

FORMULAE (ADVANCED)

DATE OF SOLUTIONS: 15/05/2018

MAXIMUM MARK: 77

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2015]

Formulae [2 Marks]

Make r the subject of the formula $A = 4\pi r^2$ where r is positive.

$$4\pi r^2 = A$$

$$r^2 = \frac{A}{4\pi}$$

(M) [DIVIDING]

$$r = \sqrt{\frac{A}{4\pi}}$$

(M) [SQUARE ROOT]

Make a the subject of $P = \sqrt{ab}$

$$P^2 = ab \quad (M1)$$

$$a = \frac{P^2}{b}$$

$$a = \frac{P^2}{b} \quad (A1)$$

Make W the subject of the formula $h = \sqrt{\frac{W}{I}}$

$$\sqrt{\frac{W}{I}} = h \Rightarrow \frac{W}{I} = h^2 \rightarrow W = h^2 I$$

(m) (A)

The formula for the curved surface area, A , of a cylinder is

$$A = 2\pi rh$$

where r is the radius and h is the height.

Calculate the value of r when $A = 19.8$ and $h = 2.1$

Give your answer correct to one decimal place.

$$A = 2\pi rh \quad (A = 19.8, h = 2.1)$$

$$19.8 = 2\pi \times r \times 2.1 \quad (m)$$

$$r = \frac{19.8}{2\pi \times 2.1} = 1.5006 \dots$$

$$r = \dots 1.5 \quad (A1)$$

Make x the subject of $3x - y = x + 7$

$$3x - x = 7 + y$$

$$2x = 7 + y$$

$$x = \frac{7 + y}{2}$$

(M1) [x ON ONE SIDE ONLY]

(M0) [SINGLE OCCURANCE OF x]

(A1)

Make h the subject of the formula $A = 2\pi r(r+h)$

$$\begin{aligned} A &= 2\pi r^2 + 2\pi rh \\ \Rightarrow 2\pi rh &= A - 2\pi r^2 \quad (M1) \\ \Rightarrow h &= \frac{A - 2\pi r^2}{2\pi r} \quad (A1) \end{aligned}$$

METHOD 2

$$r+h = \frac{A}{2\pi r} \quad (M1)$$

$$h = \frac{A}{2\pi r} - r \quad (A1)$$

Make y the subject of $3(y + 2x - 1) = x + 5y$

$$3y + 6x - 3 = x + 5y \quad (m1)$$

$$3y - 5y = x - 6x + 3$$

$$-2y = -5x + 3 \quad (m1)$$

$$2y = 5x - 3$$

$$y = \frac{5x - 3}{2}$$

$$y = \frac{5x - 3}{2} \quad (AV)$$

OR $2.5x - 1.5$

Make t the subject of $5(t - g) = 2t + 7$

$$5t - 5g = 2t + 7 \quad \text{(M1) [EXPAND BRACKETS]}$$

$$\left. \begin{aligned} 5t - 2t &= 7 + 5g \\ 3t &= 7 + 5g \end{aligned} \right\} \text{(M2) ['t' TERMS TOGETHER]}$$

$$t = \frac{7 + 5g}{3} \quad \text{(A1) [OR EQUIVALENT]}$$



Diagram **NOT**
accurately drawn

The diagram shows a fish bowl.

The water surface is a circle with a diameter of 16 cm.

- (a) Work out the area of a circle with a diameter of 16 cm.
Give your answer correct to 3 significant figures.

$$\begin{aligned} A &= \pi r^2 \quad (r = 8) \\ &= \pi \times 8^2 \quad (ml) \\ &= 201.0619\dots \end{aligned}$$

$$\begin{aligned} &\dots\dots\dots 201 \quad (AU) \\ &\dots\dots\dots \text{cm}^2 \\ &\quad (2) \end{aligned}$$

- (b) The volume of water, $V \text{ cm}^3$, in the fish bowl may be found using the formula

$$V = \frac{1}{6}\pi h(3x^2 + 3y^2 + h^2)$$

Find the value of V when $h = 16.4$
 $x = 6.5$
and $y = 8$

Give your answer correct to 3 significant figures.

$$V = \frac{1}{6} \times \pi \times 16.4 \times (3 \times 6.5^2 + 3 \times 8^2 + 16.4^2) \quad (ml)$$

$$= 5046.677\dots$$

$$\begin{aligned} &\dots\dots\dots 5050 \text{ cm}^3 \quad (AU) \\ &\dots\dots\dots \quad (2) \end{aligned}$$

$$I = kT^4$$

$$k = 5.67 \times 10^{-8}$$

$$T = 5800$$

(a) Work out the value of I .

Give your answer in standard form correct to 3 significant figures.

$$I = (5.67 \times 10^{-8}) \times 5800^4$$

$$= 64164532.32 \text{ (m)}$$

$$I = \frac{6.42 \times 10^7}{(2)} \text{ (A)}$$

(b) Rearrange the formula $I = kT^4$ to make T the subject.

$$I = kT^4$$

$$T^4 = \frac{I}{k} \text{ (m)}$$

$$T = \sqrt[4]{\frac{I}{k}} \text{ (A)}$$

Make v the subject of the formula $m(v - u) = I$

$$mv - mu = I \quad (m)$$

$$mv = I + mu \quad (m)$$

$$v = \frac{I + mu}{m} \quad (AC)$$

or

$$v = \frac{I}{m} + u$$

Make r the subject of the formula $A = 4r^2 - \pi r^2$ where r is positive.

$$4r^2 - \pi r^2 = A$$

$$r^2(4 - \pi) = A \quad \text{(M1) FACTORISING}$$

$$r^2 = \frac{A}{4 - \pi} \quad \text{(M1) DIVIDING}$$

$$r = \frac{\sqrt{\frac{A}{4 - \pi}}}{\text{(A1)}}$$

Given that y is positive, make y the subject of $y = \sqrt{ay^2 + n}$

Show clear algebraic working.

$$y^2 = ay^2 + n \quad (M1) \text{ [SQUARING]}$$

$$y^2 - ay^2 = n \quad (M1) \text{ [Y-TERMS BOTH ON LEFT]}$$

$$y^2(1-a) = n \quad (M1) \text{ [FACTORISE]}$$

$$y^2 = \frac{n}{1-a} \quad (M1) \text{ [DIVIDE BY FACTOR]}$$

$$\underline{\underline{y = \sqrt{\frac{n}{1-a}}}} \quad (M1) \text{ SQUARE ROOT}$$

Make n the subject of the formula

$$t = \sqrt{\frac{n+3}{n}}$$

$$t^2 = \frac{n+3}{n} \quad (M1) \text{ [SQUARING]}$$

$$\left. \begin{aligned} nt^2 &= n+3 \\ nt^2 - n &= 3 \end{aligned} \right\} (M2) \text{ [EITHER]}$$

$$n(t^2 - 1) = 3 \quad (M1) \text{ [FACTORISING]}$$

$$n = \frac{3}{t^2 - 1} \quad (A1)$$

Make x the subject of $y = \sqrt{\frac{2x+1}{x-1}}$

$$y^2 = \frac{2x+1}{x-1} \quad \text{(M1) [SQUARING]}$$

$$y^2 x - y^2 = 2x + 1 \quad \text{(M1) [NO DENOMINATORS]}$$

$$y^2 x - 2x = 1 + y^2$$

$$x(y^2 - 2) = 1 + y^2 \quad \text{(M1) [REARRANGE AND FACTORISE]}$$

$$x = \frac{1 + y^2}{y^2 - 2} \quad \text{(M1) [DIVIDE]}$$

$$T = \frac{n(1+e)}{(1-e)}$$

- (a) Work out the value of T when $n = 8.6$ and $e = 0.2$

$$T = \frac{8.6(1+0.2)}{1-0.2} \quad \text{(M1)}$$

$$T = \frac{12.9}{0.8} \quad \text{(A1)}$$

(2)

- (b) Make e the subject of the formula $T = \frac{n(1+e)}{(1-e)}$

$$T(1-e) = n(1+e) \quad \text{(M1) MULTIPLYING}$$

$$T - Te = n + ne \quad \text{(M1) EXPAND BRACKETS}$$

$$\left. \begin{aligned} -Te - ne &= n - T \\ Te + ne &= -n + T \end{aligned} \right\} \text{EITHER (M1) 'e' ON ONE SIDE}$$

$$e(T+n) = T-n \quad \text{(M1) FACTORISING}$$

$$e = \frac{T-n}{T+n} \quad \text{(A1)}$$

Make t the subject of the formula $m = \frac{t+1}{t-3}$

$$m = \frac{t+1}{t-3}$$

$$m t - 3m = t + 1 \quad (M1) \text{ [MULTIPLY BY DENOMINATOR]}$$

$$m t - t = 1 + 3m \quad [t\text{-TERMS ON THE SAME SIDE}]$$

$$(M1) \quad t(m-1) = 1 + 3m \quad [\text{FACTORISE TO GET SINGLE } t]$$

$$(M1) \quad t = \frac{1+3m}{m-1} \quad (A1) \text{ [DIVIDE BY BRACKET]}$$

Make g the subject of $3e + 4g = 7 + 9eg$

$$4g - 9eg = 7 - 3e \quad (M) \text{ ['g's ON LHS]}$$

$$g(4 - 9e) = 7 - 3e \quad (M) \text{ [FACTORISING]}$$

$$g = \frac{7 - 3e}{4 - 9e} \quad (A1)$$

Make x the subject of $P = \frac{100(y-x)}{x}$

$$xP = 100(y-x) \quad \text{(M1) [MULTIPLYING BY DENOMINATOR]}$$

$$xP = 100y - 100x \quad \text{(M2) [EXPANDING BRACKETS]}$$

$$xP + 100x = 100y$$

$$x(P+100) = 100y \quad \text{(M3) [FACTORISING]}$$

$$x = \frac{100y}{P+100} \quad \text{(A1)}$$

Make R the subject of the formula $A = \pi(R+r)(R-r)$

$$A = \pi(R^2 - r^2)$$

DIFFERENCE OF TWO SQUARES

$$\Rightarrow R^2 - r^2 = \frac{A}{\pi}$$

$$R^2 = \frac{A}{\pi} + r^2$$

$$R = \sqrt{\frac{A}{\pi} + r^2}$$

[ALSO ACCEPT $R = \sqrt{\frac{A + \pi r^2}{\pi}}$]

$$y = at^2 - 2at$$

$$x = 2a\sqrt{t}$$

Express y in terms of x and a .

Give your answer in the form

$$y = \frac{x^p}{ma^3} - \frac{x^q}{na}$$

where p , q , m and n are integers.

$$x = 2a\sqrt{t} \Rightarrow t = \left(\frac{x}{2a}\right)^2 = \frac{x^2}{4a^2} \quad (B1)$$

$$\Rightarrow t^2 = \frac{x^4}{16a^4} \quad (B1)$$

$$\therefore y = at^2 - 2at$$

$$\Rightarrow y = a \times \frac{x^4}{16a^4} - 2a \times \frac{x^2}{4a^2} \quad (M1)$$

$$= \frac{x^4}{16a^3} - \frac{x^2}{2a} \quad (A1)$$

Make y the subject of $\frac{y}{x} + \frac{2y}{x+4} = 3$

Show your working clearly and give your answer as simply as possible.

$$\textcircled{m1} y(x+4) + 2xy = 3x(x+4) \textcircled{m1}$$

$$\Rightarrow y(x+4+2x) = 3x(x+4) \textcircled{m1}$$

$$y(3x+4) = 3x^2 + 12 \textcircled{m1}$$

$$y = \frac{3x^2 + 12}{3x + 4} \textcircled{A1}$$

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The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or ‘best’ method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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