PROBABILITY TREES
DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 83

SOLUTIONS
GCSE (+ IGCSE) EXAM QUESTION PRACTICE

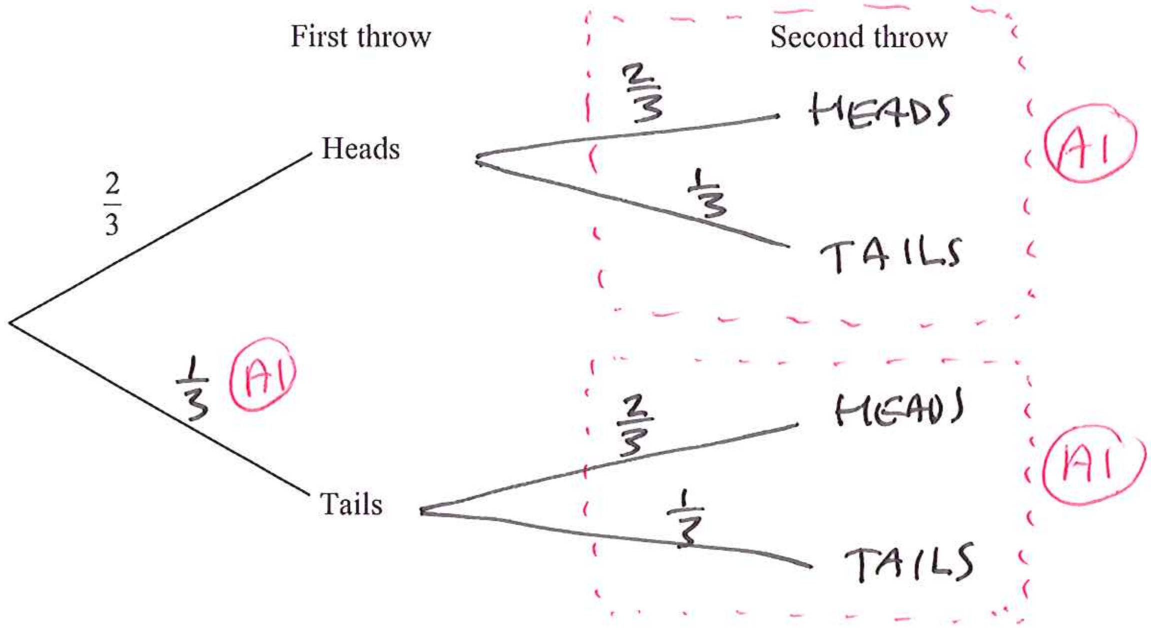
1. [Edexcel, 2008] Probability Trees **[8 Marks]**

A coin is biased.

When it is thrown, the probability that it shows Heads is $\frac{2}{3}$

Dorcas throws the coin twice.

(a) Complete the probability tree diagram.



(3)

(b) Find the probability that the coin shows Heads both times.

$P(H,H) = \frac{2}{3} \times \frac{2}{3}$ M1

$\frac{4}{9}$ A1

(2)

(c) Find the probability that the coin shows Heads at least once.

$P(H,H) + P(H,T) + P(T,H)$ OR $1 - P(T,T) = 1 - \frac{1}{3} \times \frac{1}{3}$ M1 M1

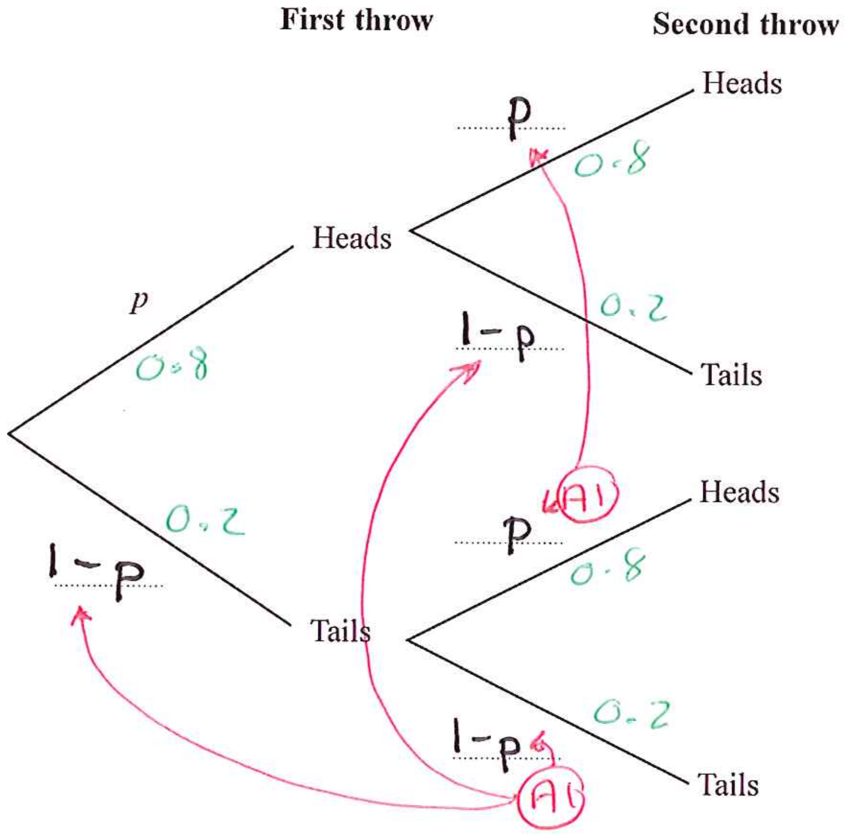
$\frac{8}{9}$ A1

(3)

2. [Edexcel, 2015] Probability Trees **[6 Marks]**

Jim has a biased coin.
The probability that Jim will throw Heads on any throw is p.
Jim throws the coin twice.

(a) Complete the probability tree diagram.
Give your probabilities in terms of p.



(2)

(b) Find an expression, in terms of p, for the probability that Jim will throw two Heads.

$P(H,H) = p \times p$

p^2 A1

(1)

Given that $p = 0.8$,

(c) work out the probability that Jim will throw exactly one Head.

$$\left.\begin{aligned} P(HT) &= 0.8 \times 0.2 = 0.16 \\ P(TH) &= 0.2 \times 0.8 = 0.16 \end{aligned}\right\} \text{TOTAL} = \underline{\underline{0.32}}$$

A1

M1 [MULTIPLY]

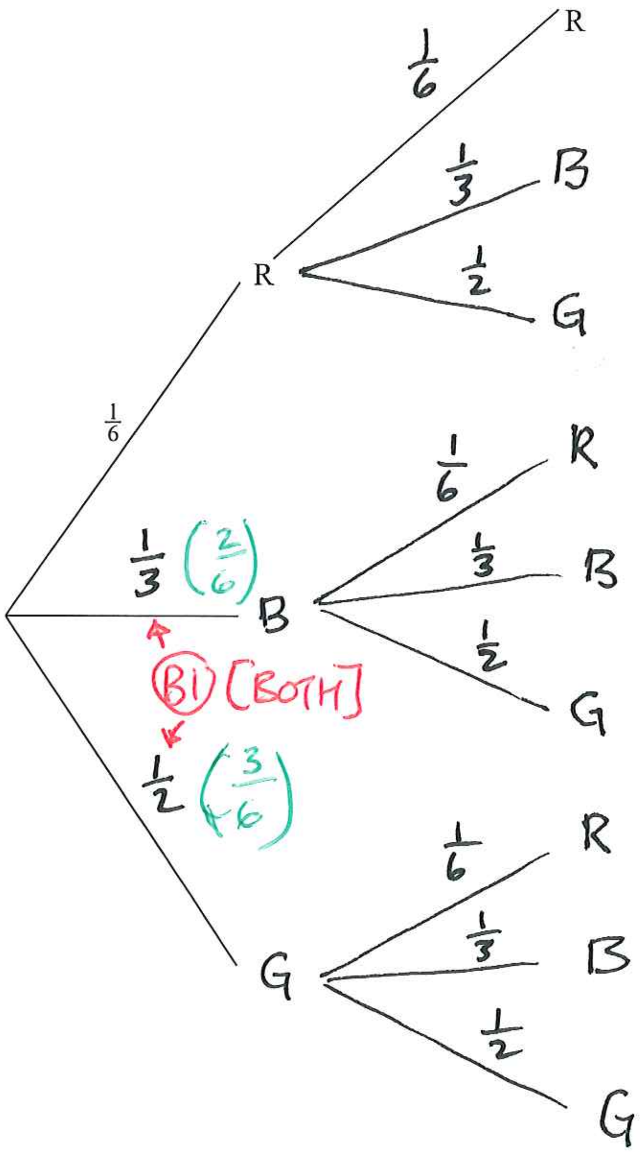
M1 [TWO ANSWERS]

A bag contains 1 red disc, 2 blue discs and 3 green discs.

PROBABILITIES DO NOT CHANGE

Xanthe chooses a disc at random from the bag. She notes its colour and replaces it.
Then Xanthe chooses another disc at random from the bag and notes its colour.

(a) Complete the probability tree diagram showing all the probabilities.



B1 [BOTH]

A1 [STRUCTURE]

A1 [ALL PROBABILITIES AND OUTCOMES CORRECT]

(3)

4. [Edexcel, 2016] Probability Trees **[5 Marks]**

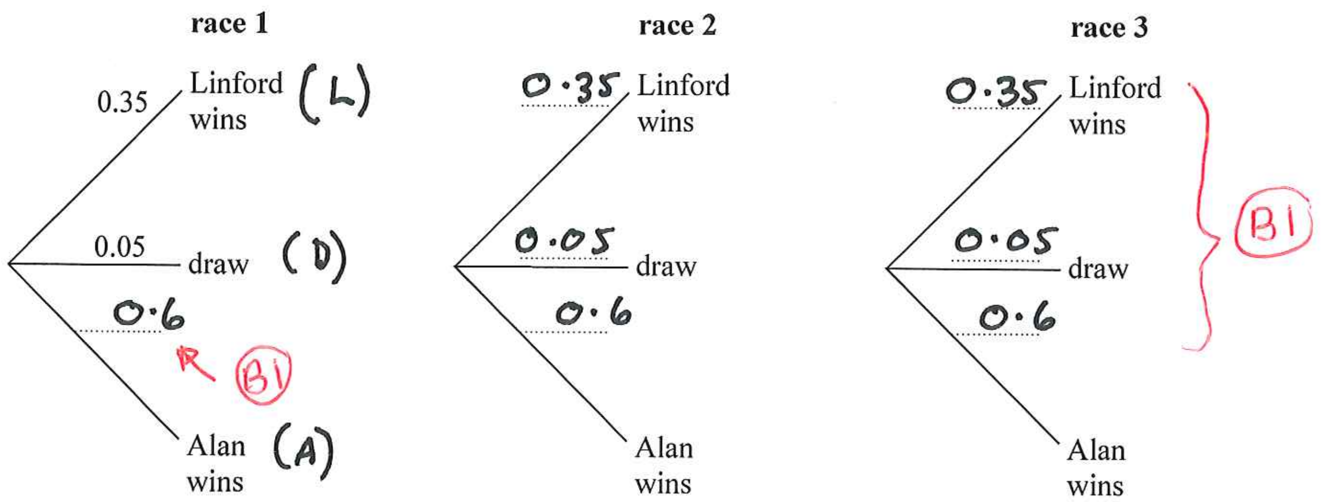
Linford and Alan race against each other in a competition.

If one of them wins a race, he wins the competition.
If the race is a draw, they run another race.

They run a maximum of three races.

Each time they race, the probability that Linford wins is 0.35
Each time they race, the probability that there is a draw is 0.05

(a) Complete the probability tree diagram.



(2)

(b) Calculate the probability that Linford wins the competition.

$P(L) = 0.35$

$P(DL) = 0.05 \times 0.35 = 0.0175$ M1

$P(DDL) = 0.05 \times 0.05 \times 0.35 = 0.000875$ M1

$\text{TOTAL} = 0.35 + 0.0175 + 0.000875$

A1

$= 0.368375$

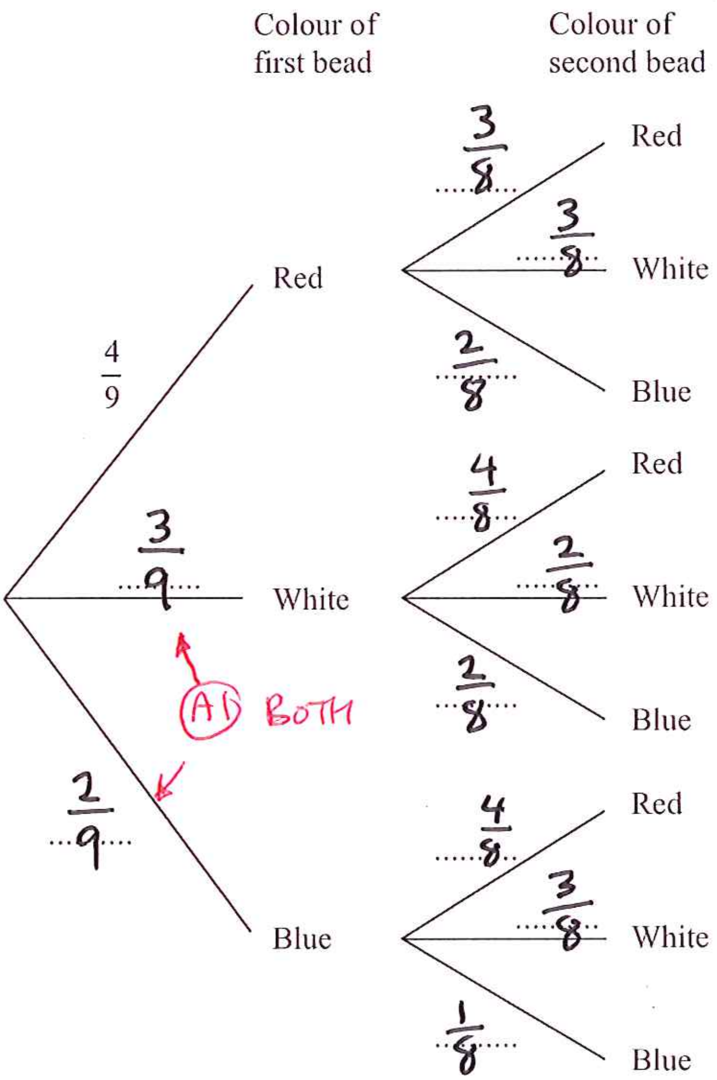
(3)

[0.368 IS OKAY]

There are 9 beads in a bag.
4 of the beads are red.
3 of the beads are white.
2 of the beads are blue.
Sanjay takes at random a bead from the bag and does not replace it.
He then takes at random a second bead from the bag.

PROBABILITIES CHANGE!

(a) Complete the probability tree diagram.



A1 ALL EIGHTHS

A1 ALL CORRECT

A1 BOTH

(3)

(b) Calculate the probability that one of Sanjay's beads is red and his other bead is blue.

$$P(R, B) = \frac{4}{9} \times \frac{2}{8} = \frac{8}{72}$$

$$P(B, R) = \frac{2}{9} \times \frac{4}{8} = \frac{8}{72}$$

M1 EITHER

M1 ADDING

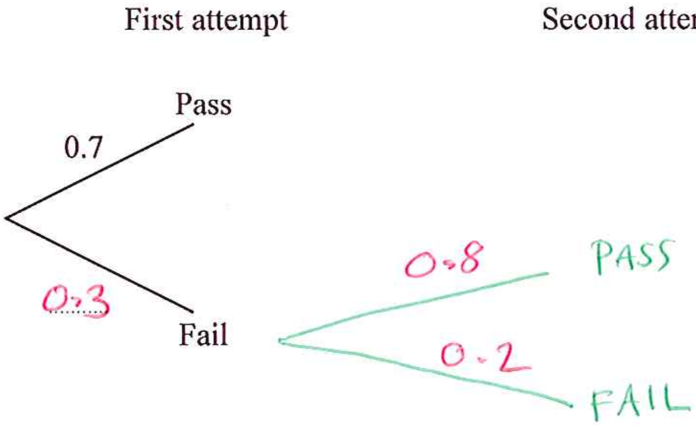
$$\text{TOTAL} = \frac{16}{72}$$

$$= \frac{2}{9} \quad \text{A1}$$

6. [Edexcel, 2014] Probability Trees **[7 Marks]**

Natasha wants to pass her driving test.
The probability that she passes at her first attempt is 0.7
When Natasha passes her driving test, she does not take it again.
If she fails, the probability that she passes at the next attempt is 0.8.

(i) Complete the probability tree diagram for Natasha's first two attempts.



(ii) Calculate the probability that Natasha needs exactly two attempts to pass her driving test.

(iii) Calculate the probability that Natasha passes her driving test at her third or fourth attempt.

(ii) $P(FP) = 0.3 \times 0.8 = \underline{\underline{0.24}}$

(iii) $P(FFP) = 0.3 \times 0.2 \times 0.8 = 0.048$

$P(FFFP) = 0.3 \times 0.2 \times 0.2 \times 0.8 = 0.0096$

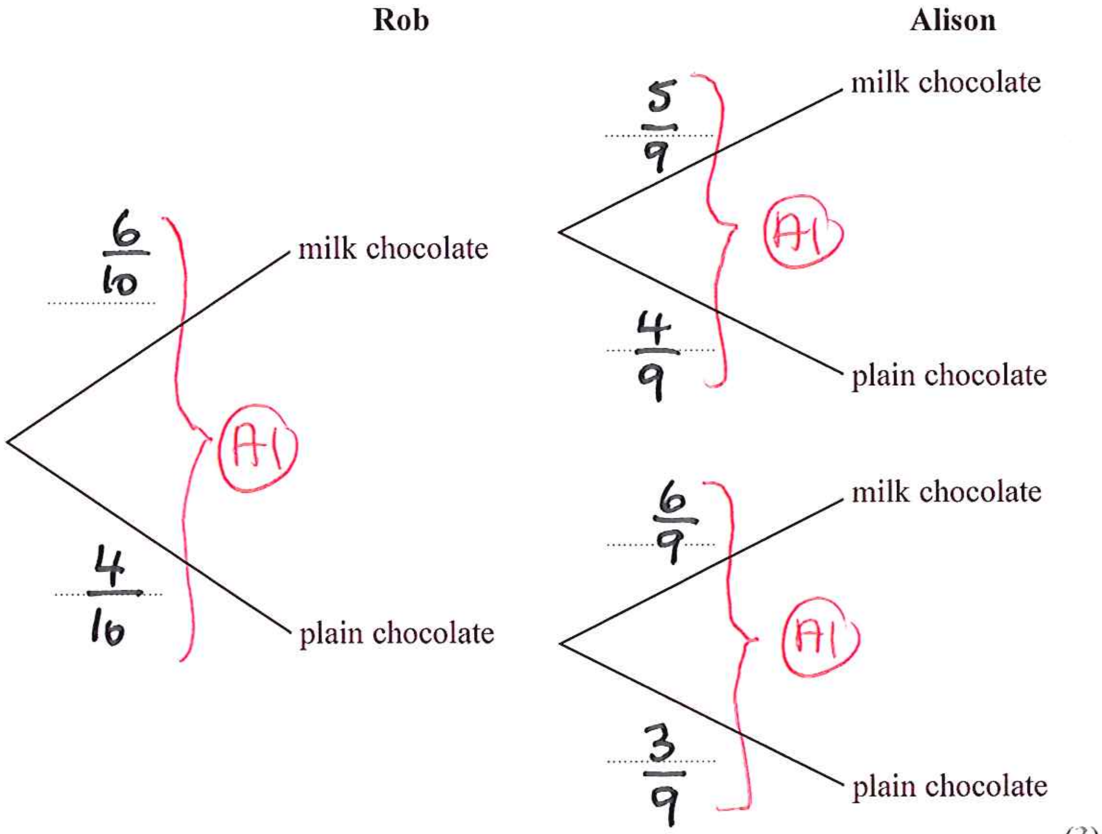
TOTAL

$\underline{\underline{0.0576}}$

There are 6 milk chocolates and 4 plain chocolates in a box.
Rob takes at random a chocolate from the box and eats it.
Then Alison takes at random a chocolate from the box and eats it.

PROBABILITIES CHANGE!

(a) Complete the probability tree diagram.



(3)

(b) Work out the probability that there are now exactly 3 plain chocolates in the box.

AT START THERE WERE 4 PLAIN CHOCOLATES.

$\therefore$ WE WANT TO EAT JUST ONE PLAIN CHOCOLATE

$$P(PM) = \frac{4}{10} \times \frac{6}{9} = \frac{24}{90}$$

$$P(MP) = \frac{6}{10} \times \frac{4}{9} = \frac{24}{90}$$

TOTAL $= \frac{48}{90}$ A1

M1 [BOTH OUTCOMES]

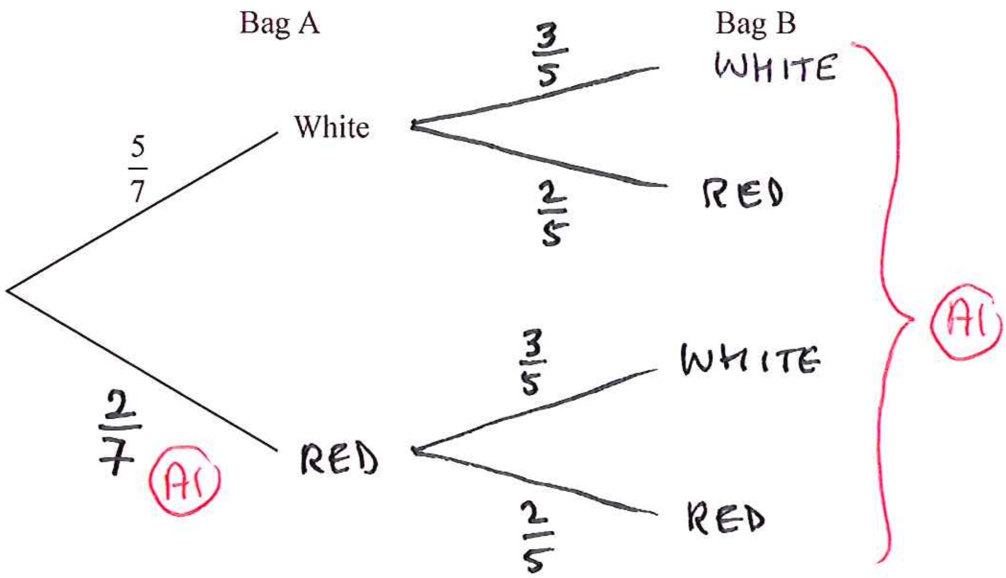
M1 [MULTIPLYING]

$= \frac{8}{15}$

Maria has two bags.
In bag A, there are 5 white counters and 2 red counters. → $\frac{5}{7}$ AND $\frac{2}{7}$
In bag B, there are 3 white counters and 2 red counters. → $\frac{3}{5}$ AND $\frac{2}{5}$

Maria is going to take at random one counter from bag A and one counter from bag B.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that both counters will be white.

$$P(WW) = \frac{5}{7} \times \frac{3}{5} \quad \text{M1}$$

$$= \frac{15}{35}$$

A1

$\frac{3}{7}$

(2)

(c) Work out the probability that exactly one of the counters will be white.

$$P(WR) = \frac{5}{7} \times \frac{2}{5} = \frac{10}{35}$$

M1 [TWO POSSIBLITIES]

$$P(RW) = \frac{2}{7} \times \frac{3}{5} = \frac{6}{35}$$

TOTAL = $\frac{16}{35}$ A1

M1

[ADDING]

9. [Edexcel, 2016] Probability Trees **[7 Marks]**

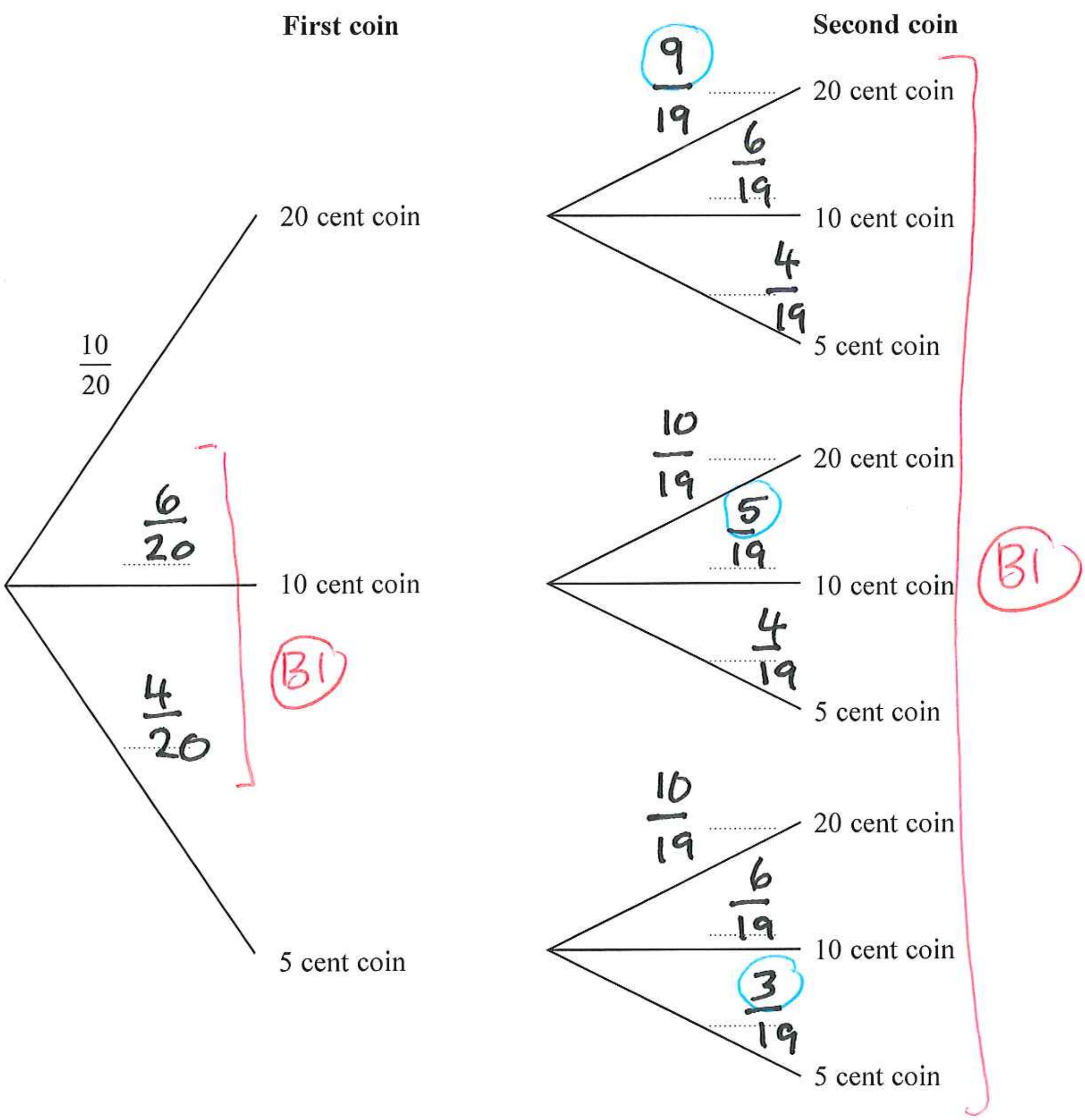
In a bag there is a total of 20 coins.

10 coins are 20 cent coins
6 coins are 10 cent coins
4 coins are 5 cent coins

Emma takes at random two of the coins from the bag. IMPLIES NON-REPLACEMENT

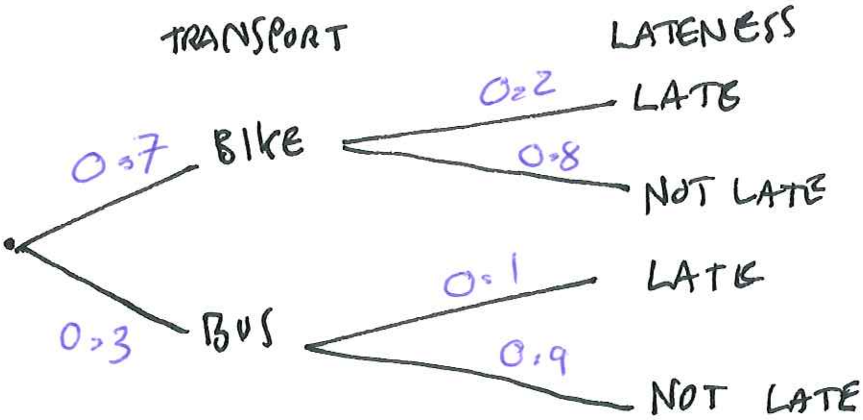
(a) Complete the probability tree diagram.

(2)



Parveen travels to school either by bicycle or by bus.
The probability that, on any day, she will travel by bicycle is 0.7
When she travels by bicycle, the probability that she will be late for school is 0.2
When she travels by bus, the probability that she will be late for school is 0.1

(a) Calculate the probability that, on a randomly chosen day, Parveen will travel by bus and be late for school.



P(BUS, LATE) = 0.3 × 0.1 = 0.03 A1

M1

(b) Calculate the probability that, on a randomly chosen day, Parveen will not be late for school.

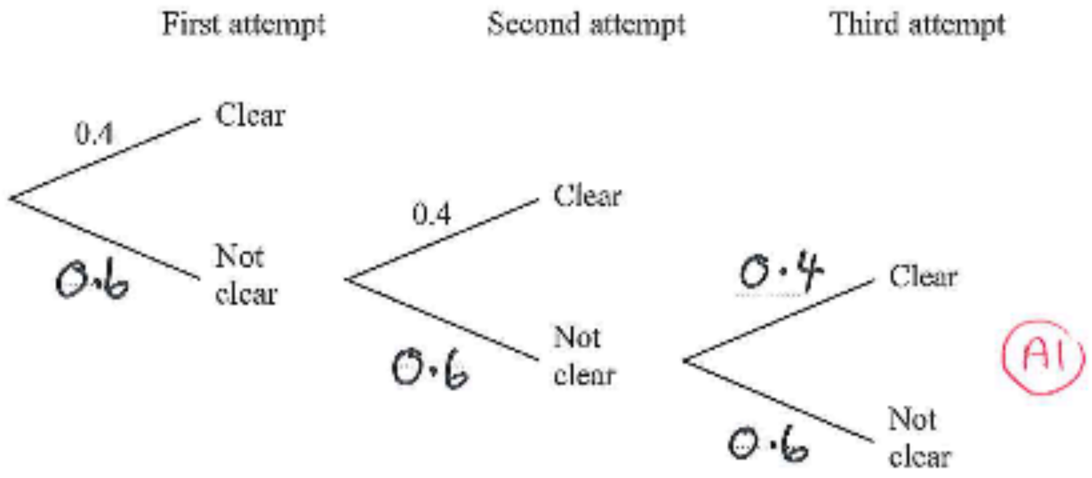
P(BUS, NOT LATE) = 0.3 × 0.9 = 0.27 M1

P(BIKE, NOT LATE) = 0.7 × 0.8 = 0.56 M1

TOTAL = 0.83 A1

Hugo competes in the high jump at a school athletics competition.
He has up to 3 attempts to clear the bar at each height.
When he clears the bar, he does not have another attempt at that height.

When the bar is set at a height of 1.60 metres, the probability that Hugo will clear the bar on any attempt is 0.4
The probability tree diagram shows the possible outcomes of Hugo's attempts at 1.60 metres.



(a) Complete the probability tree diagram to show the four missing probabilities.

(1)

(b) Work out the probability that Hugo does not clear the bar on his first two attempts and then does clear the bar on his third attempt at 1.60 metres.

$P(N, N, C) = 0.6 \times 0.6 \times 0.4$ (M1)

$= 0.144$ (A1)

Boris and Nigel play games of chess against each other in a match.
In each game, Boris wins or Nigel wins or the game is a draw.

When a player wins a game, he wins the match.
When a game is a draw, the players play another game against each other.
Boris and Nigel play a maximum of 3 games.

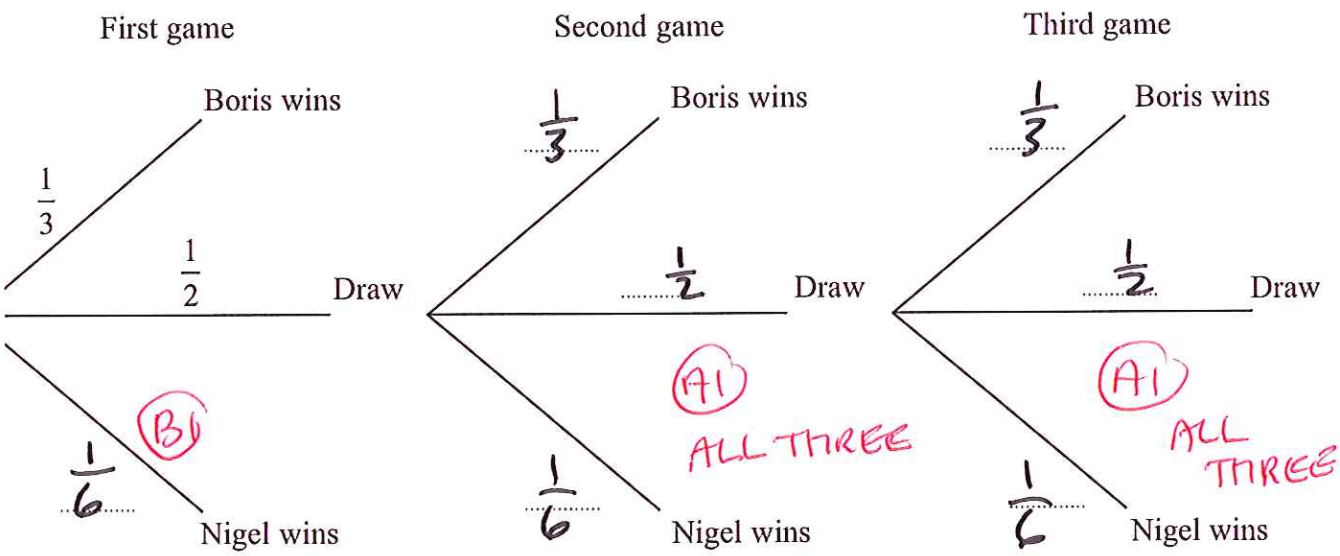
The probability that Boris wins a game is $\frac{1}{3}$

The probability that a game is a draw is $\frac{1}{2}$

$$\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$$

$$\therefore \text{P(NIGEL WINS)} = \frac{1}{6}$$

(a) Complete the probability tree diagram.



(3)

(b) Calculate the probability that Boris wins the match.

$$P(B) = \frac{1}{3}$$

$$P(DB) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6} \quad \text{M1}$$

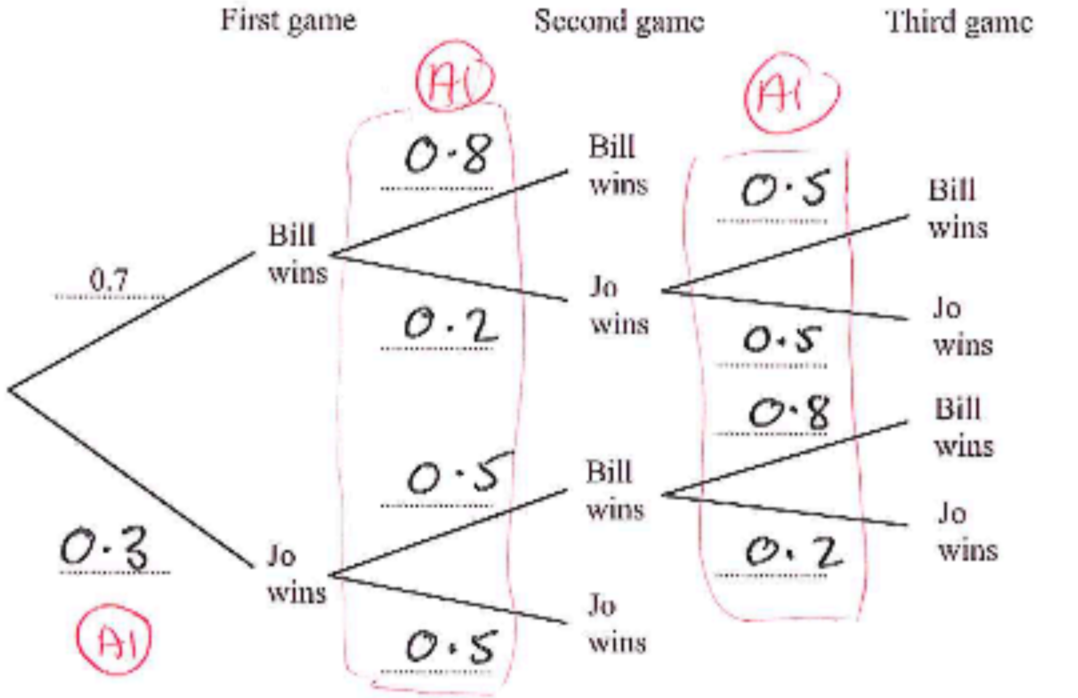
$$P(DDB) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12} \quad \text{M1}$$

$$\text{TOTAL} = \frac{7}{12} \quad \text{A1}$$

Bill and Jo play some games of table tennis.
The probability that Bill wins the first game is 0.7
When Bill wins a game, the probability that he wins the next game is 0.8
When Jo wins a game, the probability that she wins the next game is 0.5

The first person to win two games wins the match.

(a) Complete the probability tree diagram.



(3)

(b) Calculate the probability that Bill wins the match.

$P(BB) = 0.7 \times 0.8$

$P(BJB) = 0.7 \times 0.2 \times 0.5$

$P(JBB) = 0.3 \times 0.5 \times 0.8$

TOTAL = 0.75 A1

M1 [THREE POSSIBILITIES]

M1 [MULTIPLYING PROBABILITIES]

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B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site.

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words 'show that', 'show your working out', or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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