

# INTERSECTING CHORDS

DATE OF SOLUTIONS: 15/05/2018  
MAXIMUM MARK: 32

# SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2013]

Lengths of Chords and Tangents [2 Marks]

$PTR$  and  $QTS$  are chords of a circle.

$$PT = 3 \text{ cm.}$$

$$ST = 10 \text{ cm.}$$

$$RT = 15 \text{ cm.}$$

$$QT = x \text{ cm.}$$

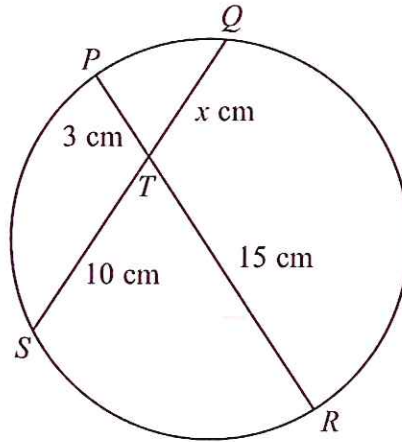


Diagram NOT  
accurately drawn

Calculate the value of  $x$ .

$$ST \times QT = PT \times RT$$

$$10 \times x = 3 \times 15 \quad (m)$$

$$x = \frac{3 \times 15}{10}$$

$$x = \dots\dots\dots 4.5 \quad (A)$$

$APC$  and  $BPD$  are chords of a circle.

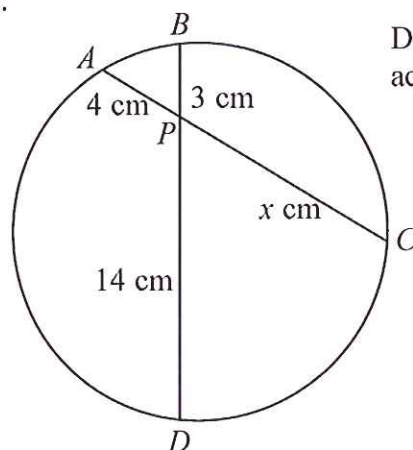
$$AP = 4 \text{ cm.}$$

$$BP = 3 \text{ cm.}$$

$$PD = 14 \text{ cm.}$$

$$PC = x \text{ cm.}$$

Diagram **NOT**  
accurately drawn

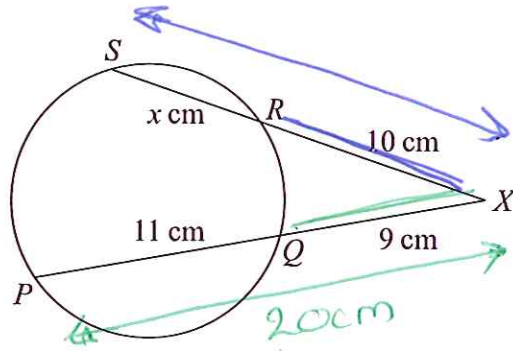


$$AP \times CP = BP \times DP$$

$$4x = 3 \times 14 \quad (\text{m1})$$

$$x = \frac{3 \times 14}{4}$$

$$x = \dots\dots\dots 10.5 \text{ cm} \quad (\text{A1})$$



The diagram shows a circle,  $PQRS$ .

$SRX$  and  $PQX$  are straight lines.

$PQ = 11$  cm.  $QX = 9$  cm.  $RX = 10$  cm.  $SR = x$  cm.

Find the value of  $x$ .

$$SX \times RX = PX \times QX$$

$$SX \times 10 = 20 \times 9$$

$$\Rightarrow SX = \frac{20 \times 9}{10}$$

$$= 18$$

(B1)

$$\begin{aligned} \therefore x &= 18 - 10 \\ &= \underline{\underline{8}} \end{aligned}$$

$$x = \underline{\underline{8}} \text{ cm} \quad \text{(A1)}$$

$AEC$  and  $DEB$  are chords of a circle.

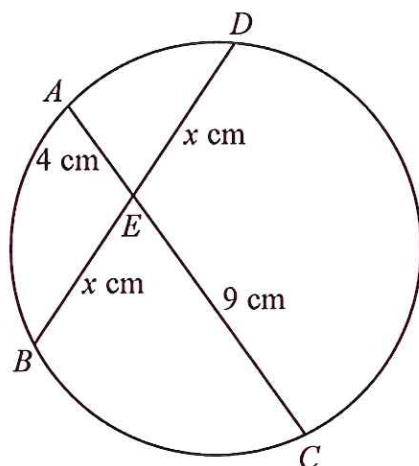


Diagram **NOT**  
accurately drawn

$$AE = 4 \text{ cm.}$$

$$CE = 9 \text{ cm.}$$

$$DE = BE = x \text{ cm.}$$

Calculate the value of  $x$ .

$$BE \times DE = AE \times CE$$

$$x^2 = 4 \times 9 \quad (m1)$$

$$x^2 = 36$$

$$x = \underline{\underline{6}} \quad (A1)$$

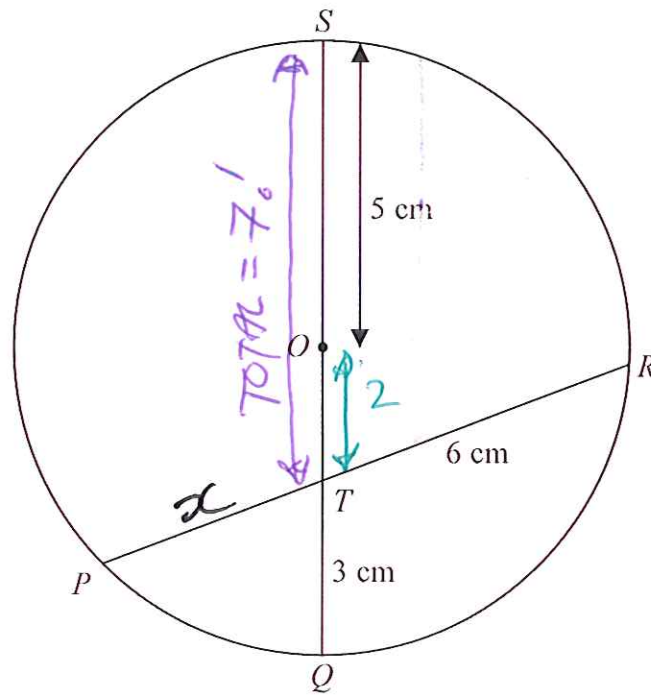


Diagram **NOT**  
accurately drawn

$P$ ,  $Q$ ,  $R$  and  $S$  are points on a circle, centre  $O$ .

$QS$  is a diameter of the circle.

$QS$  and  $PR$  intersect at the point  $T$ .

$OS = 5$  cm,  $QT = 3$  cm and  $TR = 6$  cm.

Work out the length of  $PT$ .

$$\begin{aligned}
 x \times 6 &= 7 \times 3 && \text{(M1)} \\
 x &= \frac{7 \times 3}{6} && \text{(B1)} \\
 &= \underline{\underline{3.5}} && \text{(A1)}
 \end{aligned}$$

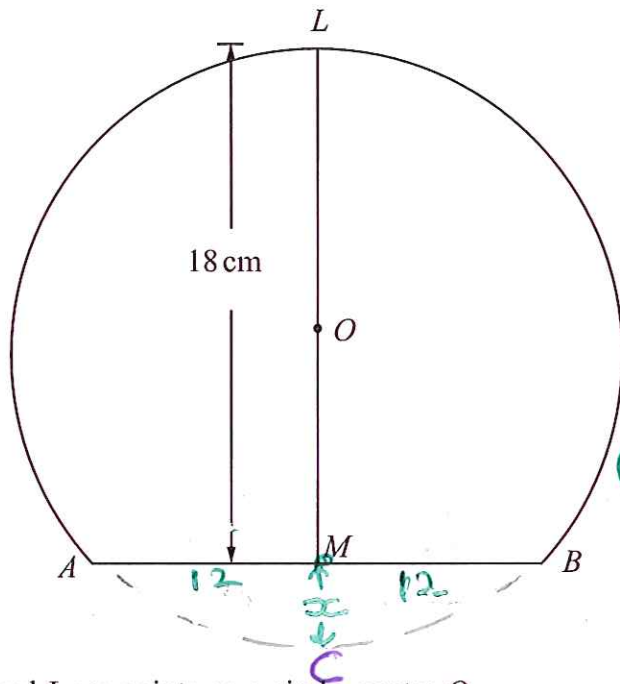


Diagram NOT  
accurately drawn

$$CM \times LM = AM \times BM$$

$A$ ,  $B$  and  $L$  are points on a circle, centre  $O$ .  
 $AB$  is a chord of the circle.  
 $M$  is the midpoint of  $AB$ .  
 $LOM$  is a straight line.  
 $AB = 24$  cm.  
 $LM = 18$  cm.

Calculate the diameter of the circle.

$$CM \times LM = AM \times BM$$

$$18x = 12 \times 12 \quad (m1)$$

$$x = \frac{12 \times 12}{18}$$

$$= 8 \quad (A1)$$

$$\therefore \text{DIAMETER} = 18 + 8 \quad (m1)$$

$$= \underline{\underline{26 \text{ cm}}} \quad (A1)$$

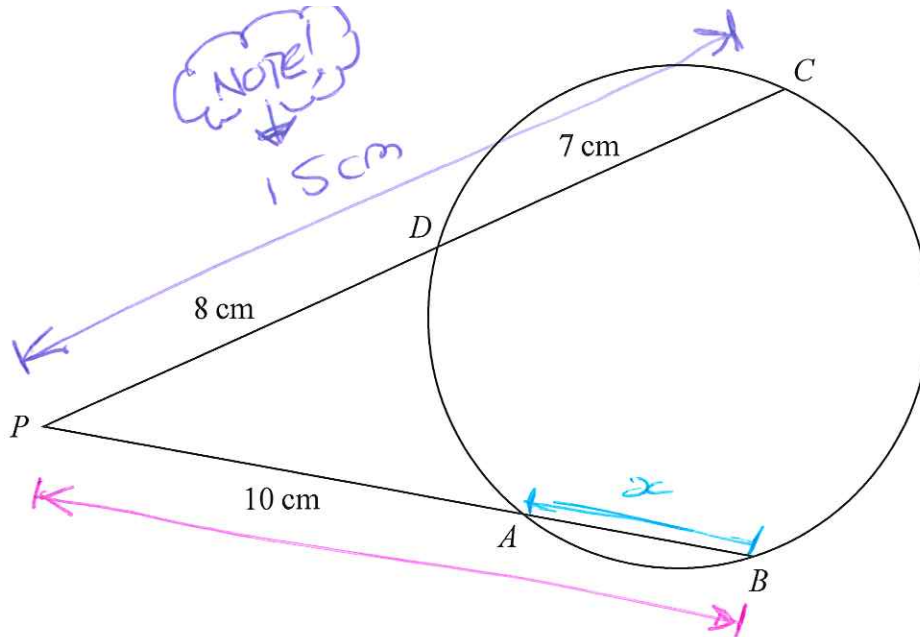


Diagram **NOT** accurately drawn

$A, B, C$  and  $D$  are points on a circle.

$PAB$  and  $PDC$  are straight lines.

$PA = 10$  cm,  $PD = 8$  cm and  $DC = 7$  cm.

Calculate the length of  $AB$ .

1ST

$$AP \times BP = DP \times CP$$

$$10 \times BP = 8 \times 15 \quad (\text{ml})$$

$$BP = \frac{8 \times 15}{10}$$

$$= 12 \quad (\text{ml})$$

$$\therefore AB = 12 - 10$$

$$= \underline{\underline{2 \text{ cm}}} \quad (\text{ml})$$

GET THIS RIGHT!!

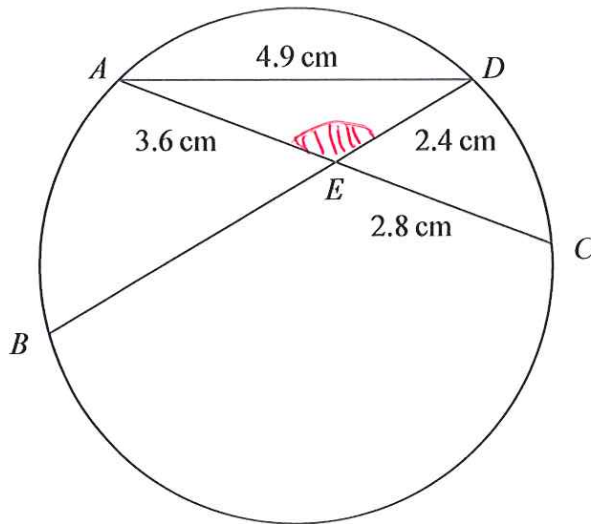


Diagram NOT  
accurately drawn

$A$ ,  $B$ ,  $C$  and  $D$  are four points on the circumference of a circle.  
The chords  $AC$  and  $BD$  intersect at  $E$ .  
 $AE = 3.6$  cm,  $CE = 2.8$  cm,  $DE = 2.4$  cm and  $AD = 4.9$  cm.

(a) Calculate the length of  $BE$ .

$$BE \cdot DE = AE \cdot CE$$

$$\Rightarrow BE \times 2.4 = 3.6 \times 2.8 \quad (m1)$$

$$\Rightarrow BE = \frac{3.6 \times 2.8}{2.4} \quad (m1) \longrightarrow \dots 4.2 \quad (A1)$$

cm  
(3)

(b) Calculate the size of angle  $AED$ .  
Give your answer correct to 3 significant figures.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Rightarrow \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{2.4^2 + 3.6^2 - 4.9^2}{2 \times 2.4 \times 3.6} \quad (m1) \quad \dots 108 \quad (A1)$$

°  
(3)

$$= -0.30613 \dots$$

$$A = \cos^{-1}(-0.30613) \quad (m1)$$

$$= 107.82 \dots$$



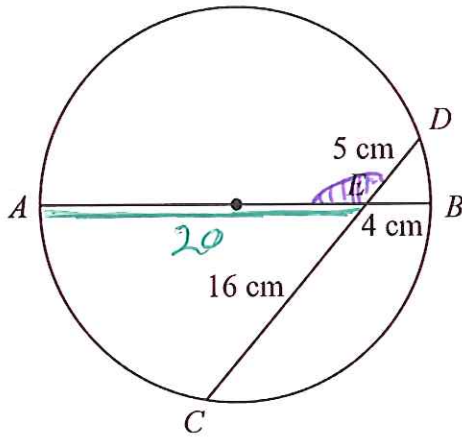


Diagram NOT  
accurately drawn

$$AE \times BE = CE \times DE$$

$AB$  is a diameter of a circle.  
 $CD$  is a chord of the circle.  
 $AB$  and  $CD$  intersect at  $E$ .  
 $BE = 4$  cm,  $CE = 16$  cm and  $DE = 5$  cm.

(a) Calculate the length of  $AE$ .

$$AE \times 4 = 16 \times 5 \quad (m1)$$

$$\Rightarrow AE = \frac{16 \times 5}{4} \quad \rightarrow \quad 20 \quad (A1)$$

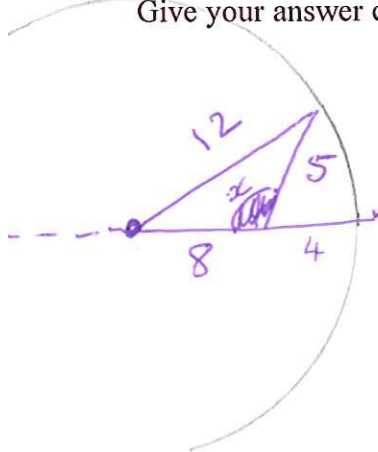
..... cm  
(2)

(b) (i) Find the radius of the circle.

$$\frac{20 + 4}{2} \quad \rightarrow \quad 12 \quad (A1)$$

..... cm

(ii) Calculate the size of angle  $AED$ .  
Give your answer correct to 1 decimal place.



$$\cos \alpha = \frac{8^2 + 5^2 - 12^2}{2 \times 8 \times 5} \quad (m1)$$

$$\cos \alpha = -0.6875 \quad (m1)$$

$$\alpha = \cos^{-1}(-0.6875) \quad (m1)$$

$$= \underline{\underline{133.4^\circ}} \quad (A1)$$

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The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or ‘best’ method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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