DIRECT AND INVERSE PROPORTION SOLUTIONS

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 72

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2006] Non-Linear Proportion (Direct and Inverse) **[8 Marks]**

A ball is dropped from a tower.
After t seconds, the ball has fallen a distance x metres.

x is directly proportional to t^2. $x = kt^2$

When $t = 2$, $x = 19.6$

(a) Find an equation connecting x and t.

$x = kt^2$ (M1) $(t = 2, x = 19.6)$

$\Rightarrow 19.6 = k \times 2^2$

$k = \frac{19.6}{2^2} = 4.9$ (M1)

$x = 4.9t^2$ (A1)

(3)

(b) Find the value of x when $t = 3$

$x = 4.9t^2 \quad (t = 3)$

$\Rightarrow x = 4.9 \times 3^2$ (M1)

$= 44.1$

$x =$ 44.1 (A1)

(2)

(c) Find how long the ball takes to fall 10 m.

$x = 4.9t^2 \quad (x = 10)$

$\Rightarrow 10 = 4.9t^2$ (M1)

$\Rightarrow t^2 = \frac{10}{4.9}$

$\Rightarrow t = \sqrt{\frac{10}{4.9}} = 1.4285...$ (M1)

1.43 seconds (A1)

(3)

2. [Edexcel, 2005] Non-Linear Proportion (Direct and Inverse) **[4 Marks]**

The amount of petrol a car uses is directly proportional to the distance it travels.
A car uses 3 litres of petrol when it travels 50 km.

(a) Work out the amount of petrol the car uses when it travels 125 km.

$L = k \times d \quad (L = 3,\ d = 50)$

$\Rightarrow 3 = k \times 50$ M1

$\Rightarrow k = \frac{3}{50}$

$\Rightarrow L = \frac{3}{50} \times d \rightarrow L = \frac{3}{50} \times 125 \rightarrow$ 7.5 litres A1

(2)

(b) Work out the distance the car travels when it uses 5.7 litres of petrol.

$L = \frac{3}{50} \times d$

$\Rightarrow 5.7 = \frac{3}{50} \times d \Rightarrow d = 5.7 \times \frac{50}{3}$ M1

$= 95$ km A1

(2)

3. [Edexcel, 2016] Non-Linear Proportion (Direct and Inverse) **[4 Marks]**

M is directly proportional to p^3
$M = 128$ when $p = 8$

$M = k \times p^3$

(a) Find a formula for M in terms of p.

$M = k \times p^3$

$128 = k \times 8^3$ M1

$\Rightarrow k = \frac{128}{8^3}$

$k = 0.25$ M1

A1

$M = 0.25p^3$

(3)

(b) Find the value of M when $p = 5$

$M = 0.25 \times (5)^3$

A1

31.25

(1)

4. [Edexcel, 2014] Non-Linear Proportion (Direct and Inverse) **[5 Marks]**

A particle moves from rest.
The speed of the particle is v m/s when it has moved a distance of x metres.

v is proportional to $\sqrt{x}$

When $v = 8$, $x = 25$

(a) Express v in terms of x.

$v = k \times \sqrt{x}$

$8 = k \times \sqrt{25}$ (M1)

$k = \frac{8}{\sqrt{25}}$

$= 1.6$ (B1)

(A1) $v = 1.6\sqrt{x}$

(3)

(b) Find the speed of the object when it has moved a distance of 56.25 metres.

$v = 1.6\sqrt{x} \quad (x = 56.25)$

$\Rightarrow v = 1.6\sqrt{56.25}$ (M1)

$= 12$ m/s (A1)

5. [Edexcel, 2007] Non-Linear Proportion (Direct and Inverse) **[6 Marks]**

The frequency, f kilohertz, of a radio wave is inversely proportional to its wavelength, w metres.

When $w = 200$, $f = 1500$

(a) (i) Express f in terms of w.

$f = \frac{k}{w}$ $(w = 200, f = 1500)$

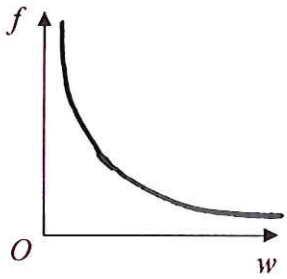
M1 EITHER

$\Rightarrow 1500 = \frac{k}{200}$ $\Rightarrow k = 1500 \times 200$

$= 300\,000$ A1

$f = \frac{300\,000}{w}$ A1

(ii) On the axes, sketch the graph of f against w.



B1

(4)

(b) The wavelength of a radio wave is 1250 m.
Calculate its frequency.

$f = \frac{300\,000}{w}$ $(w = 1250)$

$\Rightarrow f = \frac{300\,000}{1250}$ M1

240 kilohertz A1

(2)

6. [Edexcel, 2010] Non-Linear Proportion (Direct and Inverse) **[6 Marks]**

The light intensity, E, at a surface is inversely proportional to the square of the distance, r, of the surface from the light source.

$E = 4$ when $r = 50$

$E = \frac{k}{r^2}$

(a) Express E in terms of r.

$E = \frac{k}{r^2} \quad (E=4, r=50)$

$\Rightarrow 4 = \frac{k}{50^2}$ M1 $\Rightarrow k = 4 \times 50^2$

$= 10\,000$ B1

$E = \frac{10\,000}{r^2}$ A1

(3)

(b) Calculate the value of E when $r = 20$

$E = \frac{10\,000}{r^2}$

$= \frac{10\,000}{20^2}$

$E = 25$ A1

(1)

(c) Calculate the value of r when $E = 1600$

$E = \frac{10\,000}{r^2}$

$\Rightarrow 1600 = \frac{10\,000}{r^2}$ M1

$\Rightarrow r^2 = \frac{10\,000}{1600}$

$= 6.25$

$\Rightarrow r = \sqrt{6.25}$

$= 2.5$

$r = 2.5$ A1

(2)

7. [Edexcel, 2015] Non-Linear Proportion (Direct and Inverse) **[5 Marks]**

V is inversely proportional to the square of t

$V = 28$ when $t = 2.5$

$V = \frac{k}{t^2}$

(a) Express V in terms of t

$V = \frac{k}{t^2}$ $(V = 28, t = 2.5)$

$\Rightarrow 28 = \frac{k}{2.5^2}$ M1 [EITHER]

$\Rightarrow k = 28 \times 2.5^2 = 175$ M1

$V = \frac{175}{t^2}$ A1

(3)

(b) Work out the value of V when $t = 6.25$

$V = \frac{175}{t^2}$ $(t = 6.25)$

$\Rightarrow V = \frac{175}{6.25^2}$ M1

$V = 4.48$ A1

(2)

P is inversely proportional to V.
$P = 18$ when $V = 24$

$P = \frac{k}{V}$

(a) Express P in terms of V.

$P = \frac{k}{V}$ $(P = 18, V = 24)$

$18 = \frac{k}{24}$ M1

$k = 18 \times 24 = 432$ B1

$P = \frac{432}{V}$ A1

.....

(3)

(b) Find the positive value of V when $P = 3V$

$P = \frac{432}{V}$ $(P = 3V)$

$\Rightarrow 3V = \frac{432}{V}$ M1

$\Rightarrow 3V^2 = 432$

$V^2 = \frac{432}{3}$

$V = \sqrt{\frac{432}{3}} = 12$ A1

9. [Edexcel, 2004] Non-Linear Proportion (Direct and Inverse) **[5 Marks]**

An electrician has wires of the same length made from the same material.
The electrical resistance, R ohms, of a wire is inversely proportional to the square of its radius, r mm.
When $r = 2$, $R = 0.9$

(a) (i) Express R in terms of r.

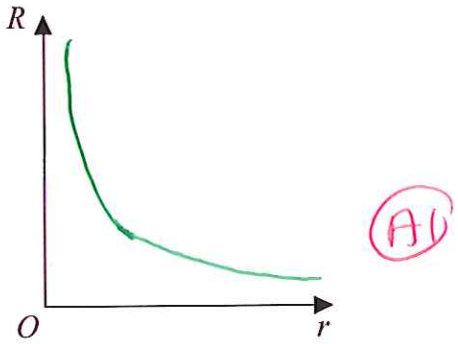
$R = \frac{k}{r^2}$ $(r=2, R=0.9)$

$0.9 = \frac{k}{2^2}$ M1 $\Rightarrow k = 0.9 \times 2^2$

$= 3.6$ B1

$R =$ $\frac{3.6}{r^2}$ A1

(ii) On the axes, sketch the graph of R against r.



A1

(4)

One of the electrician's wires has a radius of 3 mm.

(b) Calculate the electrical resistance of this wire.

$R = \frac{3.6}{r^2}$ $(r=3)$

$= \frac{3.6}{3^2}$

B1

0.4 ohms

(1)

10. [Edexcel, 2015] Non-Linear Proportion (Direct and Inverse) **[5 Marks]**

P is directly proportional to q^3
$P = 270$ when $q = 7.5$

(a) Find a formula for P in terms of q

$P = k \times q^3 \quad (P = 270,\ q = 7.5)$

$\Rightarrow 270 = k \times 7.5^3$ (M1)

$\Rightarrow k = \frac{270}{7.5^3}$

$= 0.64$ (B1) $\longrightarrow P = 0.64q^3$ (A1)

(b) Work out the positive value of q when $P = q$

$P = 0.64q^3 \quad (P = q)$

$\Rightarrow q = 0.64q^3$ (M1)

$\Rightarrow 1 = 0.64q^2$

$\Rightarrow q^2 = \frac{1}{0.64}$ $\longrightarrow q = \sqrt{\frac{1}{0.64}}$

$= 1.25$ (A1)

11. [Edexcel, 2008] Non-Linear Proportion (Direct and Inverse) **[6 Marks]**

The distance, d kilometres, of the horizon from a person is directly proportional to the square root of the person's height, h metres, above sea level.
When $h = 225$, $d = 54$

$d \propto \sqrt{h}$

(a) Find a formula for d in terms of h.

$d = k\sqrt{h} \quad (h = 225, d = 54)$

$54 = k\sqrt{225}$ M1

$\Rightarrow k = \frac{54}{\sqrt{225}} = 3.6$ A1

$d = 3.6\sqrt{h}$ A1

(3)

(b) Calculate the distance of the horizon from a person whose height above sea level is 64 metres.

$d = 3.6\sqrt{h} \quad (h = 64)$

$\Rightarrow d = 3.6 \times \sqrt{64}$

$= 28.8$

28.8 A1 kilometres

(1)

(c) Calculate the height above sea level of a person, when the distance of the horizon is 61.2 kilometres.

$d = 3.6\sqrt{h} \quad (d = 61.2)$

$\Rightarrow 61.2 = 3.6\sqrt{h}$

$\sqrt{h} = \frac{61.2}{3.6}$ M1

$= 17$

$h = 17^2$

$= 289$

289 A1 metres

(2)

12. [Edexcel, 2007] Non-Linear Proportion (Direct and Inverse) **[9 Marks]**

A wind turbine generates a power of P kilowatts when the wind speed is w m/s.

P is proportional to w^3.

$P = 300$ when $w = 12$

(a) Find a formula for P in terms of w.

$P = k \times w^3 \quad (P = 300, w = 12)$

$300 = k \times 12^3$ (M1)

$\Rightarrow k = \frac{300}{12^3}$

$= \frac{25}{144}$ (A1)

(A1) $P = \frac{25}{144} \times w^3$

.....

(3)

(b) Calculate the value of P when $w = 7.5$
Give your answer correct to 3 significant figures.

$P = \frac{25}{144} \times 7.5^3 = 73.242...$

(M1) (A1)

$P =$ 73.2

(2)

(c) When the wind speed is x m/s, the wind turbine generates twice as much power as it does when the wind speed is 10 m/s.
Calculate the value of x.
Give your answer correct to 3 significant figures.

$\frac{25}{144} \times x^3 = 2 \times \frac{25}{144} \times 10^3$ (M1 [EQUATION]) (B1)

$\Rightarrow x^3 = 2 \times 10^3$

$= 2000$ (M1 [ANY])

$x = \sqrt[3]{2000}$

$= 12.599...$

(A1)

$x =$ 12.6

(4)

13. [Edexcel, 2015] Non-Linear Proportion (Direct and Inverse) **[4 Marks]**

A, r and T are three variables.

A is proportional to T^2
A is also proportional to r^3

$A \propto T^2$ AND $A \propto r^3 \Rightarrow T^2 \propto r^3$ ← USE THIS!

$T = 47$ when $r = 0.25$

Find r when $T = 365$
Give your answer correct to 3 significant figures.

$T^2 = kr^3 \quad (T = 47, r = 0.25)$

$\Rightarrow 47^2 = k \times 0.25^3$ (M1)

$\Rightarrow k = \dfrac{47^2}{0.25^3}$

$= 141376$ (M1)

$\Rightarrow T^2 = 141376r^3$ OR $r^3 = \dfrac{T^2}{141376}$ (M1 [EITHER])

$\Rightarrow r = \sqrt[3]{\dfrac{T^2}{141376}}$

$\Rightarrow r = \sqrt[3]{\dfrac{365^2}{141376}}$

$= 0.98040\ldots$

$= 0.980$ (A1)

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B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

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Note that some questions contain the words 'show that', 'show your working out', or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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