FUNCTIONS

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 82

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2014] Functions **[8 Marks]**

The functions f and g are defined as

$$f(x) = \frac{1}{2}x + 4$$

$$g(x) = \frac{2x}{x+1}$$

(a) Work out f(6)

$f(6) = \frac{1}{2}(6) + 4$

7 A1

(1)

(b) Work out fg(–3)

1ST $g(-3) = \frac{2\times(-3)}{(-3)+1} = 3$ B1

2ND $f(3) = \frac{1}{2}(3) + 4 \rightarrow 5.5$ A1

(2)

(c) g(*a*) = –2

Work out the value of *a*.

$\frac{2a}{a+1} = -2 \Rightarrow 2a = -2a - 2$

$\Rightarrow 4a = -2$

$\Rightarrow a = -\frac{1}{2}$ A1

M1 [FOR EQUATION]

a = ...

(2)

(d) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

FORWARD

$x \rightarrow \boxed{\div 2} \rightarrow \boxed{+4} \rightarrow f(x)$

BACKWARDS

$f^{-1}(x) \leftarrow \boxed{\times 2} \leftarrow \boxed{-4} \leftarrow x$

$[\,2(x-4) \leftarrow (x-4) \leftarrow x\,]$

A1 A1

$f^{-1}(x) =$ $2(x-4)$

(3)

The functions f and g are defined as follows.

$$f(x) = \frac{1}{x+2}$$

DENOMINATORS CANNOT BE ZERO

$$g(x) = \sqrt{x-1}$$

CANNOT SQUARE ROOT A NEGATIVE

(a) (i) State which value of x cannot be included in the domain of f.

$$x+2 \neq 0$$
$$x \neq -2$$

-2 B1

(ii) State which **values** of x cannot be included in the domain of g.

$$x-1 \geq 0$$
$$x \geq 1$$

ALLOWED VALUES

[NOT ALLOWED]

$x < 1$ A1 A1

(3)

(b) Calculate fg(10)

1ST

$$g(10) = \sqrt{10-1}$$
$$= \sqrt{9}$$
$$= 3 \quad \text{B1}$$

2ND

$$f(3) = \frac{1}{(3)+2} \quad \text{M1}$$
$$= \frac{1}{5}$$

0.2 A1

(3)

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) = ...$

FORWARD $x \rightarrow \boxed{-1} \rightarrow \boxed{\text{SQRT}} \rightarrow f(x)$

BACKWARDS $f^{-1}(x) \leftarrow \boxed{+1} \leftarrow \boxed{\text{SQUARE}} \leftarrow x$

$[x^2+1 \leftarrow x^2 \leftarrow x]$

$g^{-1}(x) = x^2 + 1$ A2

3. [Edexcel, 2004] Functions **[6 Marks]**

$$f : x \mapsto 2x - 1$$

$$g : x \mapsto \frac{3}{x}, \quad x \neq 0$$

(a) Find the value of

(i) f(3),

$f(3) = 2(3) - 1$

5 B1

(ii) fg(6).

$\boxed{\text{1ST}}$ $g(6) = \frac{3}{6}$

$= 0.5$

$\boxed{\text{2ND}}$ $f(0.5) = 2(0.5) - 1$

0 B1

(2)

(b) Express the inverse function f^{-1} in the form $f^{-1} : x \mapsto$ ...

FORWARD $x \rightarrow \boxed{\times 2} \rightarrow \boxed{-1} \rightarrow f(x)$

BACKWARD $f^{-1}(x) \leftarrow \boxed{\div 2} \leftarrow \boxed{+1} \leftarrow x$

$\left[\frac{x+1}{2} \leftarrow (x+1) \leftarrow x \right]$

$\frac{x+1}{2}$ B1 B1

(2)

(c) (i) Express the composite function gf in the form $gf : x \mapsto$ ...

$g[f(x)] = \frac{3}{f(x)}$

$= \frac{3}{2x-1}$

$\frac{3}{2x-1}$ B1

(ii) Which value of x must be excluded from the domain of gf?

$\boxed{\text{DENOMINATORS CANNOT BE ZERO}}$

$2x - 1 \neq 0$

$2x \neq 1$

$x \neq \frac{1}{2}$

B1

$x =$ 0.5 [IS NOT ALLOWED]

(2)

The function f is defined as

$$f(x) = \frac{x-6}{2}$$

(a) Find $f(8)$

$$f(8) = \frac{(8)-6}{2} = 1$$

1 B1

(1)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

FORWARD: $x \rightarrow \boxed{-6} \rightarrow \boxed{\div 2} \rightarrow f(x)$

BACKWARDS: $f^{-1}(x) \leftarrow \boxed{+6} \leftarrow \boxed{\times 2} \leftarrow x$

$[\,2x+6 \leftarrow 2x \leftarrow x\,]$

B1 B1

$f^{-1}(x) =$ $2x + 6$

(2)

The function g is defined as

$$g(x) = \sqrt{x-4}$$ ← CANNOT SQUARE ROOT A NEGATIVE

(c) Which values of x cannot be included in a domain of g?

$\left.\begin{aligned} x - 4 &\geq 0 \\ x &\geq 4 \end{aligned}\right\}$ ALLOWED

NOT ALLOWED

B1 B1

$x < 4$

(2)

(d) Express the function gf in the form $gf(x) = \ldots$
Give your answer as simply as possible.

$$g[f(x)] = \sqrt{f(x) - 4}$$

$$= \sqrt{\frac{x-6}{2} - 4} \quad \text{M1}$$

$$= \sqrt{\frac{x-6-8}{2}}$$

$$= \sqrt{\frac{x-14}{2}}$$

$gf(x) =$ $\sqrt{\dfrac{x-14}{2}}$ A1

(2)

The function f is defined as $f(x) = \dfrac{x}{x-1}$.

DENOMINATORS CANNOT BE ZERO

(a) Find the value of

(i) f(3),

$$f(3) = \frac{(3)}{(3)-1} = \frac{3}{2}$$

1.5 (B1)

(ii) f(–3).

$$f(-3) = \frac{(-3)}{(-3)-1} = \frac{-3}{-4}$$

0.75 (B1)

(2)

(b) State which value(s) of x must be excluded from the domain of f.

$$x - 1 \neq 0$$
$$x \neq 1$$

(B1) $x = 1$ [NOT ALLOWED]

(1)

(c) (i) Find ff(x).
Give your answer in its most simple form.

$$f[f(x)] = \frac{f(x)}{f(x)-1} = \frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} = \frac{\frac{x}{x-1}}{\frac{x-(x-1)}{x-1}}$$

(M1) (M1) [OR EQUIVALENT]

$$= \frac{x}{x-(x-1)} = \frac{x}{x-x+1}$$

ff(x) = x (A1)

(ii) What does your answer to (c)(i) show about the function f?

IF $ff(x) = x$ THE $f(x)$ IS ITS OWN INVERSE!

(B1)

(4)

$$f(x) = \frac{2x}{x-1}$$

DENOMINATORS CANNOT BE ZERO

(a) Find the value of f(11)

$$f(11) = \frac{2(11)}{(11)-1} = \frac{22}{10} \rightarrow 2.2$$

B1

(1)

(b) State which value of x must be excluded from any domain of f

$$x - 1 \neq 0$$
$$x \neq 1 \rightarrow x = 1$$

B1 [NOT ALLOWED]

(1)

(c) Find $f^{-1}(x)$

$$\text{LET } y = \frac{2x}{x-1}$$

[SWAP x AND y]

$$x = \frac{2y}{y-1}$$

M1

[REARRANGE TO GET $y = \ldots$]

$$x(y-1) = 2y$$
$$xy - x = 2y$$
$$xy - 2y = x$$
$$y(x-2) = x$$

M1 [ANY]

$$y = \frac{x}{x-2}$$

$$\Rightarrow f^{-1}(x) = \frac{x}{x-2}$$

A1

(3)

(d) State the value which cannot be in any range of f

RANGE OF f IS THE DOMAIN OF f^{-1}

For f^{-1}, $x - 2 \neq 0$
$$x \neq 2 \rightarrow x = 2$$

B1 [NOT ALLOWED]

(1)

f is the function such that

$$f(x) = \frac{x}{3x+1}$$

DENOMINATORS CANNOT BE ZERO

(a) Find f(0.5)

$$f(0.5) = \frac{(0.5)}{3(0.5)+1} = \frac{0.5}{2.5} = \frac{1}{5}$$

0.2 B1

(1)

(b) Find ff(−1)

1ST $f(-1) = \frac{(-1)}{3(-1)+1} = \frac{-1}{-2} = 0.5$ B1

2ND $f(0.5) = 0.2$ B1 [ALREADY DONE THIS!]

(2)

(c) Find the value of x that cannot be included in any domain of f

$3x+1 \neq 0$

$3x \neq -1$

$x \neq -\frac{1}{3}$

$x = -\frac{1}{3}$ B1 [NOT ALLOWED]

(1)

(d) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$
Show clear algebraic working.

LET $y = \frac{x}{3x+1}$

[SWAP x AND y]

$x = \frac{y}{3y+1}$ M1

[REARRANGE TO GET $y = \ldots$]

$x(3y+1) = y$

$3xy + x = y$

$3xy - y = -x$

$y(3x-1) = -x$

M1 [ANY]

$y = \frac{-x}{3x-1}$

A1 [EITHER]

$f^{-1}(x) = \frac{x}{1-3x}$

(3)

$f(x) = (x-1)^2$

(a) Find f(8)

$f(8) = (8-1)^2 = 7^2 \longrightarrow$ 49 B1

(1)

(b) The domain of f is all values of x where $x \geqslant 7$
Find the range of f.

IF $x \geqslant 7$

$\Rightarrow f(x) \geqslant (7-1)^2$ M1 $\longrightarrow$ $f(x) \geqslant 36$ A1

(2)

$g(x) = \dfrac{x}{x-1}$

(c) Solve the equation $g(x) = 1.2$

$\dfrac{x}{x-1} = 1.2$

M1 [FOR EQUATION]

$x = 1.2x - 1.2$

$\Rightarrow 0.2x = 1.2$

$x = \underline{\underline{6}}$ A1

(2)

(d) (i) Express the inverse function g^{-1} in the form $g^{-1}(x) = \ldots\ldots$

LET $y = \dfrac{x}{x-1}$

[SWAP x AND y]

$x = \dfrac{y}{y-1}$ M1

[REARRANGE TO GET $y = \ldots$]

$x(y-1) = y$

$xy - x = y$

$xy - y = x$

$y(x-1) = x$

M1 [ANY]

$y = \dfrac{x}{x-1}$ A1

$g^{-1}(x) = \dfrac{x}{x-1}$

This is $g(x)$ so g is its own inverse!

(ii) Hence write down $gg(x)$ in terms of x.

$gg(x) = x$ B1

OWN INVERSE MEANS...

$gg(x) = gg^{-1}(x) = x$!

9. [Edexcel, 2010] Functions **[5 Marks]**

(a) $f(x) = 2x + 1$

Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

FORWARD $\quad x \to \boxed{\times 2} \to \boxed{+1} \to f(x)$

BACKWARDS $\quad f^{-1}(x) \xleftarrow{\frac{x-1}{2}} \boxed{\div 2} \xleftarrow{x-1} \boxed{-1} \leftarrow x$

$$f^{-1}(x) = \frac{x-1}{2}$$ A1 A1

(b) $g(x) = 2 + x$
$h(x) = x^2$

Solve the equation $hg(x) = h(x)$.

$$h[g(x)] = h[2+x]$$
$$= (2+x)^2$$

$\therefore$ $hg(x) = h(x)$ BECOMES $(2+x)^2 = x^2$ M1 [EQUATION]

$$\Rightarrow (2+x)(2+x) = x^2$$
$$\Rightarrow 4 + 4x + x^2 = x^2$$
$$\Rightarrow 4 + 4x = 0$$ M1 [EITHER]
$$\Rightarrow 4x = -4$$
$$\Rightarrow x = \underline{\underline{-1}}$$ A1

The function f is defined as $f(x) = \dfrac{3}{\boxed{4 + x}}$

DENOMINATORS CANNOT BE ZERO!

(a) Find the value of f(1)

$$f(1) = \frac{3}{4+(1)} = \frac{3}{5}$$

B1

0.6

(1)

(b) State which value of x must be excluded from any domain of f.

B1

$$x + 4 \neq 0 \Rightarrow x \neq -4$$

$x = -4$

[NOT ALLOWED]

The function g is defined as $g(x) = 5 + x$

(c) Given that $g(a) = 7$, find the value of a.

$$g(a) = 7$$
$$5 + a = 7$$
$$a = 7 - 5$$
$$= \underline{\underline{2}}$$

A1

$a =$ 2

(1)

(d) Calculate fg(1)

1ST

$$g(1) = 5 + (1)$$
$$= \underline{\underline{6}}$$

M1

2ND

$$f(6) = \frac{3}{4+(6)}$$

A1

0.3

(2)

(e) Find fg(x)
Simplify your answer.

$$f[g(x)] = \frac{3}{4 + (5+x)}$$

M1

$$= \underline{\underline{\frac{3}{9+x}}}$$

A1

f is a function such that

$$f(x) = \frac{1}{x^2 + 1}$$

(a) Find $f(\frac{1}{2})$

$$f(0.5) = \frac{1}{(0.5)^2+1} = \frac{1}{0.25+1} = \frac{1}{1.25} \rightarrow 0.8$$ B1

(1)

g is a function such that

$g(x) = \sqrt{x-1} \quad x \geqslant 1$

(b) Find $fg(x)$
Give your answer as simply as possible.

$$f[g(x)] = \frac{1}{(\sqrt{x-1})^2+1}$$ M1

$$= \frac{1}{x-1+1} = \frac{1}{x} \rightarrow$$

$fg(x) = \frac{1}{x}$ A1

(2)

12. [Edexcel, 2017] Functions **[4 Marks]**

The function f is such that

$$f(x) = 4x - 1$$

(a) Find $f^{-1}(x)$

FORWARD $x \rightarrow \boxed{\times 4} \rightarrow \boxed{-1} \rightarrow f(x)$

BACKWARDS $f^{-1}(x) \leftarrow \boxed{\div 4} \leftarrow \boxed{+1} \leftarrow x$

$f^{-1}(x) =$ $\dfrac{x+1}{4}$ B1 B1

(2)

The function g is such that

$$g(x) = kx^2 \text{ where } k \text{ is a constant.}$$

Given that $fg(2) = 12$

(b) work out the value of k

$g(2) = k \times 2^2$

$= 4k$

$f(4k) = 4(4k) - 1$

$= 16k - 1$ B1

$\therefore\ 16k - 1 = 12$

$\Rightarrow\ 16k = 13$

$k = \dfrac{13}{16}$ A1

13. [Edexcel, 2009] Functions **[5 Marks]**

$$f(x) = x^2$$
$$g(x) = x - 3$$

(a) (i) Find gf(x)

$g[f(x)] = (x^2) - 3 \longrightarrow x^2 - 3$ (B1)

(ii) Find $g^{-1}(x)$

FORWARD $x \longrightarrow \boxed{-3} \longrightarrow g(x)$

BACKWARDS $g^{-1}(x) \longleftarrow \boxed{+3} \longleftarrow x$

$x+3$ (B1)

(2)

(b) Solve the equation $gf(x) = g^{-1}(x)$

$x^2 - 3 = x + 3$ (M1) [EQUATION]

$\Rightarrow x^2 - x - 6 = 0$ (M1) [RHS = 0]

$(x-3)(x+2) = 0$

$x - 3 = 0$ $\quad$ $x + 2 = 0$

$x = 3$ $\quad$ $x = -2$

$x = 3, \; x = -2$ (A1) [BOTH]

(3)

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Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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