 mm.

$$\text{(M1)} \quad \frac{1+8}{20} = \frac{9}{20}$$

$$\text{(A1)} \quad \underline{0.45}$$

(4)

(b) Jamila puts all 20 nails into a bag.

She takes at random one of the nails and records its length.

She **replaces** the nail in the bag.

She then takes at random a second nail from the bag and records its length.

Calculate the probability that the two nails she takes

(i) each have a length of 60 mm,

$$P(60,60) = \frac{2}{20} \times \frac{2}{20} \quad \text{(M1)}$$

$$\text{(A1)} \quad \underline{0.01}$$

(ii) have a total length of 80 mm.

$$P(40,40) = \frac{4}{20} \times \frac{4}{20} = \frac{16}{400}$$

$$P(30,50) = \frac{8}{20} \times \frac{5}{20} = \frac{40}{400}$$

$$P(50,30) = \frac{5}{20} \times \frac{8}{20} = \frac{40}{400}$$

$$\left. \begin{array}{l} \frac{16}{400} \\ \frac{40}{400} \\ \frac{40}{400} \end{array} \right\} \text{total} = \frac{96}{400}$$

$$\text{(A1)} \quad \underline{\underline{0.24}}$$

(M1) [THREE POSSIBILITIES]

(M1) [MULTIPLY]

Two bags contain discs.

Bag A contains 12 discs.

5 of the discs are red, 6 are blue and 1 is white.

$$\left. \begin{array}{l} P(R) = \frac{5}{12}, P(B) = \frac{6}{12}, P(W) = \frac{1}{12} \end{array} \right\}$$

Bag B contains 25 discs.

n of the discs are red and the rest are blue.

$$\left. \begin{array}{l} P(R) = \frac{n}{25}, P(B) = \frac{25-n}{25} \end{array} \right\}$$

James takes at random a disc from Bag A.

Lucy takes at random a disc from Bag B.

Given that the probability that James and Lucy both take a red disc is $\frac{2}{15}$

(i) find the value of n , the number of red discs in Bag B.

$$P(RR) = \frac{5}{12} \times \frac{n}{25} = \frac{2}{15} \quad (M1)$$

$$\Rightarrow \frac{5n}{300} = \frac{2}{15} \Rightarrow n = \frac{2}{15} \times \frac{300}{5}$$

$$n = \dots\dots\dots 8 \quad (A1)$$

(ii) Hence calculate the probability that James and Lucy take discs of different colours.

$$P(BB) = \frac{6}{12} \times \frac{(25-8)}{25}$$

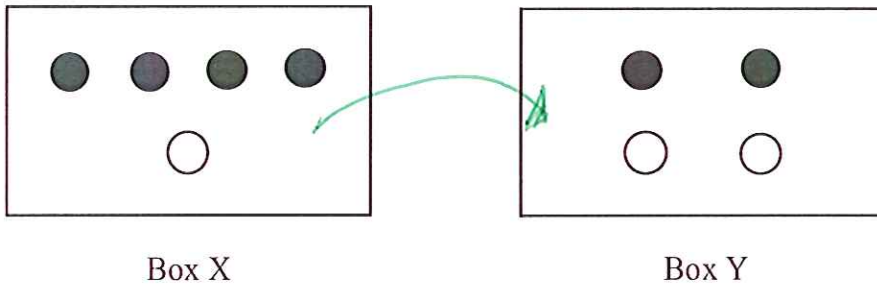
$$= \frac{17}{50} \quad (M1)$$

$$\therefore P(\text{SAME COLOUR}) = \frac{2}{15} + \frac{17}{50}$$

$$= \frac{71}{150} \quad (M1)$$

$$\therefore P(\text{DIFFERENT COLOURS}) = 1 - \frac{71}{150}$$

$$= \frac{79}{150} \quad (A1)$$



In Box X, there are 4 black discs and 1 white disc.

In Box Y, there are 2 black discs and 2 white discs.

Vikram takes at random a disc from Box X and puts it in Box Y.

He then takes at random a disc from Box Y.

- (a) Calculate the probability that the disc he takes from Box X and the disc he takes from Box Y will both be black discs.

$$P(BB) = \frac{4}{5} \times \frac{3}{5} \quad \text{(mi)}$$

$$= \frac{12}{25}$$

$$\frac{12}{25} \quad \text{(AI)}$$

(2)

- (b) Calculate the probability that the disc he takes from Box Y will be a white disc.

$$P(BW) = \frac{4}{5} \times \frac{2}{5} = \frac{8}{25}$$

$$P(WW) = \frac{1}{5} \times \frac{3}{5} = \frac{3}{25}$$

(mi) EITHER

$$\frac{11}{25} \quad \text{(AI)}$$

(mi) FOR ADDING.

$\frac{1}{3}$ of the people in a club are men.

The number of men in the club is n .

(a) Write down an expression, in terms of n , for the number of people in the club.

$$\frac{1}{3} \times p = n \quad \longrightarrow \quad p = 3n \quad \text{(A1)} \quad (1)$$

Two of the people in the club are chosen at random.

The probability that both these people are men is $\frac{1}{10}$

(b) Calculate the number of people in the club.

$$\frac{n}{p} \times \frac{n-1}{p-1} = \frac{1}{10} \quad \text{(M1) [AN APPROPRIATE EQUATION]}$$

$$\Rightarrow \frac{n^2 - n}{p^2 - p} = \frac{1}{10}$$

$$\Rightarrow 10n^2 - 10n = p^2 - p \quad \text{(M1) [QUADRATIC WITHOUT DENOMINATORS]}$$

$$\Rightarrow 10n^2 - 10n = 9n^2 - 3n$$

$$\Rightarrow 10n - 10 = 9n - 3 \quad \text{(M1) [LINEAR EQUATION]}$$

$$\Rightarrow 10n - 9n = -3 + 10$$

$$n = 7 \quad \text{(A1) [FINDING n]}$$

$$\begin{aligned} \therefore p &= 3 \times 7 \\ &= \underline{\underline{21}} \quad \text{(A1)} \end{aligned}$$

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The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or ‘best’ method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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