SOLUTIONS

1. [New Question, by Maths4Everyone.com] Differentiation (Inc Velocity and Acceleration) **[2 Marks]**

$y = 2x^3 + 3x^2 - 5$

Find $\frac{dy}{dx}$

B1 B1

$6x^2 + 6x$

(2)

A curve has equation $y = x^2 - 4x + 1$.

(a) For this curve find

(i) $\frac{dy}{dx}$,

$\frac{dy}{dx} = 0$

$2x - 4$ B1 [BOTH]

(ii) the coordinates of the turning point.

$2x - 4 = 0$ M1

$\Rightarrow x = \underline{\underline{2}}$ A1

$y = (2)^2 - 4(2) + 1$

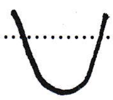
$= \underline{\underline{-3}}$ A1

$(2, -3)$

(4)

(b) State, with a reason, whether the turning point is a maximum or a minimum.

MINIMUM. IT'S A POSITIVE QUADRATIC AND THESE ONLY HAVE A MINIMUM

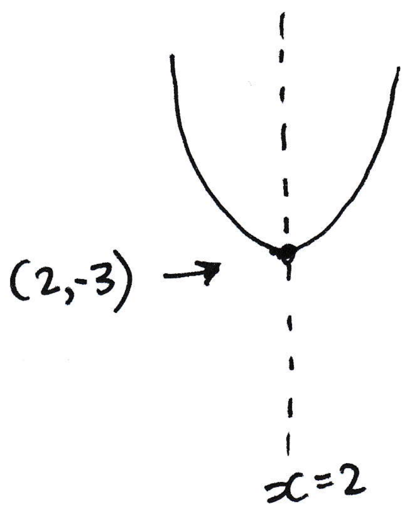


(2) B2 [NEED ANSWER AND EXPLANATION]

(c) Find the equation of the line of symmetry of the curve $y = x^2 - 4x + 1$

QUADRATICS ARE SYMMETRICAL ABOUT THEIR TURNING POINT → $x = 2$ B2

(2)



[ANY $x = \ldots$ EXPRESSION GETS ONE MARK]

For the curve with equation $y = 4x^3 - 2x + 5$

(i) find $\frac{dy}{dx}$

A1 A1

$12x^2 - 2$

(ii) find the coordinates of the two points on the curve where the gradient of the curve is 1

$\frac{dy}{dx}$

$12x^2 - 2 = 1$ M1

$12x^2 = 3$

$x^2 = \frac{3}{12}$

$x = \pm\sqrt{\frac{3}{12}}$

$x = \underline{\underline{0.5}}$ A1

$x = \underline{\underline{-0.5}}$ A1

$\Rightarrow y = 4 \times 0.5^3 - 2 \times 0.5 + 5$

$= \underline{\underline{4.5}}$

$y = 4 \times (-0.5)^3 - 2 \times (-0.5) + 5$

$= \underline{\underline{5.5}}$

$\underline{\underline{(0.5, 4.5)}}$

$\underline{\underline{(-0.5, 5.5)}}$

A1 [BOTH]

A curve has equation $y = x^3 - 5x^2 + 8x - 7$

(a) Find the gradient of the curve at (2, –3).

$$\frac{dy}{dx} = 3x^2 - 10x + 8$$ A2

M1

AT $x = 2$, $\frac{dy}{dx} = 3 \times 2^2 - 10 \times 2 + 8$ $= 0$ A1

(4)

(b) What does your answer to part (a) tell you about the point (2, −3)?

IT IS A TURNING POINT A1

(1)

A particle moves along a straight line.
The fixed point O lies on this line.
The displacement of the particle from O at time t seconds is s metres, where

DIFFERENTIATE

$$s = t^3 - 6t + 3$$

(a) Find an expression for the $\boxed{\text{velocity}}$, v m/s, of the particle at time t seconds.

$$v = \frac{ds}{dt}$$

A1 A1

$v = 3t^2 - 6$

(2)

DIFFERENTIATE AGAIN!

(b) Find the $\boxed{\text{acceleration}}$ of the particle at time 5 seconds.

$$a = \frac{dv}{dt} = 6t \quad \text{B1}$$

$$= 6 \times 5$$

A1

30 m/s^2

(2)

6. [New Question, by Maths4Everyone.com] Differentiation (Inc Velocity and Acceleration) **[3 Marks]**

Differentiate with respect to x

(a) $5x^2$

B1

$10x$

(b) $\frac{3}{x}$ $= 3x^{-1}$

B1

$-3x^{-2}$ $\left(\frac{-3}{x^2}\right)$

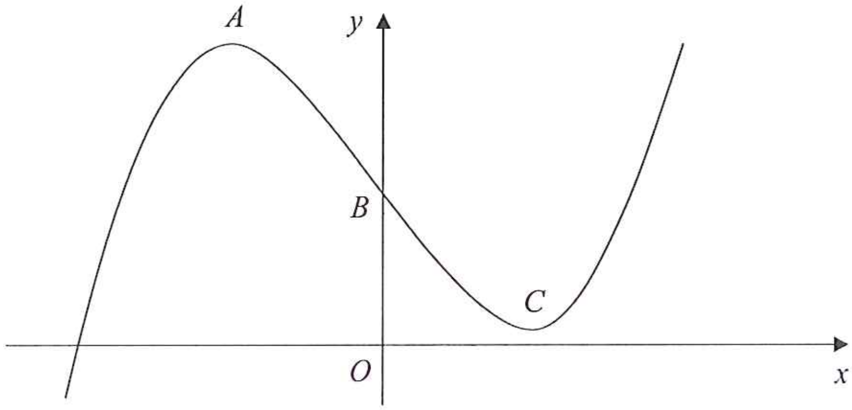
(c) $\sqrt{x}$ $= x^{\frac{1}{2}}$

B1

$\frac{1}{2}x^{-\frac{1}{2}}$ $\left(\frac{1}{2\sqrt{x}}\right)$

(3)

The diagram shows the graph of $y = x^3 - 12x + 17$
A is the maximum point on the curve.
C is the minimum point on the curve.
The curve crosses the y axis at B.



For the equation $y = x^3 - 12x + 17$

(a) find $\frac{dy}{dx}$,

A1 A1

$3x^2 - 12$

(2)

(b) find the gradient of the curve at B, $(x = 0)$

$3 \times 0^2 - 12$ M1

-12 A1

(2)

(c) find the coordinates of A and C. (TURNING POINTS)

$3x^2 - 12 = 0$ M1

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$ A1

$y = (2)^3 - 12(2) + 17$

$= 1$ A1

$y = (-2)^3 - 12 \times (-2) + 17$

$= 33$ A1

$A(-2, 33)$

$C(2, 1)$

8. [Edexcel, 2013]	Differentiation (Inc Velocity and Acceleration) **[4 Marks]**

A particle is moving in a straight line which passes through a fixed point O.
The displacement, s metres, of the particle from O at time t seconds is given by

$$s = 10 + 9t^2 - t^3$$

(a) Find an expression for the $\boxed{\text{velocity,}}$ v m/s, of the particle at time t seconds.

DIFFERENTIATE

B1 B1

$v = 18t - 3t^2$

(2)

(b) Find the time at which the $\boxed{\text{acceleration}}$ of the particle is $\boxed{\text{zero.}}$

DIFFERENTIATE AGAIN!

$a = 18 - 6t$

WHEN a = ZERO

$18 - 6t = 0$ M1

$\Rightarrow -6t = -18$

$t = \frac{-18}{-6}$

$= 3$ SECS

A1

Differentiate with respect to x

(a) $\frac{5}{x}$ $= 5x^{-1}$

B1

$-5x^{-2}$ $\left(\frac{-5}{x^2}\right)$

(b) $3\sqrt{x}$ $= 3x^{\frac{1}{2}}$

B1

$1.5x^{-\frac{1}{2}}$ $\left(\frac{1.5}{\sqrt{x}}\right)$

(c) $x(x+4)$ $= x^2 + 4x$ M1

B1 [BOTH]

$2x + 4$

(4)

A curve has equation $y = x^3 + 3x^2 - 24x$

(a) Find $\frac{dy}{dx}$

A1 A1 A1

$3x^2 + 6x - 24$

(3)

$\frac{dy}{dx} = 0$

(b) Find the coordinates of the turning points of the curve.

$3x^2 + 6x - 24 = 0$ M1

$\Rightarrow x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$ M1 [FACTORISE]

$x = -4$ A1 $x = 2$

$y = (2)^3 + 3(2)^2 - 24(2)$

$= -28$

$y = (-4)^3 + 3(-4)^2 - 24(-4)$ M1 [SUBSTITUTE]

$= 80$

A1 [BOTH]

$(-4, 80)$ AND $(2, -28)$

(5)

A particle moves in a straight line through a fixed point O.
The displacement, s metres, of the particle from O at time t seconds is given by

$$s = t^3 - 5t^2 + 8$$

LINKED BY DIFFERENTIATION!

(a) Find an expression for the velocity, v m/s, of the particle after t seconds.

$$v = \frac{ds}{dt}$$

A1 A1

$v =$ $3t^2 - 10t$

(2)

(b) Find the time at which the acceleration of the particle is 20 m/s².

$$a = \frac{dv}{dt}$$

$$= 6t - 10$$

M1

$$6t - 10 = 20$$

$$6t = 30$$

$$t = \frac{30}{6}$$

A1

$$= \underline{\underline{5}} \text{ secs}$$

12. [New Question, by Maths4Everyone.com] Differentiation (Inc Velocity and Acceleration) **[3 Marks]**

Differentiate with respect to x

(a) $x(x^2 - x)$ $= x^3 - x^2$

B1 B1

$3x^2 - 2x$

(b) $\dfrac{1}{x}$ $= x^{-1}$

B1

$-1x^{-2}$ $\left(\dfrac{-1}{x^2}\right)$

(3)

$y = x^2 - \dfrac{16}{x}$

(a) Find $\dfrac{dy}{dx}$

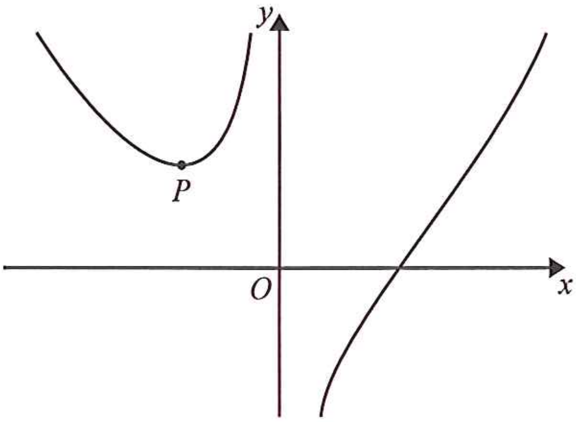
$y = x^2 - 16x^{-1}$ M1

A1 A1

$\dfrac{dy}{dx} = 2x + 16x^{-2}$

(3)

THIS CAN BE WRITTEN AS $\dfrac{16}{x^2}$



The graph shows part of the curve with equation $y = x^2 - \dfrac{16}{x}$

The point P is the $\boxed{\text{turning point}}$ of the curve.

$\dfrac{dy}{dx} = 0$

(b) Work out the coordinates of P.

$2x + \dfrac{16}{x^2} = 0$ M1

$\Rightarrow 2x^3 + 16 = 0$

$\Rightarrow x^3 + 8 = 0$

$\Rightarrow x^3 = -8$

$\Rightarrow x = \underline{\underline{-2}}$ A1

$y = (-2)^2 - \dfrac{16}{(-2)}$ M1

$= 4 + 8$

$= \underline{\underline{12}}$ A1

COORDINATES ARE $(-2, 12)$

A particle moves in a straight line through a fixed point O.
The displacement of the particle from O at time t seconds is s metres, where

$$s = t^2 - 6t + 10$$

(a) Find $\frac{ds}{dt}$

A1 A1

$2t - 6$ ....

(2)

(b) Find the velocity of the particle when $t = 5$

$v = 2t - 6 \quad (t = 5)$

$= 2 \times 5 - 6$ M1

$= \underline{\underline{4}}$

4 .... m/s A1

(2)

(c) Find the acceleration of the particle.

$a = \frac{dv}{dt}$ M1

$= \underline{\underline{2}}$

2 .... m/s^2 A1

(2)

Differentiate with respect to x

(a) $x^2\left(2x+\frac{5}{x}\right) = 2x^3 + 5x$ M1

B1 [BOTH]

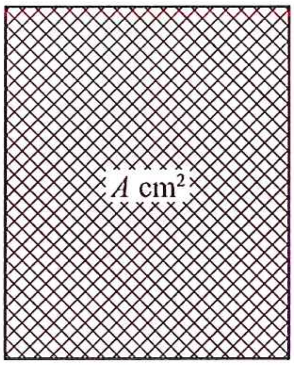
$6x^2 + 5$

(c) $5\sqrt{x} = 5x^{\frac{1}{2}}$

B1

$2.5x^{-\frac{1}{2}} \quad \left(\frac{2.5}{\sqrt{x}}\right)$

(3)



The diagram shows a rectangular photo frame of area A cm^2.
The width of the photo frame is x cm.
The height of the photo frame is y cm.
The perimeter of the photo frame is 72 cm.

$\Rightarrow 2x + 2y = 72$ B1

$2y = 72 - 2x$

$\boxed{y = 36 - x}$ B1

(a) Show that $A = 36x - x^2$

$A = x \times y$

$= x \times (36 - x)$ B1

$= 36x - x^2$

(3)

(b) Find $\frac{dA}{dx}$

A1 A1

$36 - 2x$

(2)

(c) Find the maximum value of A.

AT TURNING POINTS $\frac{dA}{dx} = 0$

M1

$36 - 2x = 0$

$\Rightarrow 36 = 2x$

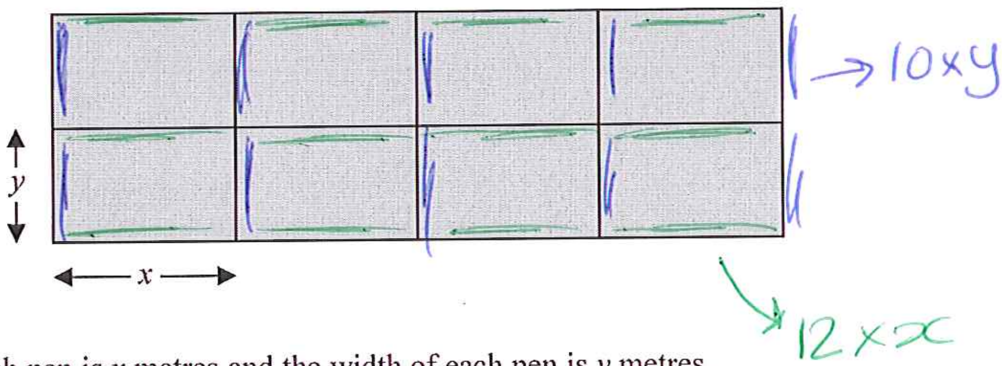
$\Rightarrow x = 18$ A1

$\therefore$ MAXIMUM AREA, $A = 36x - x^2$ $(x = 18)$

$= 36 \times 18 - 18^2$

$= 324 \text{ cm}^2$ A1

A farmer has 180 metres of fencing. TOTAL = 180
With the 180 metres of fencing, he makes an enclosure divided into eight equal, rectangular pens.
The fencing is used for the perimeter of each pen.



The length of each pen is x metres and the width of each pen is y metres.

(a) (i) Show that $y = 18 - 1.2x$

$10y + 12x = 180$ M1

$\Rightarrow 10y = 180 - 12x$ M1

$y = 18 - 1.2x$

The total area of the enclosure is A m^2.

(ii) Show that $A = 144x - 9.6x^2$

$A = 4x \times 2y = 8xy$ M1

$= 8x(18 - 1.2x)$

$= 144x - 9.6x^2$

(3)

(b) Find $\frac{dA}{dx}$

A1 A1

$144 - 19.2x$

(2)

(c) Find the maximum value of A.

$144 - 19.2x = 0$ M1

$\Rightarrow -19.2x = -144$

$x = \frac{-144}{-19.2}$

$= 7.5$ B1

$A = 144 \times 7.5 - 9.6 \times 7.5^2$

$= 540 \text{ m}^2$ A1

HW11

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B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

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A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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