

HISTOGRAMS

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 60

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2015]

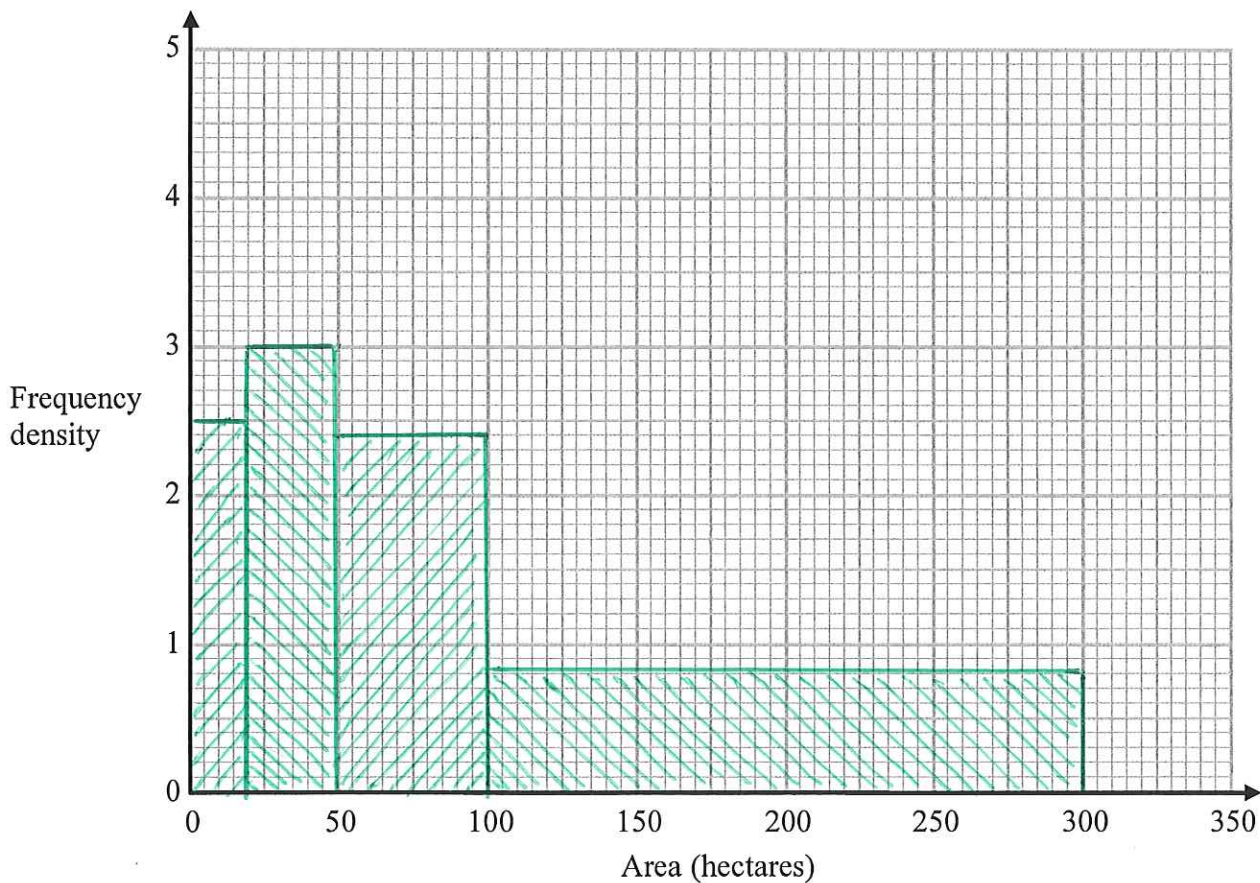
Histograms [3 Marks]

The table gives information about the areas of some farms in France.

Area (A hectares)	Frequency	WIDTH	FREQ. DEN
$0 < A \leq 20$	50	20	2.5
$20 < A \leq 50$	90	30	3
$50 < A \leq 100$	120	50	2.4
$100 < A \leq 300$	160	200	0.8

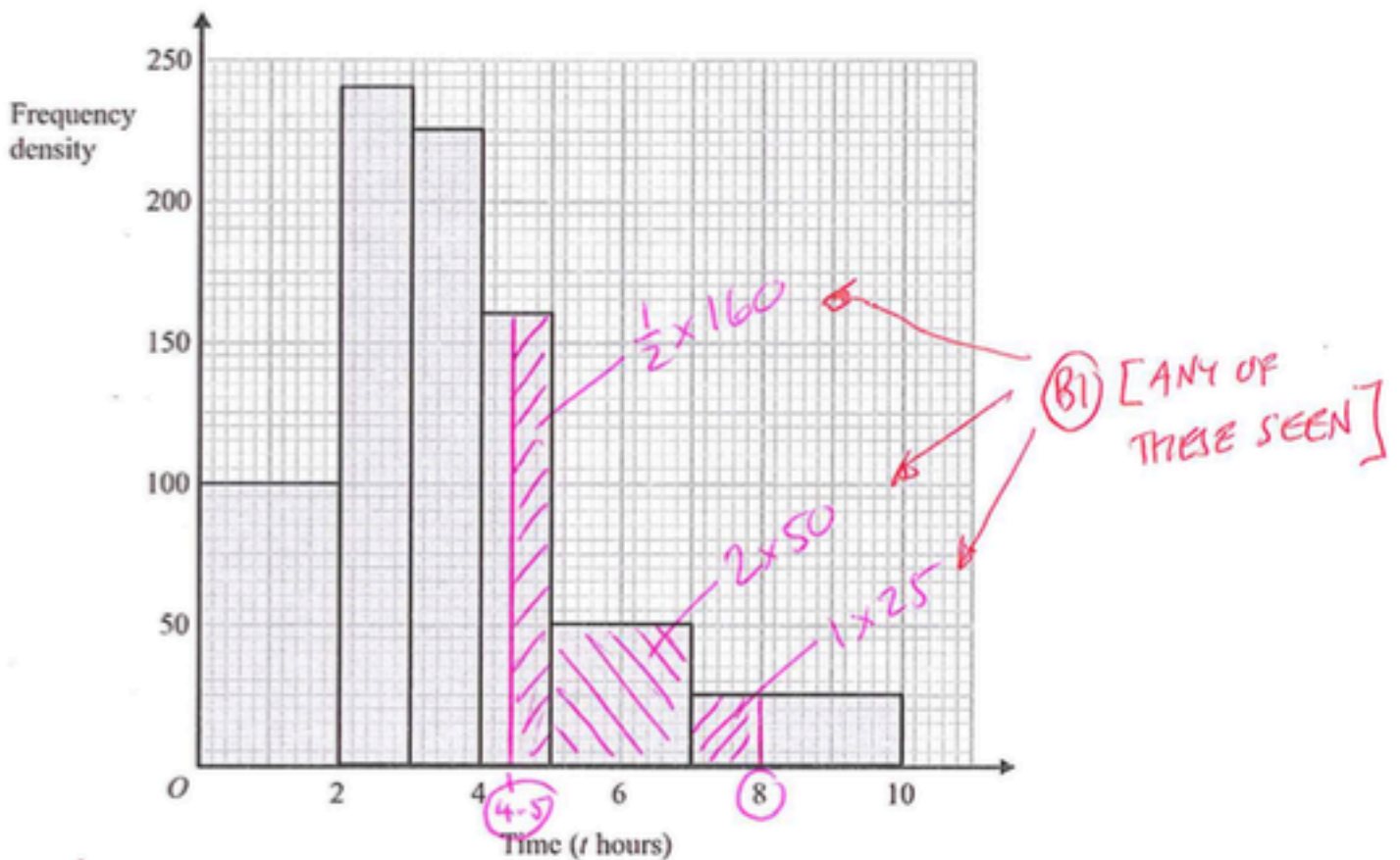
USE
$$\text{FREQUENCY DENSITY} = \frac{\text{FREQUENCY}}{\text{WIDTH}}$$

On the grid, draw a histogram to show this information.



TWO CORRECT BARS \Rightarrow (A1)
THREE " " \Rightarrow (A2)
ALL FOUR CORRECT \Rightarrow (A3)

The histogram shows information about the times, t hours, for which some cars were left in a car park.



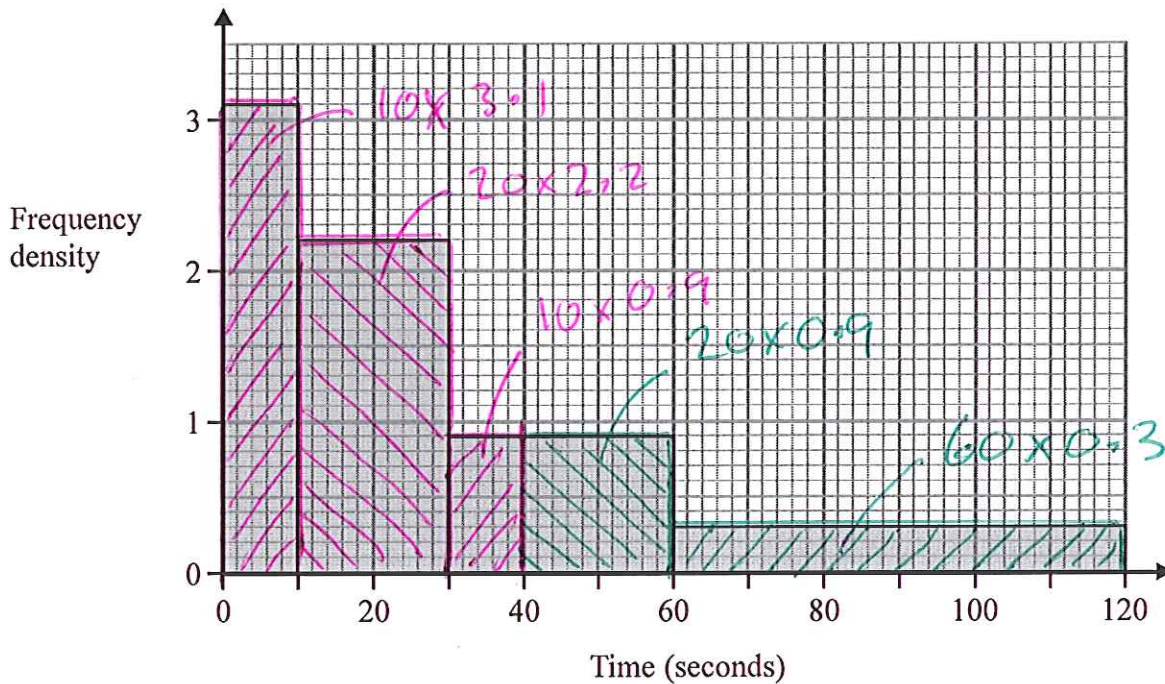
Calculate an estimate for the number of cars which were left in the car park for between 4.5 hours and 8 hours.

NUMBER OF CARS = AREA OF BARS

$$= \frac{1}{2} \times 160 + 2 \times 50 + 1 \times 25 \quad (m6)$$

$$= \underline{\underline{205}} \quad (A1)$$

The histogram shows information about the times taken by a telephone call centre to answer incoming calls.



Work out an estimate for the percentage of calls that are answered in less than 40 seconds.

CALLS ANSWERED IN LESS THAN 40 SECONDS

$$10 \times 3.1 + 20 \times 2.2 + 10 \times 0.9 = \underline{\underline{84}} \text{ (B1)}$$

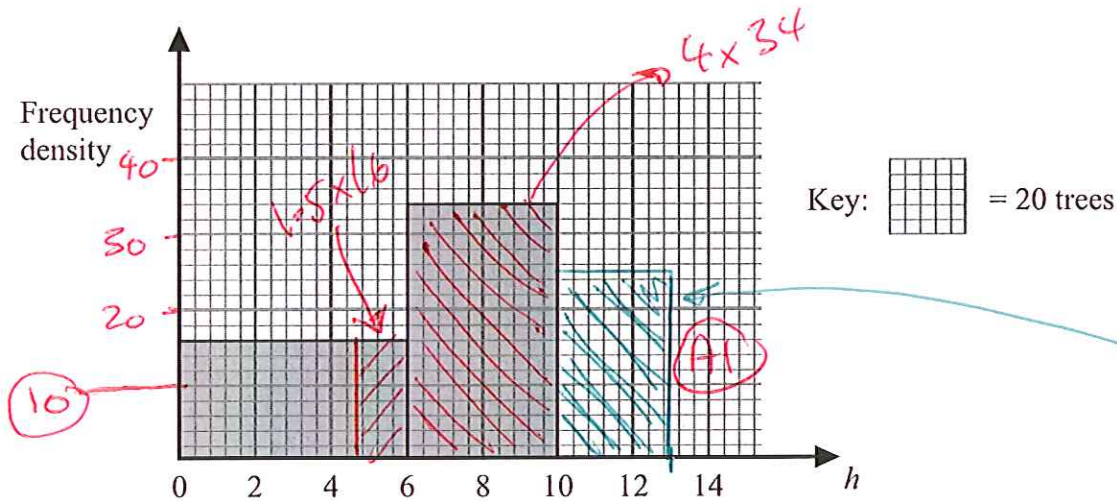
TOTAL CALLS ANSWERED

$$10 \times 3.1 + \dots + \dots + 60 \times 0.3 = \underline{\underline{120}} \text{ (B1)}$$

$$\% = \frac{84}{120} \times 100$$

$$= \underline{\underline{70\%}} \text{ (A1)}$$

The unfinished histogram shows information about the heights, h metres, of some trees.
A key is also shown.



- (a) Calculate an estimate for the number of trees with heights in the interval $4.5 < h \leq 10$

$$1.5 \times 16 + 4 \times 34$$

(mi) (mi)

$$\underline{\quad 160 \quad} \text{ (AI)}$$

(3)

- (b) There are 75 trees with heights in the interval $10 < h \leq 13$
Use this information to complete the histogram.

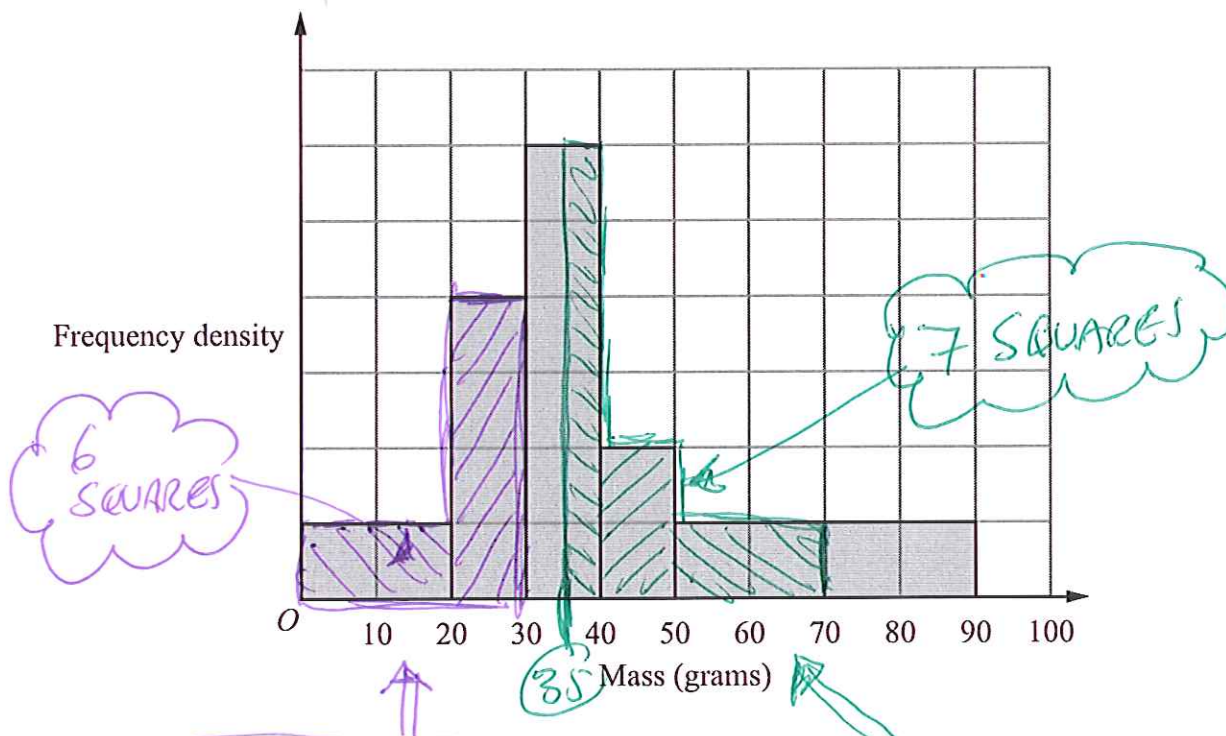
←

$$\text{WIDTH} = 3$$

(2)

$$\begin{aligned} \Rightarrow \text{FREQ DENSITY} &= \frac{75}{3} \text{ (mi)} \\ &= \underline{\underline{25}} \end{aligned}$$

The histogram shows information about the masses, in grams, of some stones.



There are 120 stones with masses less than 30 g.

Calculate an estimate of the number of stones with masses between 35 g and 70 g.

1ST

6 SQUARES REPRESENT 120 STONES.

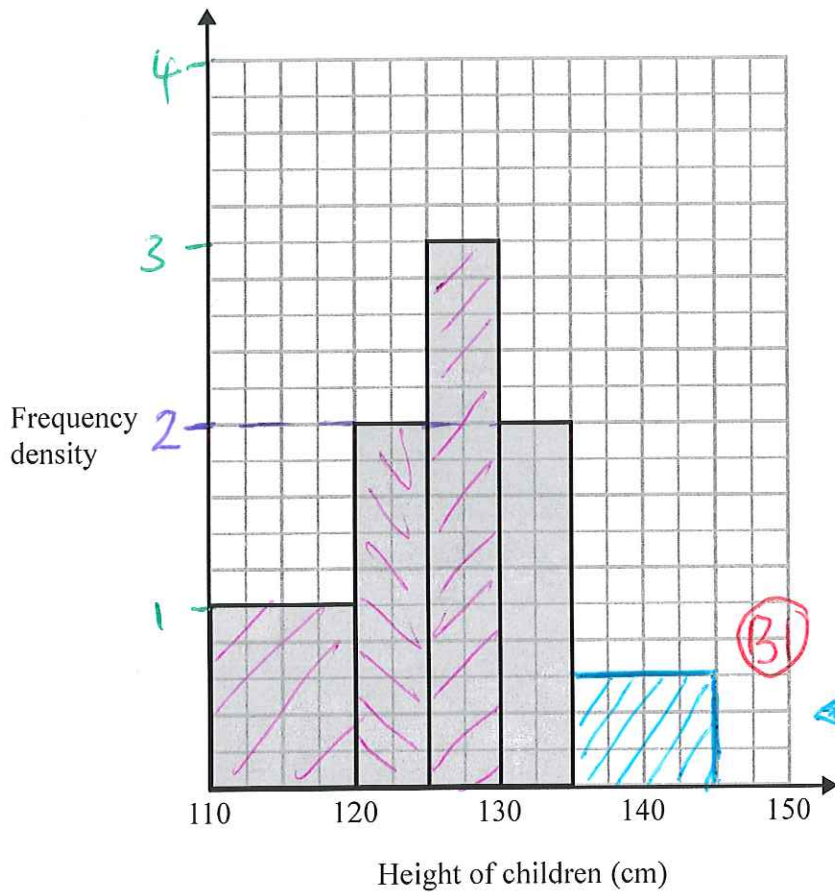
\therefore 1 SQUARE REPRESENTS 20 STONES

2ND

THERE ARE 7 SQUARES BETWEEN
35 AND 70

\therefore NUMBER OF STONES = 7×20
= 140

The incomplete histogram shows information about the heights of a group of children.



There were 10 children with heights between 130cm and 135cm. → Area = 10!

(a) How many children had heights between 110cm and 130cm?

Handwritten calculations for part (a):

- A diagram shows a rectangle with width 10 (from 130 to 135) and height 2. It is labeled "1ST" at the top and "10" inside. Below it are the numbers "130 135".
- Equation: $HEIGHT = \frac{10}{5} = 2$
- Equation: $10 \times 1 + 5 \times 2 + 5 \times 3$
- Result: 35
- Notes: "2ND" in a pink box, "MI" in a red circle, "AI" in a red circle, and "[ALLOW COUNTING SQUARES]" in red.

There were 6 children with heights between 135cm and 145cm.

(b) Show this information on the histogram.

Handwritten calculations for part (b):

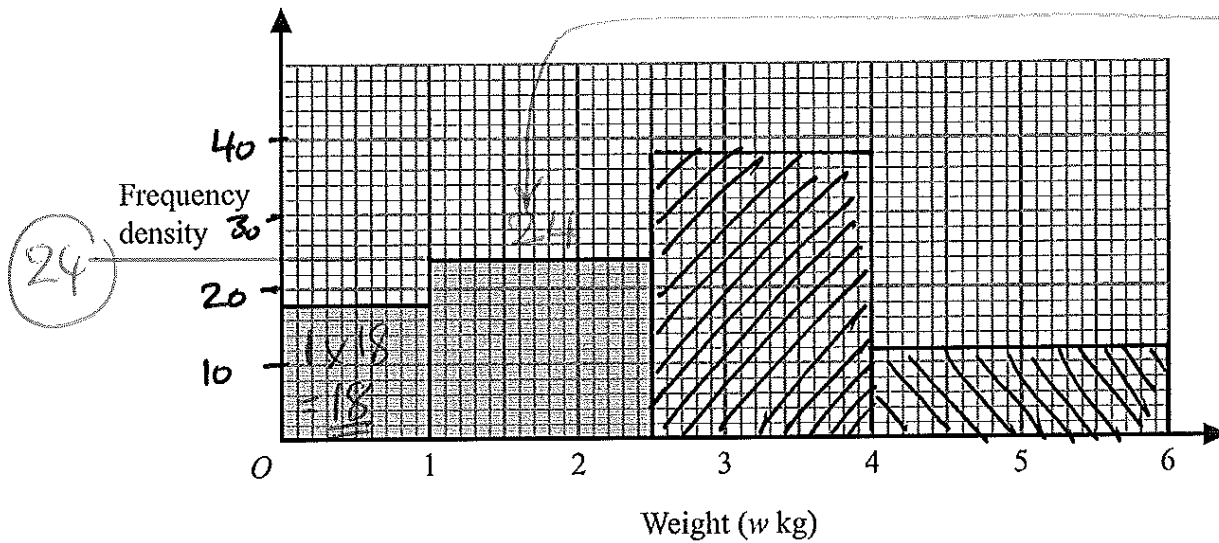
- Equation: $WIDTH = 145 - 135$
- Equation: $= 10$
- Equation: $HEIGHT OF BAR = \frac{6}{10}$
- Result: $\frac{6}{10}$
- Note: "(1)" in a red circle.

The incomplete table and histogram show information about the weights of some books.

Weight (w kg)	Frequency
$0 < w \leq 1$	18
$1 < w \leq 2.5$	36
$2.5 < w \leq 4$	57
$4 < w \leq 6$	24

WIDTH	F. DENSITY
1	
1.5	$\frac{36}{1.5} = 24$
1.5	$\frac{57}{1.5} = 38$
2	$\frac{24}{2} = 12$

ON
HISTOGRAM



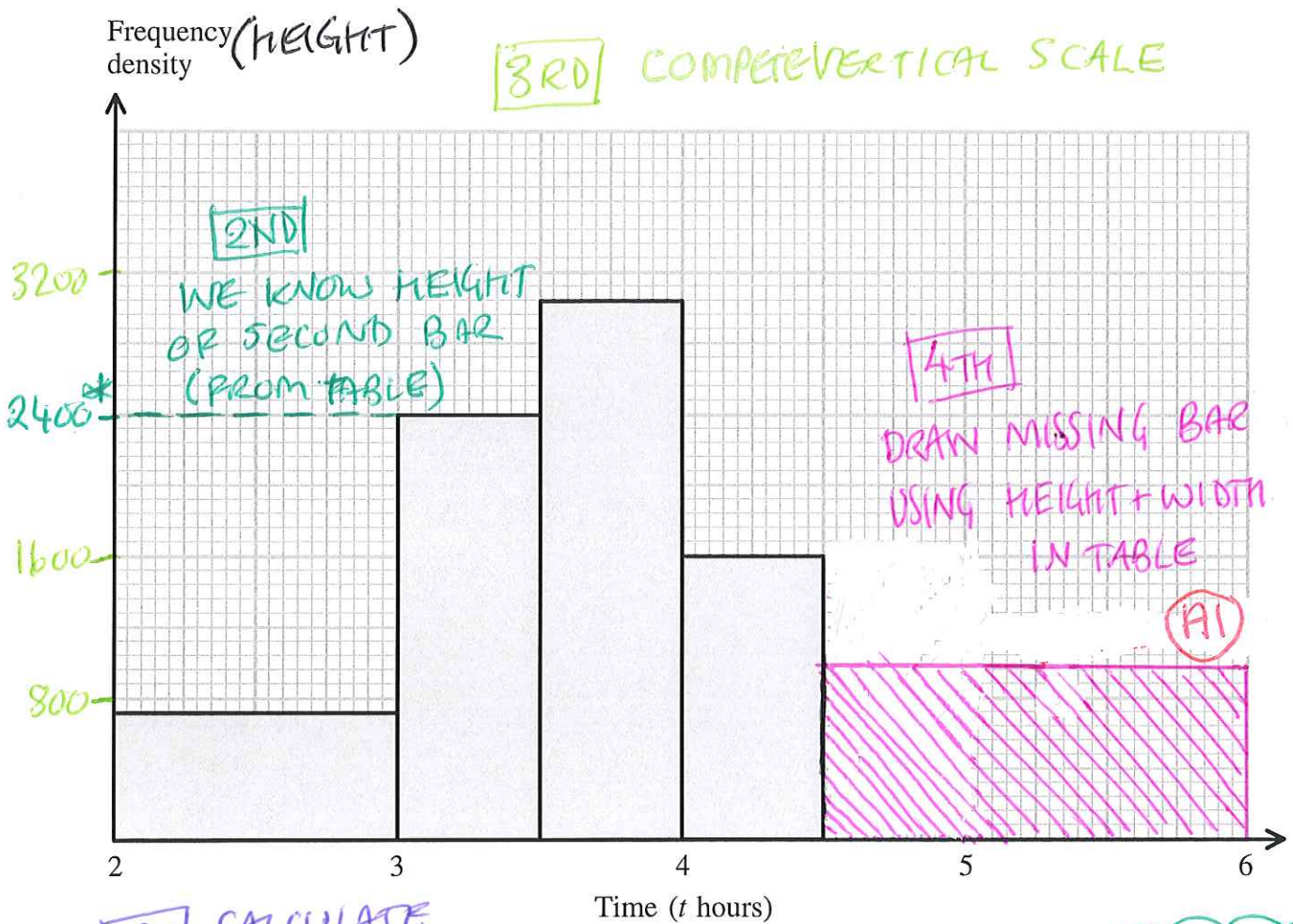
(a) Use the information in the histogram to complete the table.

(1)

(b) Use the information in the table to complete the histogram.

(2)

The unfinished histogram and table give information about the times, in hours, taken by runners to complete the Mathstown Marathon.



1ST] CALCULATE WIDTHS & HEIGHTS

WIDTH	Time (t hours)	(AREA) Frequency	HEIGHT
1	$2 \leq t < 3$	720 (B1)	720
0.5	$3 \leq t < 3.5$	1200	2400*
0.5	$3.5 \leq t < 4$	1520 (B1)	3040
0.5	$4 \leq t < 4.5$	800	1600
* 1.5	$4.5 \leq t < 6$	1440	960*

$$H = \frac{\text{AREA}}{\text{WIDTH}}$$

5TH] COMPLETE MISSING HEIGHTS & AREAS

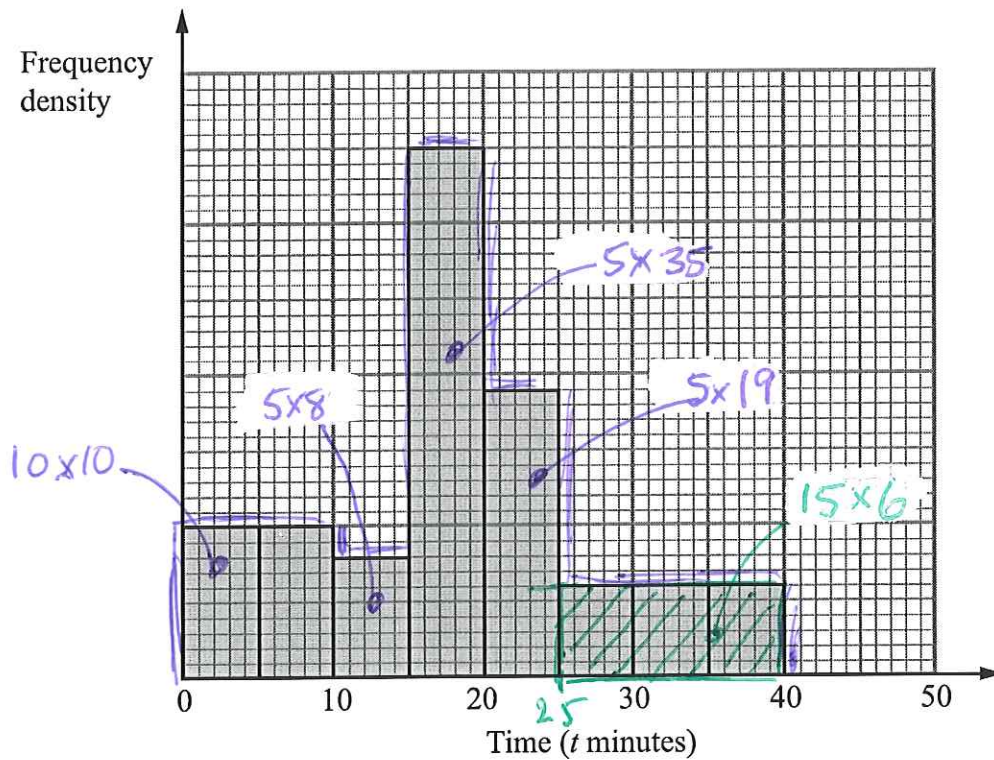
(a) Use the histogram to complete the table.

(2)

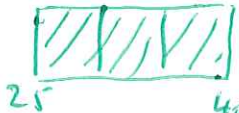
(b) Use the table to complete the histogram.


(1)

The histogram shows information about the times, t minutes, patients spent at a doctors' surgery on one day.
No patient spent more than 40 minutes at the surgery.



- (a) Calculate the percentage of the patients who spent between 25 and 40 minutes at the surgery.


 $AREA = 15 \times 6 = \underline{90} \text{ (M1)}$


 $TOTAL \text{ AREA} = 100 + 40 + 175 + 95 + 90 = \underline{500} \text{ (M1)}$

$\left. \begin{array}{l} 90 \\ 500 \end{array} \right\} \times 100$
 \downarrow
 $\underline{18} \text{ (A1)} \%$
 (3)

- (b) 16 patients spent between 10 and 15 minutes at the surgery.

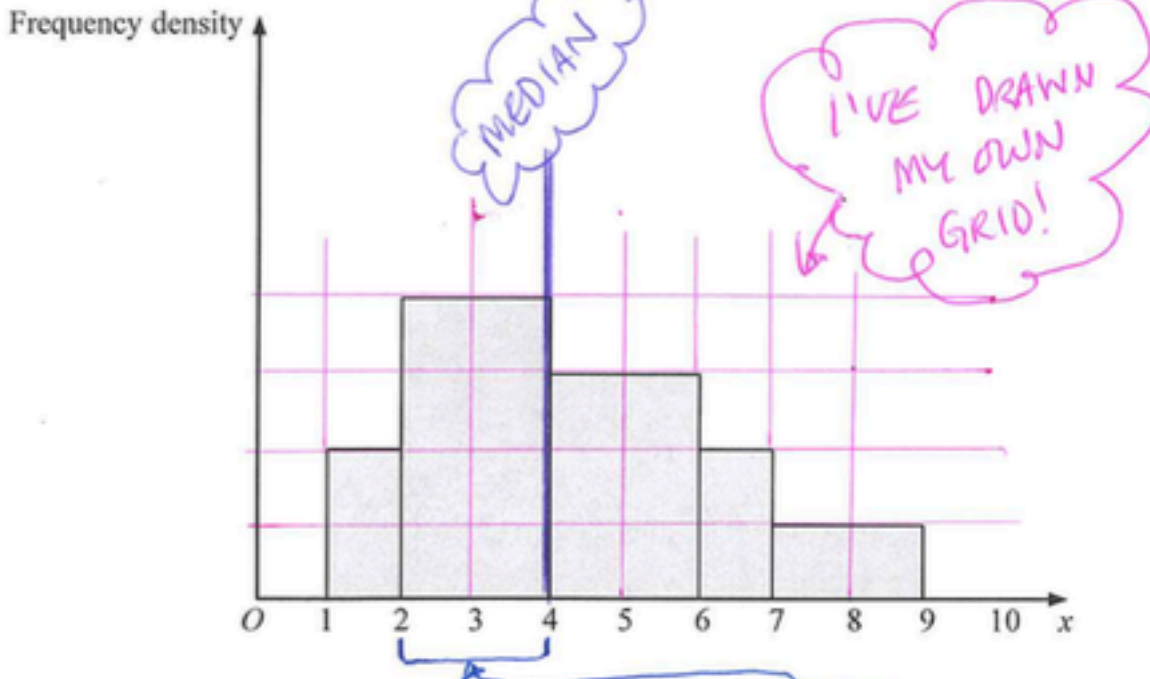
Calculate the total number of patients at the surgery that day.

$AREA \text{ BETWEEN } 10 \text{ AND } 15 \text{ MINUTES} = 40$

$\therefore AREA \text{ OF } 40 \text{ IS } 16 \text{ PATIENTS}$

$\therefore AREA \text{ OF } 500 \text{ IS } \frac{16}{40} \times 500 = \underline{200} \text{ (A1)}$
 (M1)

- The histogram shows information about the heights, x cm, of some plants.
The histogram is drawn accurately.



- (a) Calculate the percentage of values of x that lie between 2 and 4.

$$= \frac{\text{SQUARES BETWEEN 2 AND 4}}{\text{TOTAL NUMBER OF SQUARES}} \times 100$$

$$= \frac{8}{20} \times 100 \quad \text{(3)}$$

$$= 40\% \quad \text{(4)}$$

- (b) Find the median of x .

20 SQUARES IN TOTAL

\therefore MEDIAN SPLITS HISTOGRAM 10:10

[SEE PURPLE LINE]

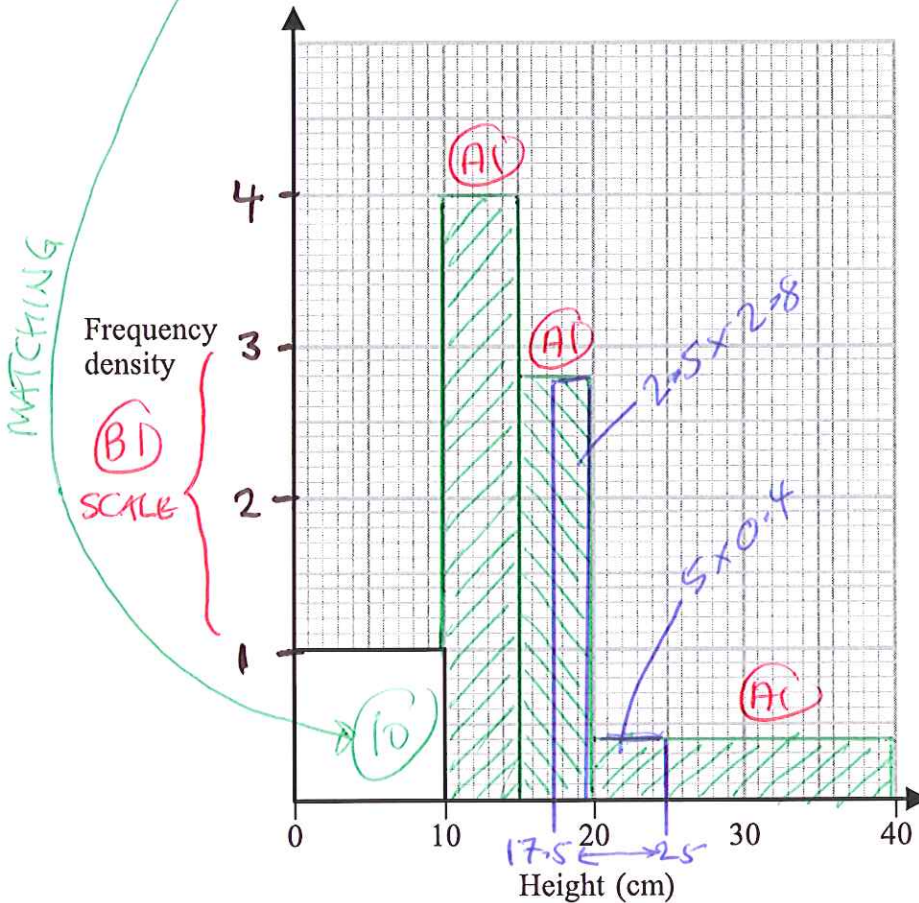
(5)

$$= 4 \quad \text{(6)}$$

In an experiment, 52 plants were grown and their heights were measured. The results are summarised in the table.

WIDTH	10	5	5	20
Height	$0 \leq h < 10$	$10 \leq h < 15$	$15 \leq h < 20$	$20 \leq h < 40$
Number of plants	10	20	14	8
FREQ. DEN	1	4	2.8	0.4

(a) Complete the histogram for these results.



(4)

The plants with heights from 17.5 cm to 25 cm are chosen for a display.

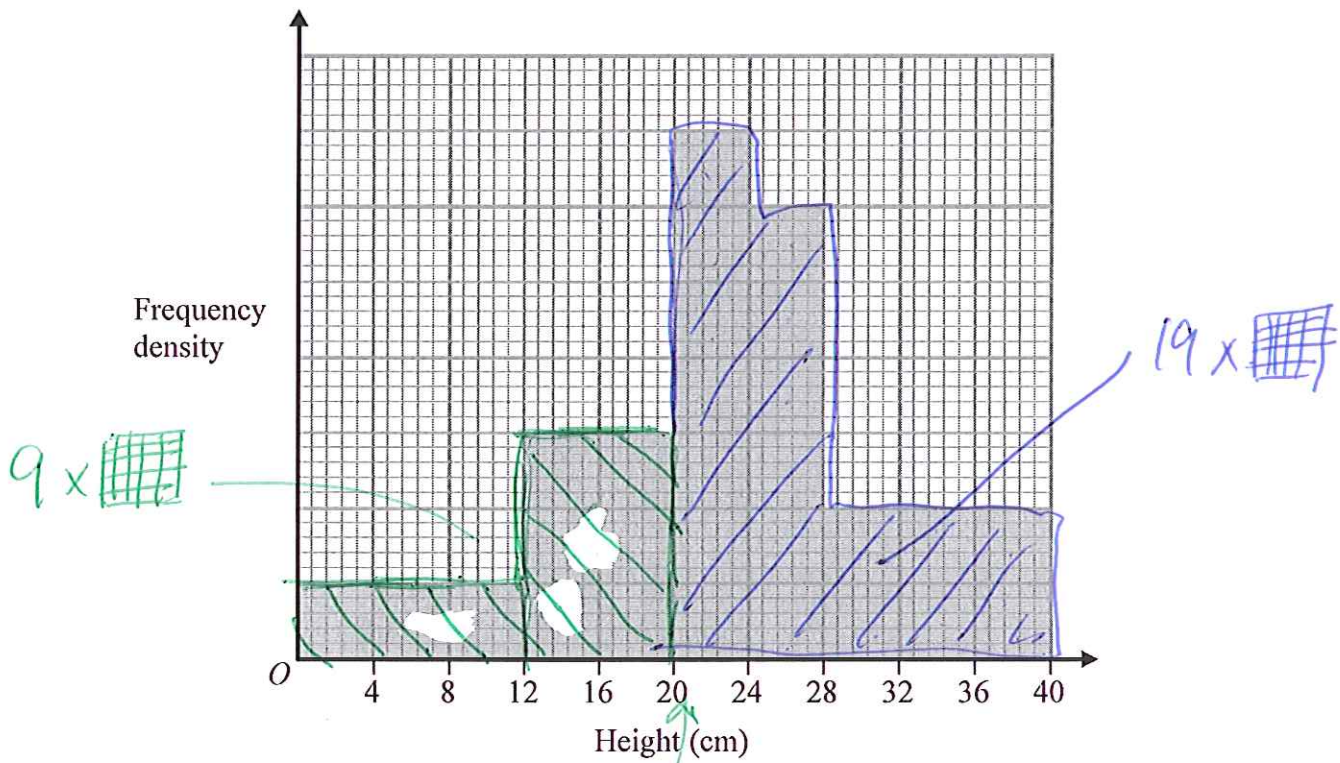
(b) Calculate an estimate of the number of plants chosen for the display.

$$2.5 \times 2.8 + 5 \times 0.4$$

(M) IDENTIFYING TWO RECTANGLES

$$\dots\dots\dots 9 \text{ (A)}$$

(2)



The histogram gives information about the heights of some plants.

There are 360 plants with a height of 20 cm or less.

Work out the number of plants with a height of more than 20 cm.

$$9 \times \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array} = 360 \quad \therefore \quad 1 \times \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array} = \frac{360}{9} \\ = 40 \text{ PLANTS} \quad \text{(B1)}$$

[NO NEED TO WORK OUT ACTUAL

FREQ DENSITIES BECAUSE ALL BARS CONTAIN
A WHOLE NUMBER OF SQUARES!]

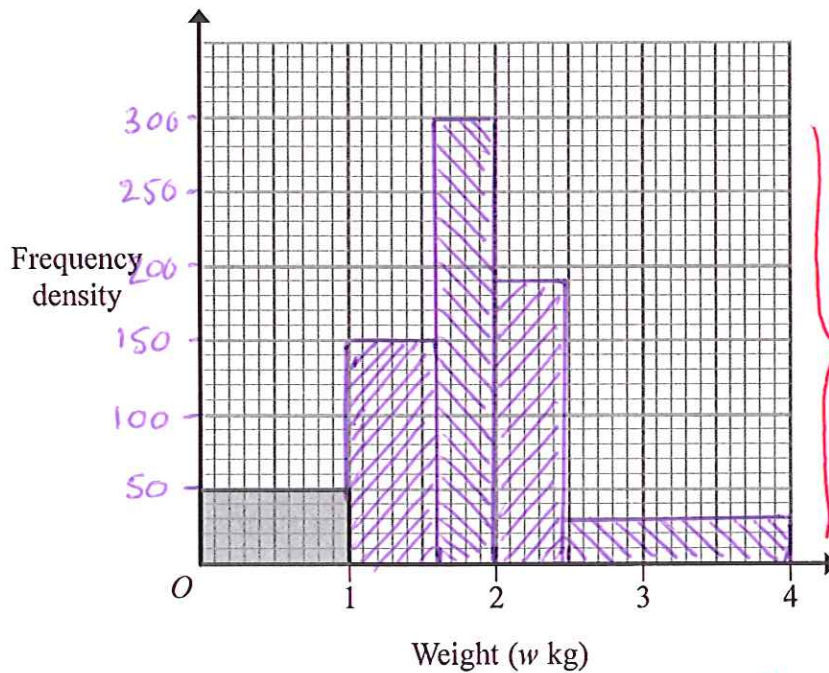
$$\text{PLANTS WITH HEIGHT} > 20 = 19 \times 40 \quad \text{(M1)} \\ = \underline{\underline{760}} \quad \text{(A1)}$$

400 people are asked to guess the weight of a large cake.
The table shows information about the weights they guess.

Weight (w kg)	Number of guesses	WIDTH	HEIGHT
$0 < w \leq 1$	50	1	50
$1 < w \leq 1.6$	90	0.6	150
$1.6 < w \leq 2$	120	0.4	300
$2 < w \leq 2.5$	95	0.5	190
$2.5 < w \leq 4$	45	1.5	30

Use the information in the table to complete the histogram.

↑
(M)

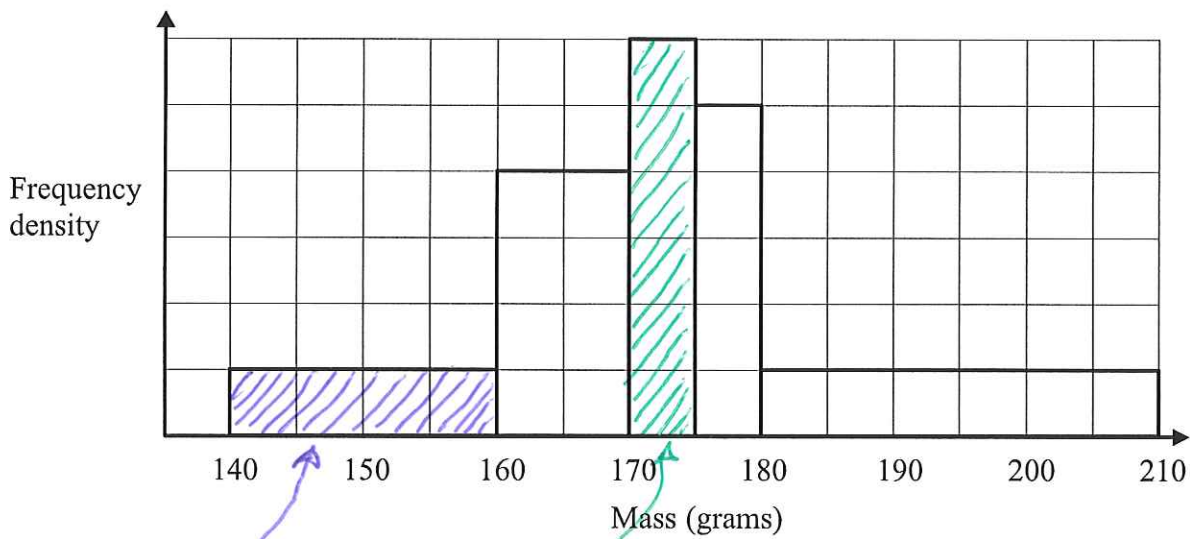


(A1) [CORRECT HEIGHTS]

Weight (w kg)

(A1) [CORRECT WIDTHS]

The histogram gives information about the masses of some stones.



The number of stones in the 170 g – 175 g class is 24 more than the number of stones in the 140 g – 160 g class.

Calculate the total number of stones.

$$\text{TWO SQUARES} = 24 \text{ STONES}$$

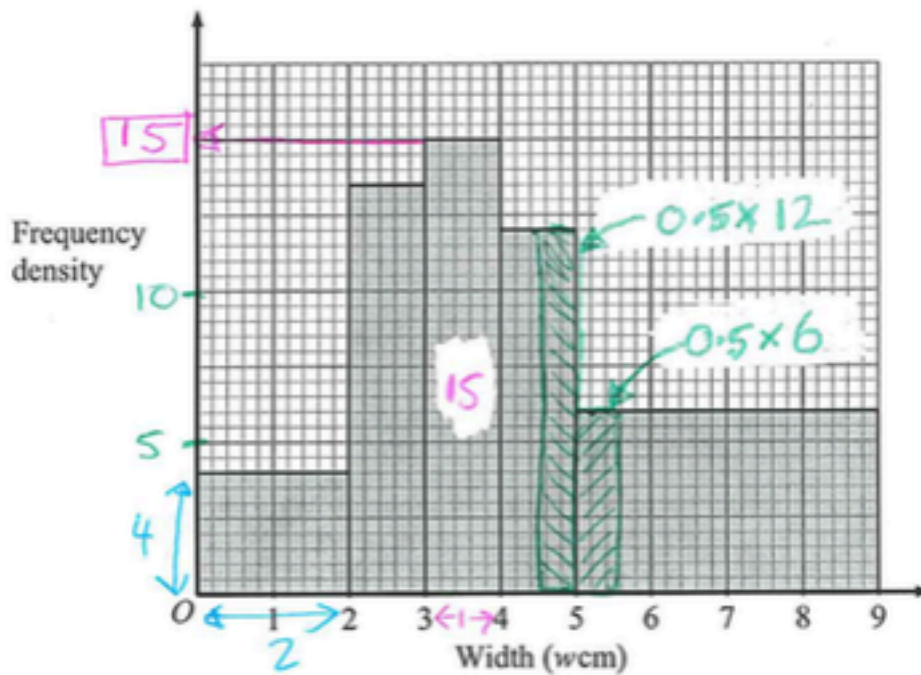
$$\therefore 1 \text{ SQUARE} = \underline{12} \text{ STONES (m)}$$

THERE IS A TOTAL OF 29 SQUARES IN ALL THE BARS

$$\therefore \text{TOTAL STONES} = 29 \times 12 \text{ (m)}$$

$$= \underline{348} \text{ (A)}$$

The histogram shows information about the widths, w centimetres, of some leaves.



The number of leaves with widths in the class $3 < w \leq 4$ is 15

(a) Find the number of leaves with widths in the class $0 < w \leq 2$

$$2 \times 4 \quad (m)$$

$$\begin{array}{r} 8 \quad (A) \\ \hline \end{array} \quad (2)$$

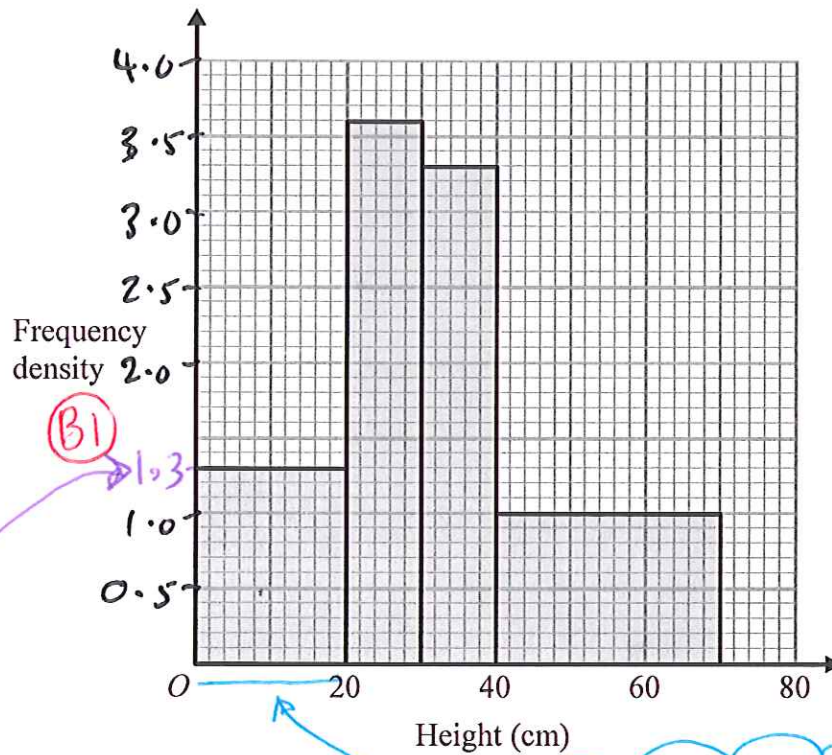
(b) Find an estimate of the number of leaves with widths in the range

$$4.5 < w \leq 5.5$$

$$\frac{0.5 \times 12}{(m)} + \frac{0.5 \times 6}{(m)}$$

$$\begin{array}{r} 9 \quad (A) \\ \hline \end{array} \quad (3)$$

The histogram shows information about the heights of some tomato plants.



26 plants have a height of less than 20 cm.

Work out the total number of tomato plants.

1ST AREA = WIDTH \times HEIGHT

$$\begin{aligned} \Rightarrow \text{HEIGHT} &= \frac{\text{AREA}}{\text{WIDTH}} \\ &= \frac{26}{20} \\ &= \underline{\underline{1.3}} \end{aligned}$$

AREA OF ALL THE BARS! [2ND]

$$\begin{aligned} 20 \times 1.3 &= 26 \\ 10 \times 3.6 &= 36 \\ 10 \times 3.3 &= 33 \\ 30 \times 1.0 &= 30 \end{aligned}$$

$$\text{TOTAL } \underline{\underline{125}}$$

AREA OF 1ST BAR IS 26!

(m)

(A)

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The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or ‘best’ method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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