



POWERS AND ROOTS

CHANGE OF BASE

NO CALCULATOR

Ref: G133.6R1

A1 Write 5 ⁸ as a power of 25	A2 Write 8 ⁶ as a power of 2	A3 Write 8 ⁶ as a power of 4	A4 Write $8^2 \times 4^3$ as a power of 2
B1 Write 2 as a power of 8	B2 Write 4 as a power of 8	B3 Write $\sqrt{8}$ as a power of 2	B4 Express $3\sqrt{3}$ as a power of 9
C1 Express $\frac{1}{81}$ as a single power of 3	C2 Express $\frac{1}{\sqrt{3}} \text{ as a single power of 9}$	C3 Express $\frac{1}{4\sqrt{2}}$ as a single power of 2	C4 Express $3^7 + 9^4 + 15 \times 27^2 \text{ as a power of 3}$
D1 Solve: $32^x = \frac{1}{16}$	D2 Solve: $2^{x} = 8^{\frac{1}{4}} \times 16^{\frac{1}{3}}$	D3 Find the value of <i>x</i> if: $9^{\frac{3}{4}} \times 27^x = 81^{\frac{2}{3}}$	D4 Find the value of <i>m</i> and <i>n</i> if: $6 \times 12^m = 9^4 \times 2^n$





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A1	Write 5 ⁸	as a power of 25

$$5^8 = 5^{2 \times 4}$$

$$= 25^4$$

A2 Write
$$8^6$$
 as a power of 2

$$8^6 = (2^3)^6 = 2^{18}$$

$$8^{6} = (2^{3})^{6} = 2^{18}$$
$$= 2^{2 \times 9}$$
$$= 4^{9}$$

A4 Write
$$8^2 \times 4^3$$
 as a power of 2

$$8^{2} \times 4^{3} = \left(2^{3}\right)^{2} \times \left(2^{2}\right)^{3}$$
$$= 2^{6} \times 2^{6}$$
$$= 2^{12}$$

$$2 = \sqrt[3]{8} \\
= 8^{\frac{1}{3}}$$

$$4 = 2^{2} = \left(8^{\frac{1}{3}}\right)^{3}$$
$$= 8^{\frac{2}{3}}$$

B3 Write
$$\sqrt{8}$$
 as a power of 2

$$\sqrt{8} = 8^{\frac{1}{2}} = \left(2^{3}\right)^{\frac{1}{2}}$$
$$= 2^{\frac{5}{2}}$$

B4 Express
$$3\sqrt{3}$$
 as a power of 9

$$3\sqrt{3} = 3 \times 3^{\frac{1}{2}} = 3^{\frac{3}{2}}$$
$$= \left(9^{\frac{1}{2}}\right)^{\frac{3}{2}} = 9^{\frac{3}{4}}$$

$$\frac{1}{81}$$
 as a single power of 3

$$\frac{1}{81} = \frac{1}{3^4} = 3^{-4}$$

$$\frac{1}{\sqrt{3}} = \frac{1}{3^{\frac{1}{2}}} = 3^{-\frac{1}{2}}$$
$$= \left(9^{\frac{1}{2}}\right)^{-\frac{1}{2}} = 9^{-\frac{1}{4}}$$

$$\frac{1}{4\sqrt{2}} = \frac{1}{2^2 \times 2^{\frac{1}{2}}} = \frac{1}{2^{\frac{5}{2}}}$$
$$= 2^{-\frac{5}{2}}$$

$$3^{7} + 9^{4} + 15 \times 27^{2} = 3^{7} + (3^{2})^{4} + 15(3^{3})^{2}$$

$$= 3^{7} + 3^{8} + 5 \times 3(3^{6})$$

$$= 3^{7} + 3^{8} + 5 \times (3^{7})$$

$$= 3^{7}(1 + 3 + 5)$$

$$= 3^{7} \times 9$$

$$= 3^{7} \times 3^{2} = 3^{9}$$

$$32^{x} = \frac{1}{16}$$

$$\Rightarrow (2^{5})^{x} = \frac{1}{2^{4}}$$

$$\Rightarrow 2^{5x} = 2^{-4}$$

$$\Rightarrow x = -\frac{4}{5}$$

$$2^{x} = 8^{\frac{1}{4}} \times 16^{\frac{1}{3}}$$

$$\Rightarrow 2^{x} = \left(2^{3}\right)^{\frac{1}{4}} \times \left(2^{4}\right)^{\frac{1}{3}}$$

$$\Rightarrow 2^{x} = 2^{\frac{3}{4}} \times 2^{\frac{4}{3}}$$

$$\Rightarrow 2^{x} = 2^{\frac{3}{4}} \times 2^{\frac{4}{3}}$$

$$= \frac{25}{12}$$

$$\begin{array}{ccc}
\mathbf{D3} & 9^{\frac{3}{4}} \times 27^{x} = 81^{\frac{2}{3}} \\
\Rightarrow \left(3^{2}\right)^{\frac{3}{4}} \times \left(3^{3}\right)^{x} = \left(3^{4}\right)^{\frac{2}{3}} & 3x = \frac{8}{3} - \frac{6}{4} \\
\Rightarrow 3^{\frac{6}{4}} \times 3^{3x} = 3^{\frac{8}{3}} & \Rightarrow 3x = \frac{7}{6} \\
\Rightarrow x = \frac{7}{18}
\end{array}$$

$$6 \times 12^{m} = 9^{4} \times 2^{n}$$

$$\Rightarrow 3 \times 2 \times (3 \times 2^{2})^{m} = 3^{2 \times 4} \times 2^{n}$$

$$\Rightarrow 3^{m+1} \times 2^{2m+1} = 3^{8} \times 2^{n}$$

$$m+1=8$$
 $n=2m+1$
 $\Rightarrow m=7$ = 15