

HCF AND LCM

DATE OF SOLUTIONS: 15/05/2018

MAXIMUM MARK: 44

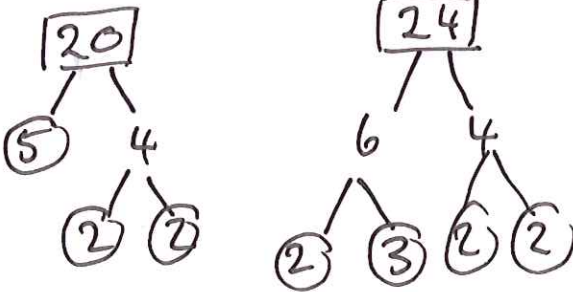
SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2014]

HCF and LCM [2 Marks]

Find the Lowest Common Multiple (LCM) of 20 and 24

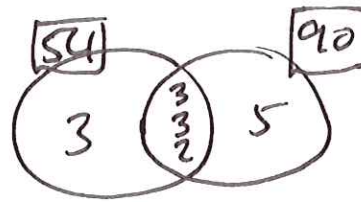
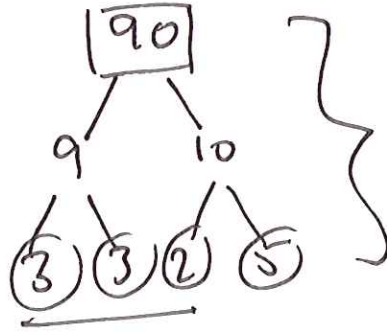
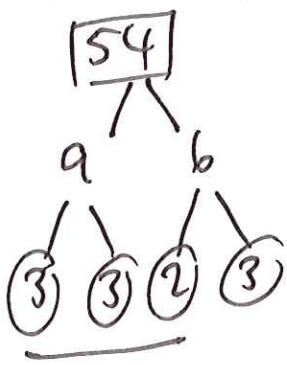


$$\begin{aligned} 20 &= 2^2 \times 5 \\ 24 &= 2^3 \times 3 \end{aligned}$$

$$= 2^3 \times 3 \times 5$$
$$= \underline{\underline{120}}$$

Handwritten notes: A red bracket groups the 5 in the first equation and the 3 in the second equation, with a circled 'M' next to it. A circled 'A1' is next to the final answer 120.

(a) Find the Highest Common Factor (HCF) of 54 and 90



$$\text{HCF} = 3 \times 3 \times 2$$

(mi)

$$\begin{array}{r} 18 \\ \hline (2) \end{array}$$

(AI)

(b) Find the Lowest Common Multiple (LCM) of 54 and 90



$$\text{LCM} = 3 \times (3 \times 3 \times 2) \times 5$$

(mi)

$$\begin{array}{r} 270 \\ \hline (2) \end{array}$$

(AI)

The highest common factor (HCF) of 140 and x is 20

The lowest common multiple (LCM) of 140 and x is 420

Find the value of x .

FOR ANY TWO NUMBERS
 $A \times B = \text{HCF} \times \text{LCM}$

$$140x = 20 \times 420 \quad (\text{ml})$$

$$\Rightarrow x = \frac{20 \times 420}{140}$$

$$= \underline{\underline{60}} \quad (\text{AI})$$

[LOTS OF OTHER METHODS ARE POSSIBLE!]

$$S = 2^4 \times 3 \times 7^2$$

$$T = 2 \times 5^3 \times 7^3$$

Find the Highest Common Factor (HCF) of S and T .

$$\begin{array}{r} S = 2^4 \times 3 \qquad \qquad \qquad 7^2 \\ T = 2 \qquad \qquad \times 5^3 \times 7^3 \\ \hline 2 \qquad \qquad \qquad \qquad \qquad 7^2 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{PRIME FACTORS} \\ \text{MATCHED UP} \end{array}$$

\leftarrow COMMON FACTORS

$$\begin{aligned} \text{HCF} &= 2 \times 7^2 \text{ (m)} \\ &= \underline{\underline{98}} \text{ (A)} \end{aligned}$$

$$A = 2^3 \times 3^2 \times 5^4 = 2 \times 2 \times 2 \times \textcircled{3} \times \textcircled{3} \times 5 \times \textcircled{5} \times 5 \times 5$$

$$B = 3^5 \times 5 \times 7^3 = 3 \times 3 \times \textcircled{3} \times \textcircled{3} \times 3 \times 5 \times 7 \times 7 \times 7$$

Find the Highest Common Factor (HCF) of A and B .

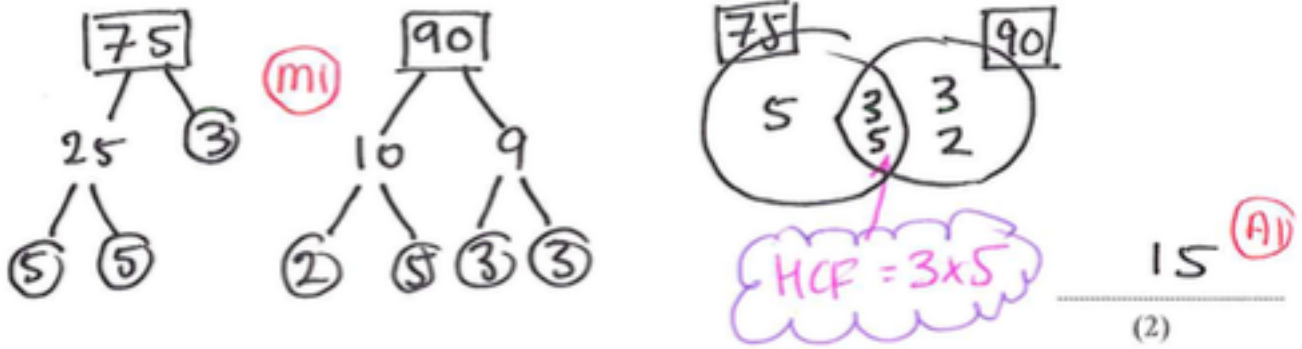
COMMON FACTORS ARE
(MATCHING)

3, 3, 5

$$\therefore \text{HCF} = 3 \times 3 \times 5$$

45 AI

(a) Find the Highest Common Factor (HCF) of 75 and 90



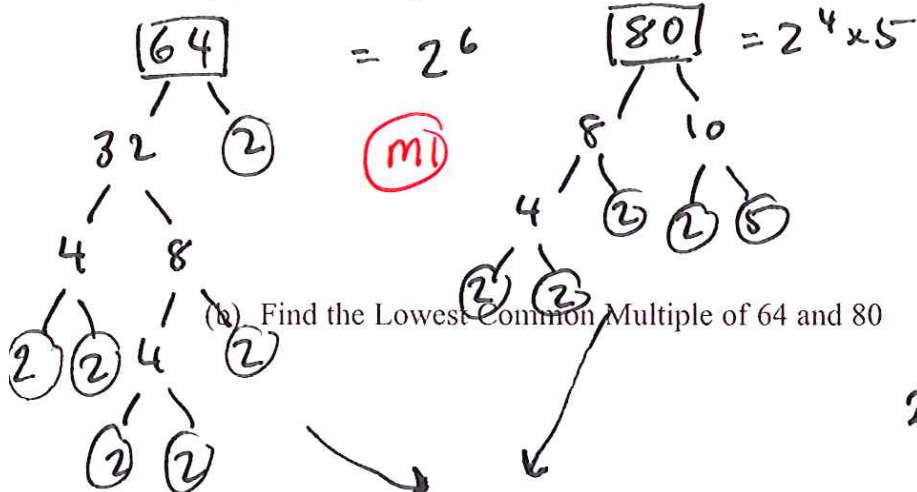
(b) Find the Lowest Common Multiple (LCM) of 75 and 90

Handwritten formula in a pink cloud: $HCF \times LCM = A \times B$

Handwritten calculation: $\Rightarrow LCM = \frac{75 \times 90}{15}$ (m1)

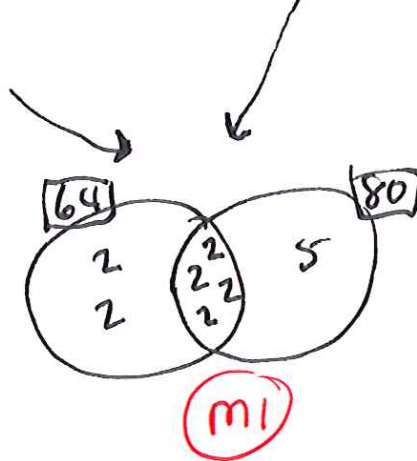
Handwritten calculation: $\frac{450}{15}$ (A1) over a horizontal line with (2) below it.

(a) Find the Highest Common Factor of 64 and 80



$$\frac{2^4 = 16}{(2)} \quad (AI)$$

(b) Find the Lowest Common Multiple of 64 and 80



$$2 \times 2 \times (2 \times 2 \times 2 \times 2) \times 5$$

$$\frac{320}{(2)} \quad (AI)$$

x is an integer.

The Lowest Common Multiple (LCM) of x and 12 is 120

The Highest Common Factor (HCF) of x and 12 is 4

Work out the value of x .

$$\text{HCF} \times \text{LCM} = A \times B$$

$$12 \times x = \text{HCF} \times \text{LCM}$$

$$\Rightarrow 12x = 4 \times 120 \quad (m1)$$

$$12x = 480$$

$$x = \frac{480}{12}$$

$$= \underline{\underline{40}} \quad (A1)$$

Given that $A = 2^3 \times 3$ and $B = 2^2 \times 3^2$

find the Lowest Common Multiple (LCM) of A and B .

$$\text{HCF} = 2^2 \times 3$$

$$\therefore \text{LCM} = \frac{2^3 \times 3 \times 2^2 \times 3^2}{2^2 \times 3}$$

(M1) [FOR ANY
USEFUL
METHOD]

$$= 2^3 \times 3^2$$

$$= \underline{\underline{72}} \leftarrow \text{(A1) EITHER}$$

$$A = 2^4 \times 3^2 \times 7^3$$

$$B = 2^2 \times 3^5 \times 5^2$$

- (a) Find the highest common factor (HCF) of A and B

$$\begin{array}{r}
 2^4 \times 3^2 \times 7^3 \\
 2^2 \times 3^5 \times 5^2 \\
 \hline
 \text{Lowest Powers } 2^2 \times 3^2 \text{ (m)} \rightarrow \text{(HCF)} \\
 \text{Highest Powers } 2^4 \times 3^5 \times 5^2 \times 7^3 \text{ (LCM)} \dots\dots\dots 36 \text{ (A)} \\
 \text{(m)} \dots\dots\dots (2)
 \end{array}$$

- (b) Find the lowest common multiple (LCM) of A and B

$$\begin{array}{r}
 \dots\dots\dots \text{ (A)} \\
 33339600 \\
 \hline
 \dots\dots\dots (2)
 \end{array}$$

$$3780 = 2^2 \times 3^3 \times 5 \times 7$$

$$3240 = 2^3 \times 3^4 \times 5$$

- (a) Find the highest common factor (HCF) of 3780 and 3240
Give your answer as a product of prime factors.

COMMON PRIME FACTORS:-

$$3780 = 2^2 \times 3^3 \times 5 \times 7$$

$$3240 = 2^3 \times 3^4 \times 5$$

[SMALLEST POWERS] $2^2 \times 3^3 \times 5$

[LARGEST POWERS] $2^3 \times 3^4 \times 5 \times 7$

I LIKE THIS METHOD!



HCF = 540

(2)

- (b) Find the lowest common multiple (LCM) of 3780 and 3240
Give your answer as a product of prime factors.

$3780 \times 3240 = \text{HCF} \times \text{LCM}$

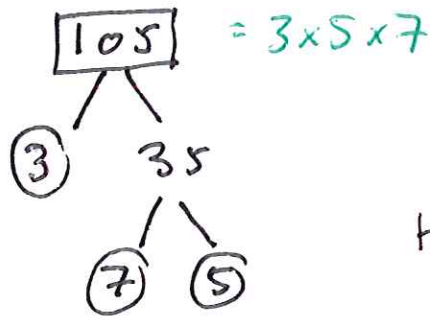
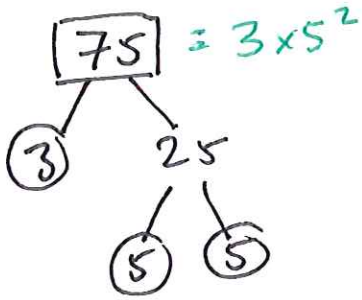
$$\text{LCM} = \frac{3780 \times 3240}{\text{HCF}}$$

$$= \frac{3780 \times 3240}{540}$$

= 22680

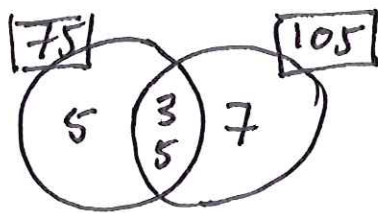
(2)

(a) Find the Highest Common Factor of 75 and 105.



$$\begin{aligned} \text{HCF} &= 3 \times 5 \quad (\text{ms}) \\ &= \underline{15} \quad (\text{As}) \\ &\quad (2) \end{aligned}$$

(b) Find the Lowest Common Multiple of 75 and 105.



$$\text{LCM} = 5 \times (3 \times 5) \times 7 \quad (\text{ms})$$

$$= \underline{\underline{525}} \quad (\text{As})$$

$$267\,300 = 2^2 \times 3^5 \times 5^2 \times 11$$

$$246\,960 = 2^4 \times 3^2 \times 5 \times 7^3$$

- (a) Find the highest common factor (HCF) of 267 300 and 246 960
Give your answer as a product of prime factors.

LOWEST OF
EACH POWER

$$= 2^2 \times 3^2 \times 5 \times 7^0 \times 11^0$$

$$\underline{2^2 \times 3^2 \times 5} \quad \text{(A2)}$$

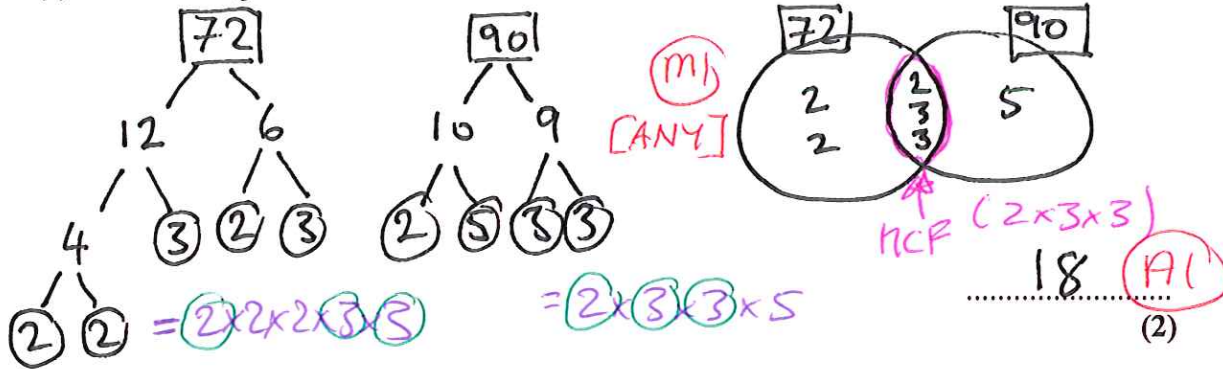
(2)

- (b) Find the lowest common multiple (LCM) of 267 300 and 246 960
Give your answer as a product of prime factors.

HIGHEST OF
EACH POWER

$$= \underline{\underline{2^4 \times 3^5 \times 5^2 \times 7^3 \times 11}} \quad \text{(A2)}$$

(a) Find the Highest Common Factor of 72 and 90



(b) Find the Lowest Common Multiple of 72 and 90

Handwritten solution for finding the Lowest Common Multiple (LCM) of 72 and 90:

Formula: $HCF \times LCM = A \times B$

Calculation: $\Rightarrow LCM = \frac{72 \times 90}{18}$

Final answer: 360 (2)

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While reasonable endeavours have been used to verify the accuracy of these solutions, these solutions are provided on an “as is” basis and no warranties are made of any kind, whether express or implied, in relation to these solutions.

There is no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

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The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or ‘best’ method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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