

FUNCTIONS

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 82

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2014]

Functions [8 Marks]

The functions f and g are defined as

$$f(x) = \frac{1}{2}x + 4$$

$$g(x) = \frac{2x}{x+1}$$

(a) Work out $f(6)$

$$f(6) = \frac{1}{2}(6) + 4$$

$$\frac{7}{(1)} \quad \text{(AI)}$$

(b) Work out $fg(-3)$

1ST $g(-3) = \frac{2 \times (-3)}{(-3)+1} = 3$ (B6)

2ND $f(3) = \frac{1}{2}(3) + 4 = 5.5$ (AI)

(c) $g(a) = -2$

Work out the value of a .

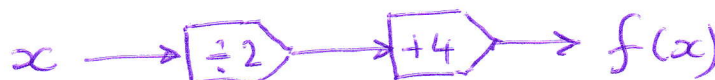
$$\frac{2a}{a+1} = -2 \Rightarrow 2a = -2a - 2$$
$$\Rightarrow 4a = -2$$
$$\Rightarrow a = -\frac{1}{2} \quad \text{(AI)}$$

(mi) [FOR EQUATION]

$$a = \dots \quad \text{(2)}$$

(d) Express the inverse function f^{-1} in the form $f^{-1}(x) = \dots$

FORWARD



BACKWARDS



$$[2(x-4) \leftarrow (x-4) \leftarrow x]$$

$$f^{-1}(x) = \frac{2(x-4)}{(3)} \quad \text{(AI)}$$

The functions f and g are defined as follows.

$$f(x) = \frac{1}{x+2}$$

DENOMINATORS CANNOT BE ZERO

$$g(x) = \sqrt{x-1}$$

CANNOT SQUARE ROOT A NEGATIVE

(a) (i) State which value of x cannot be included in the domain of f .

$$x+2 \neq 0$$

$$x \neq -2$$

-2 (B1)

(ii) State which values of x cannot be included in the domain of g .

$$x-1 \geq 0$$

$$x \geq 1$$

ALLOWED VALUES

[NOT ALLOWED]

$$x < 1$$

(A1) (A1) (3)

(b) Calculate $fg(10)$.

1ST

$$g(10) = \sqrt{10-1}$$

$$= \sqrt{9}$$

$$= 3$$

(B1)

2ND

$$f(3) = \frac{1}{(3)+2}$$

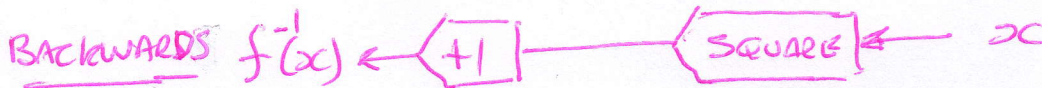
(M1)

$$= \frac{1}{5}$$

$$= 0.2$$

(A1) (3)

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) = \dots$



$$[x^2+1 \leftarrow x^2 \leftarrow x]$$

$$g^{-1}(x) = x^2 + 1$$

(A2)

$$f: x \mapsto 2x - 1$$

$$g: x \mapsto \frac{3}{x}, x \neq 0$$

(a) Find the value of

(i) $f(3)$,

$$f(3) = 2(3) - 1$$

$$\underline{\underline{5}} \quad \text{(BI)}$$

(ii) $fg(6)$.

$$\text{[IST]} \quad g(6) = \frac{3}{6}$$

$$= \underline{\underline{0.5}}$$

$$\text{[2ND]} \quad f(0.5) = 2(0.5) - 1$$

$$\underline{\underline{0}} \quad \text{(BT)}$$

(2)

(b) Express the inverse function f^{-1} in the form $f^{-1}: x \mapsto \dots$

FORWARD $x \rightarrow \boxed{\times 2} \rightarrow \boxed{-1} \rightarrow f(x)$

BACKWARD $f(x) \leftarrow \boxed{\div 2} \leftarrow \boxed{+1} \leftarrow x$

$$\left[\frac{x+1}{2} \leftarrow (x+1) \leftarrow x \right]$$

$$\frac{x+1}{2} \leftarrow \text{(BT)}$$

$$\underline{\underline{\frac{x+1}{2}}} \leftarrow \text{(BI)}$$

(2)

(c) (i) Express the composite function gf in the form $gf: x \mapsto \dots$

$$g[f(x)] = \frac{3}{f(x)}$$

$$= \frac{3}{2x-1}$$

$$\frac{3}{2x-1} \quad \text{(BI)}$$

(ii) Which value of x must be excluded from the domain of gf ?

DENOMINATORS CANNOT BE ZERO

$$\hookrightarrow 2x - 1 \neq 0$$

$$2x \neq 1$$

$$x \neq \frac{1}{2}$$

$$x = \underline{\underline{0.5}} \quad \text{(BI)}$$

[IS NOT ALLOWED]

(2)

The function f is defined as

$$f(x) = \frac{x-6}{2}$$

(a) Find $f(8)$

$$f(8) = \frac{(8)-6}{2} = 1$$

$$\frac{1}{1} \quad \text{(B1)}$$

(1)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = \dots$

FORWARD: $x \rightarrow -6 \rightarrow \div 2 \rightarrow f(x)$

BACKWARDS: $f^{-1}(x) \leftarrow +6 \leftarrow \times 2 \leftarrow x$

$$[2x+6 \leftarrow 2x \leftarrow x]$$

$$f^{-1}(x) = \frac{2x+6}{2} \quad \text{(B1)} \quad \text{(B1)}$$

(2)

The function g is defined as

$$g(x) = \sqrt{x-4}$$

CANNOT SQUARE ROOT
A NEGATIVE

(c) Which values of x cannot be included in a domain of g ?

$$\left. \begin{array}{l} x-4 \geq 0 \\ x \geq 4 \end{array} \right\} \text{ALLOWED}$$

NOT ALLOWED

$$x < 4 \quad \text{(B1)} \quad \text{(B1)}$$

(2)

(d) Express the function gf in the form $gf(x) = \dots$
Give your answer as simply as possible.

$$g[f(x)] = \sqrt{f(x)-4}$$

$$= \sqrt{\frac{x-6}{2} - 4} \quad \text{(M1)}$$

$$= \sqrt{\frac{x-6-8}{2}}$$

$$= \sqrt{\frac{x-14}{2}}$$

$$gf(x) = \sqrt{\frac{x-14}{2}} \quad \text{(A1)}$$

(2)

The function f is defined as $f(x) = \frac{x}{x-1}$.

DENOMINATORS CANNOT BE ZERO

(a) Find the value of

(i) $f(3)$, $f(3) = \frac{(3)}{(3)-1} = \frac{3}{2}$ \rightarrow 1.5 (B1)

(ii) $f(-3)$, $f(-3) = \frac{(-3)}{(-3)-1} = \frac{-3}{-4}$ \rightarrow 0.75 (B1)
(2)

(b) State which value(s) of x must be excluded from the domain of f .

$$\begin{aligned} x-1 &\neq 0 \\ x &\neq 1 \end{aligned} \rightarrow x=1 \text{ [NOT ALLOWED]} \text{ (1)}$$

(c) (i) Find $ff(x)$.

Give your answer in its most simple form.

$$\begin{aligned} f[f(x)] &= \frac{f(x)}{f(x)-1} = \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{\frac{x}{x-1}}{\frac{x-(x-1)}{x-1}} \\ &= \frac{x}{x-(x-1)} = \frac{x}{x-x+1} \end{aligned}$$

$ff(x) = x$ (A1)

(M1) [OR EQUIVALENT]

(ii) What does your answer to (c)(i) show about the function f ?

IF $ff(x) = x$ THE $f(x)$ IS ITS OWN INVERSE!
(B1) (4)

$$f(x) = \frac{2x}{x-1}$$

DENOMINATORS CANNOT BE ZERO

(a) Find the value of $f(11)$

$$f(11) = \frac{2(11)}{(11)-1} = \frac{22}{10}$$

2.2

(BI)

(1)

(b) State which value of x must be excluded from any domain of f

$$x-1 \neq 0$$

$$x \neq 1$$

$x = 1$ [NOT ALLOWED]

(1)

(c) Find $f^{-1}(x)$

$$\text{LET } y = \frac{2x}{x-1}$$

[SWAP x AND y]

$$x = \frac{2y}{y-1} \quad (\text{mi})$$

[REARRANGE TO GET $y = \dots$]

$$x(y-1) = 2y$$

$$xy - x = 2y$$

$$xy - 2y = x$$

$$y(x-2) = x$$

$$y = \frac{x}{x-2}$$

$$\Rightarrow f^{-1}(x) = \frac{x}{x-2} \quad (\text{AI})$$

(3)

(d) State the value which cannot be in any range of f

RANGE OF f IS THE DOMAIN OF f^{-1}

$$\text{For } f^{-1}, x-2 \neq 0$$

$$x \neq 2$$

$x = 2$

[NOT ALLOWED]

(1)

f is the function such that

$$f(x) = \frac{x}{3x+1}$$

DENOMINATORS CANNOT BE ZERO

(a) Find $f(0.5)$

$$f(0.5) = \frac{(0.5)}{3(0.5)+1} = \frac{0.5}{2.5}$$

$$= \frac{1}{5}$$

0.2 (M1)
(1)

(b) Find $ff(-1)$

1ST

$$f(-1) = \frac{(-1)}{3(-1)+1}$$

$$= \frac{-1}{-2}$$

$$= 0.5 \text{ (M1)}$$

2ND

$$f(0.5) = 0.2 \text{ (M1)}$$

[ALREADY DONE THIS!]

(2)

(c) Find the value of x that cannot be included in any domain of f

$$3x+1 \neq 0$$

$$3x \neq -1$$

$$x \neq -\frac{1}{3}$$

$$x = -\frac{1}{3} \text{ (M1) [NOT ALLOWED]}$$

(1)

(d) Express the inverse function f^{-1} in the form $f^{-1}(x) = \dots$
Show clear algebraic working.

LET $y = \frac{x}{3x+1}$

[SWAP x AND y]

$$x = \frac{y}{3y+1} \text{ (M1)}$$

[REARRANGE TO GET $y = \dots$]

$$x(3y+1) = y$$

$$3xy + x = y$$

$$3xy - y = -x$$

$$y(3x-1) = -x$$

$$y = \frac{-x}{3x-1}$$

$$f^{-1}(x) = \frac{x}{1-3x} \text{ (M1) [NEITHER]}$$

(3)

$$f(x) = (x - 1)^2$$

(a) Find $f(8)$

$$f(8) = (8 - 1)^2 = 7^2 \longrightarrow 49 \quad \text{(BI)}$$

(1)

(b) The domain of f is all values of x where $x \geq 7$
Find the range of f .

IF $x \geq 7$

$$\Rightarrow f(x) \geq (7 - 1)^2 \quad \text{(MI)} \longrightarrow f(x) \geq 36 \quad \text{(AI)}$$

(2)

$$g(x) = \frac{x}{x - 1}$$

(c) Solve the equation $g(x) = 1.2$

$$\frac{x}{x - 1} = 1.2 \quad \text{(MI) [FOR EQUATION]}$$

$$\begin{aligned} x &= 1.2x - 1.2 \\ \Rightarrow 0.2x &= 1.2 \\ x &= \underline{\underline{6}} \quad \text{(AI)} \end{aligned}$$

(2)

(d) (i) Express the inverse function g^{-1} in the form $g^{-1}(x) = \dots$

LET $y = \frac{x}{x - 1}$

[SWAP x AND y]

$$x = \frac{y}{y - 1} \quad \text{(MI)}$$

[REARRANGE TO GET $y = \dots$]

$$x(y - 1) = y$$

$$\begin{aligned} xy - x &= y \\ xy - y &= x \\ y(x - 1) &= x \\ y &= \frac{x}{x - 1} \quad \text{(AI)} \end{aligned}$$

(MI) [CARRY]

$$g^{-1}(x) = \frac{x}{x - 1}$$

} This is $g(x)$
so g is ITS OWN INVERSE!

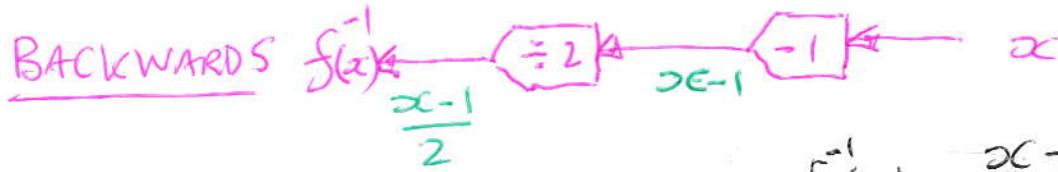
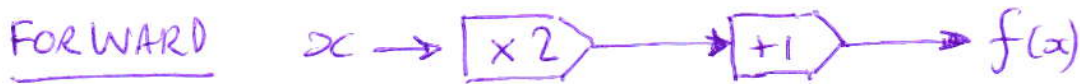
(ii) Hence write down $gg(x)$ in terms of x .

OWN INVERSE MEANS...

$$gg(x) = gg^{-1}(x) = x!$$

$$gg(x) = \dots x \quad \text{(BI)}$$

(a) $f(x) = 2x + 1$

Express the inverse function f^{-1} in the form $f^{-1}(x) = \dots$ 

$$f^{-1}(x) = \frac{x-1}{2} \quad \text{(A1)}$$

(A1)

(b) $g(x) = 2 + x$
 $h(x) = x^2$

Solve the equation $hg(x) = h(x)$.

$$\begin{aligned} h[g(x)] &= h[2+x] \\ &= (2+x)^2 \end{aligned}$$

$\therefore hg(x) = h(x)$ BECOMES $(2+x)^2 = x^2$ (M1) [EQUATION]

$$\Rightarrow (2+x)(2+x) = x^2$$

$$\Rightarrow 4 + 4x + x^2 = x^2 \quad \left. \begin{array}{l} \text{(M1)} \\ \text{[EITHER]} \end{array} \right\}$$

$$\Rightarrow 4 + 4x = 0$$

$$\Rightarrow 4x = -4$$

$$\Rightarrow x = \underline{\underline{-1}} \quad \text{(A1)}$$

The function f is defined as $f(x) = \frac{3}{4+x}$

DENOMINATORS CANNOT BE ZERO!

(a) Find the value of $f(1)$

$$f(1) = \frac{3}{4+(1)} = \frac{3}{5}$$

0.6
(1) (B1)

(b) State which value of x must be excluded from any domain of f .

$$x+4 \neq 0 \Rightarrow x \neq -4$$

$x = -4$ (B1)

[NOT ALLOWED]

The function g is defined as $g(x) = 5+x$

(c) Given that $g(a) = 7$, find the value of a .

$$g(a) = 7$$

$$5+a = 7$$

$$a = 7-5$$

$$= \underline{\underline{2}}$$

$a = 2$ (A1)
(1)

(d) Calculate $fg(1)$

1ST

$$g(1) = 5+(1) = \underline{\underline{6}}$$

(M1)

2ND

$$f(6) = \frac{3}{4+(6)}$$

0.3 (A1)

(2)

(e) Find $fg(x)$

Simplify your answer.

$$f[g(x)] = \frac{3}{4+(5+x)} \quad (M1)$$

$$= \underline{\underline{\frac{3}{9+x}}} \quad (A1)$$

f is a function such that

$$f(x) = \frac{1}{x^2 + 1}$$

(a) Find $f\left(\frac{1}{2}\right)$

$$f(0.5) = \frac{1}{(0.5)^2 + 1} = \frac{1}{0.25 + 1} = \frac{1}{1.25} \rightarrow 0.8 \quad \text{(B1)}$$

(1)

g is a function such that

$$g(x) = \sqrt{x-1} \quad x \geq 1$$

(b) Find $fg(x)$

Give your answer as simply as possible.

$$f[g(x)] = \frac{1}{(\sqrt{x-1})^2 + 1} \quad \text{(M1)}$$

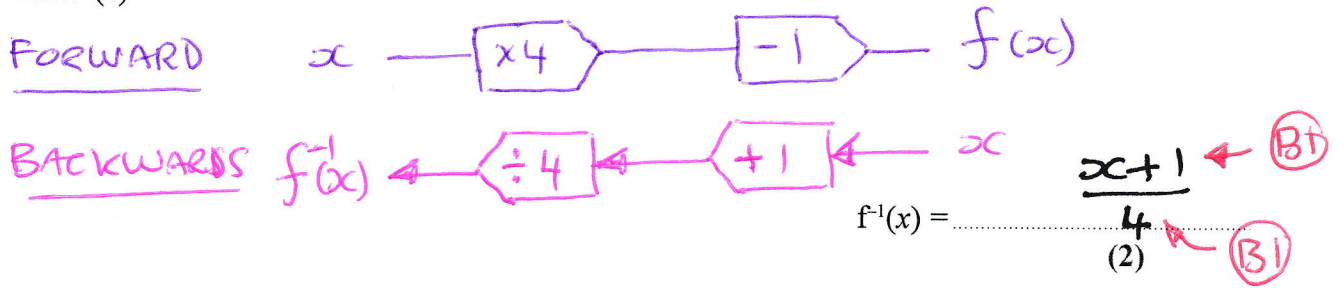
$$= \frac{1}{x-1+1} = \frac{1}{x} \rightarrow fg(x) = \frac{1}{x} \quad \text{(A1)}$$

(2)

The function f is such that

$$f(x) = 4x - 1$$

(a) Find $f^{-1}(x)$



The function g is such that

$$g(x) = kx^2 \text{ where } k \text{ is a constant.}$$

Given that $fg(2) = 12$

(b) work out the value of k

$$g(2) = k \times 2^2$$

$$= \underline{4k}$$

$$f(4k) = 4(4k) - 1$$

$$= \underline{16k - 1} \quad (B1)$$

$$\therefore 16k - 1 = 12$$

$$\Rightarrow 16k = 13$$

$$k = \underline{\underline{\frac{13}{16}}} \quad (A1)$$

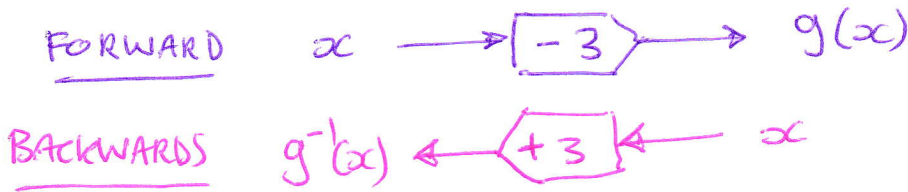
$$f(x) = x^2$$

$$g(x) = x - 3$$

(a) (i) Find $gf(x)$

$$g[f(x)] = (x^2) - 3$$

$x^2 - 3$ (BI)

(ii) Find $g^{-1}(x)$ 

$x + 3$ (BI)

(2)

(b) Solve the equation $gf(x) = g^{-1}(x)$

$$x^2 - 3 = x + 3 \quad \text{(mi) [EQUATION]}$$

$$\Rightarrow x^2 - x - 6 = 0 \quad \text{(mi) [RHS = 0]}$$

$$(x-3)(x+2) = 0$$

$$\begin{array}{l} x-3=0 \\ x=3 \\ \underline{\underline{3}} \end{array} \quad \begin{array}{l} x+2=0 \\ x=-2 \\ \underline{\underline{-2}} \end{array}$$

$x = 3, x = -2$ (AI) [Both]

(3)

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There is no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

Please report any errors or omissions that You find*. These solutions will be updated to correct errors that are discovered. It is recommended that You always check that You have the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or ‘best’ method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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