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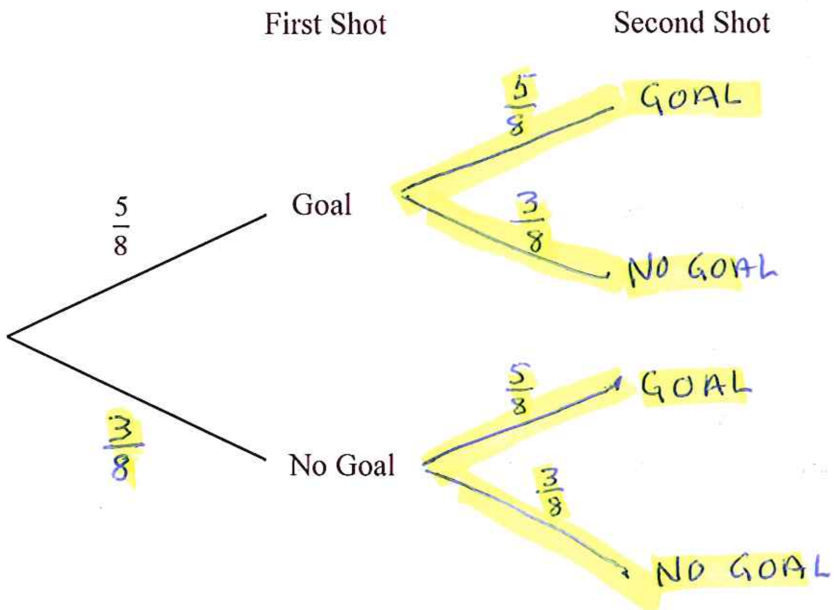
WORKBOOK

PROBABILITY TREES (STANDARD)

1. Each time that Sam takes a shot at goal, the probability that he will score is $\frac{5}{8}$

Sam takes two shots.

(a) Complete the probability tree.



(b) What is the probability that Sam

(i) Scores twice

$P(G,G) = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$

(ii) Scores exactly once

$P(G,\bar{G}) = \frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$

$P(\bar{G},G) = \frac{3}{8} \times \frac{5}{8} = \frac{15}{64}$

TOTAL $= \frac{30}{64}$ $\left(\frac{15}{32}\right)$

(iii) Scores at least once

$1 - P(\bar{G},\bar{G})$

$1 - \frac{3}{8} \times \frac{3}{8} = 1 - \frac{9}{64} = \frac{55}{64}$

* NOTATION:

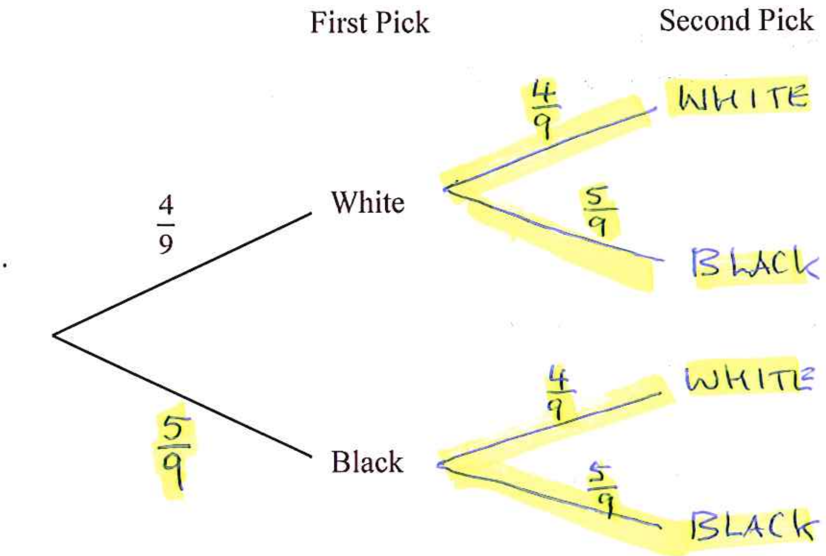
$\bar{G}$ MEANS 'NO GOAL'

2. A bag contains 4 white beads and 5 black beads.

Amy picks at random a bead from the bag and **replaces** it.

The beads are mixed and she then picks at random another bead from the bag.

(a) Complete the probability tree of the problem.



(b) What is the probability that Amy picks:

(i) two black beads

$$P(B,B) = \frac{5}{9} \times \frac{5}{9} = \frac{25}{81}$$

(ii) a black bead in his second draw.

$$P(B,B) = \frac{5}{9} \times \frac{5}{9} = \frac{25}{81}$$

$$P(W,B) = \frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$$

TOTAL $= \frac{45}{81}$ $\left(\frac{5}{9}\right)$

(iii) beads of different colours

$$P(W,B) = \frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$$

$$P(B,W) = \frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$$

TOTAL $= \frac{40}{81}$

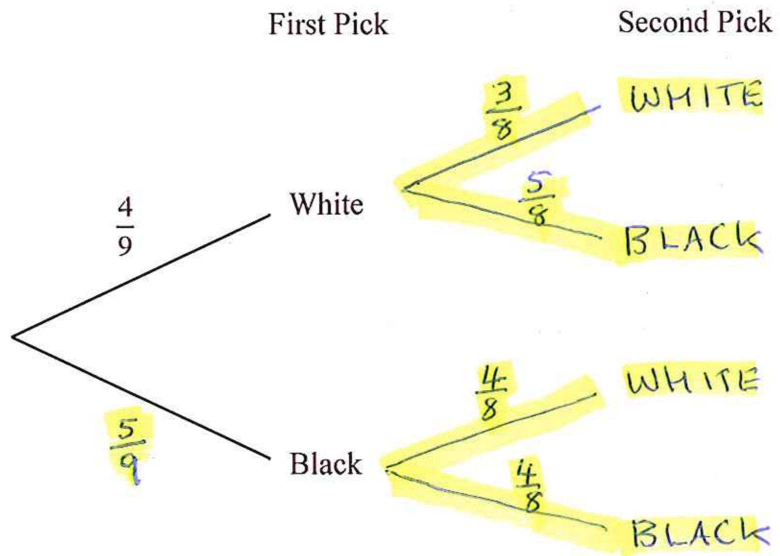
3. A bag contains 4 white beads and 5 black beads.

Shanaiya picks at random a bead from the bag and **<u>does not replace</u>** it.

The beads are mixed and she then picks at random another bead from the bag.

* PROBABILITIES CHANGE

(a) Complete the probability tree of the problem.



(b) What is the probability that Shanaiya picks:

(i) two black beads

$P(B,B) = \frac{5}{9} \times \frac{4}{8} = \frac{20}{72} \quad \left(\frac{5}{18}\right)$

(ii) a black bead in her second draw.

$$\left.\begin{aligned} P(B,B) &= \frac{5}{9} \times \frac{4}{8} = \frac{20}{72} \\ P(W,B) &= \frac{4}{9} \times \frac{5}{8} = \frac{20}{72} \end{aligned}\right\} \text{TOTAL} = \frac{40}{72} \quad \left(\frac{5}{9}\right)$$

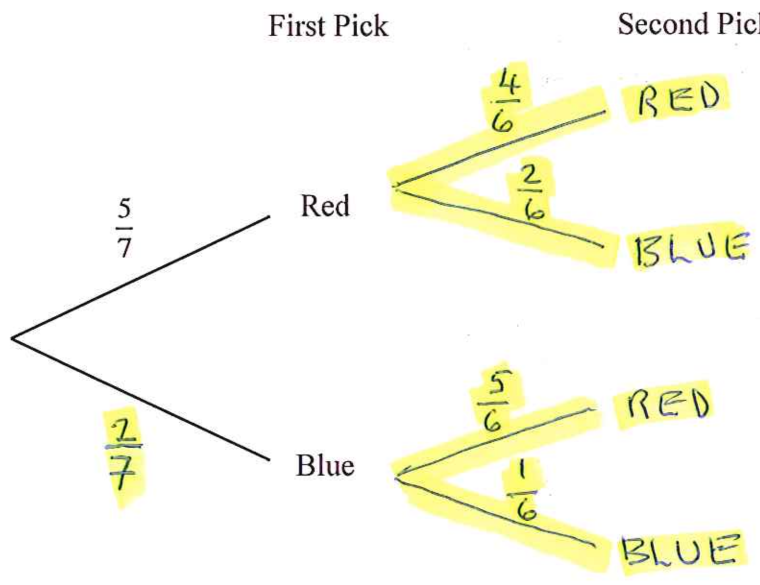
(iii) beads of different colours

$$\left.\begin{aligned} P(W,B) &= \frac{4}{9} \times \frac{5}{8} = \frac{20}{72} \\ P(B,W) &= \frac{5}{9} \times \frac{4}{8} = \frac{20}{72} \end{aligned}\right\} \text{TOTAL} = \frac{40}{72} \quad \left(\frac{5}{9}\right)$$

4. A packet contains 5 red and 2 blue sweets. Pardeep picks a sweet from the packet, eats it and then picks a second sweet and eats it.

PROBABILITIES CHANGE

(a) Complete the probability tree.



(b) Calculate the probability that Pardeep eats:

(i) one red sweet and one blue sweet

$$P(R,B) = \frac{5}{7} \times \frac{2}{6} = \frac{10}{42}$$

$$P(B,R) = \frac{2}{7} \times \frac{5}{6} = \frac{10}{42}$$

$$\text{TOTAL} = \frac{20}{42} \quad \left(\frac{10}{21}\right)$$

(ii) two sweets of the same colour.

$$P(R,R) = \frac{5}{7} \times \frac{4}{6} = \frac{20}{42}$$

$$P(B,B) = \frac{2}{7} \times \frac{1}{6} = \frac{2}{42}$$

$$\text{TOTAL} = \frac{22}{42} \quad \left(\frac{11}{21}\right)$$

(iii) at least one red sweet

$$1 - P(B,B) = 1 - \frac{2}{7} \times \frac{1}{6}$$

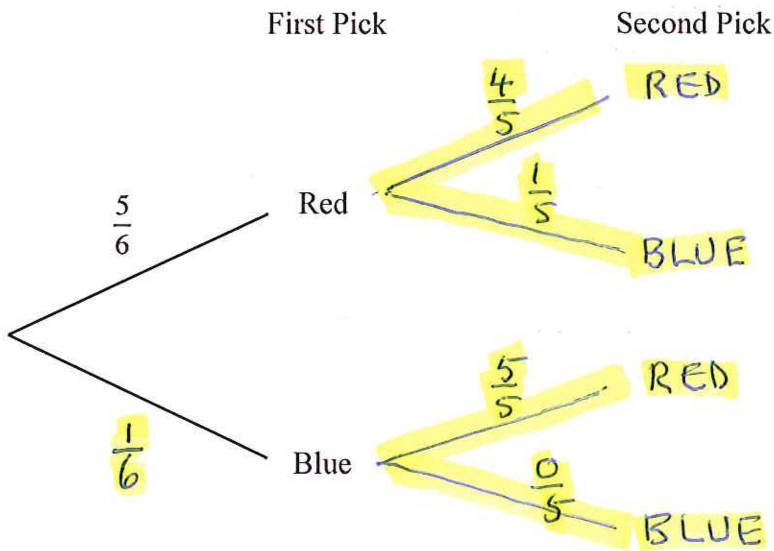
$$= 1 - \frac{2}{42}$$

$$= \frac{40}{42} \quad \left(\frac{20}{21}\right)$$

5. A packet contains 5 red and 1 blue sweets. Maddie picks a sweet from the packet, eats it and then picks a second sweet and eats it. *

PROBABILITIES CHANGE

(a) Complete the probability tree.



(b) Calculate the probability that Maddie eats:

(i) one red sweet and one blue sweet

$$\left.\begin{array}{l} P(R,B) = \frac{5}{6} \times \frac{1}{5} = \frac{5}{30} \\ P(B,R) = \frac{1}{6} \times \frac{5}{5} = \frac{5}{30} \end{array}\right\} \text{TOTAL} = \frac{10}{30} \quad \left(\frac{1}{3}\right)$$

(ii) two sweets of the same colour.

$$P(R,R) = \frac{5}{6} \times \frac{4}{5} = \frac{20}{30} \quad \left(\frac{2}{3}\right)$$

$P(B,B) = 0$ [ONLY ONE BLUE SWEET!]

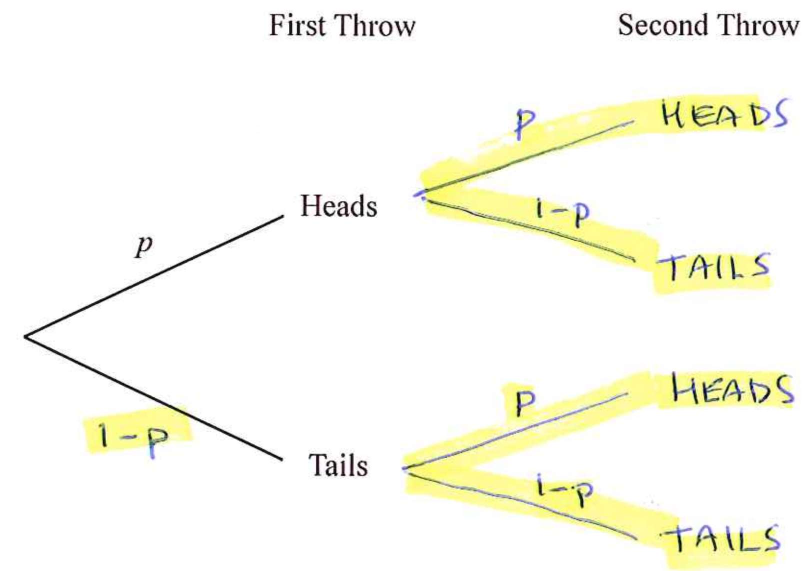
(iii) at least one red sweet

$$1 - P(B,B) = 1$$

6. Michael has a biased coin.
The probability that Michael will throw a Head on any throw is p.
Michael throws the coin twice.

PROBABILITIES DON'T CHANGE

(a) Complete the probability tree.
Give the probabilities in terms of p.



(b) Find expressions, in terms of p, for the probability that Michael will throw:

(i) two heads

$P(H, H) = p \times p = p^2$

(ii) two tails

$P(T, T) = (1-p)(1-p) = (1-p)^2$

(iii) exactly one head

$$\left.\begin{array}{l} P(H, T) = p(1-p) \\ P(T, H) = p(1-p) \end{array}\right\} \text{TOTAL} = 2p(1-p)$$

[OR $2p - 2p^2$]

Given that $p = 0.7$

(c) Work out the probability that Michael will throw a head on his second throw.

$$\left.\begin{array}{l} P(H, H) = p \times p = p^2 \\ P(T, H) = p(1-p) = p - p^2 \end{array}\right\} \text{TOTAL} = p^2 + p - p^2$$

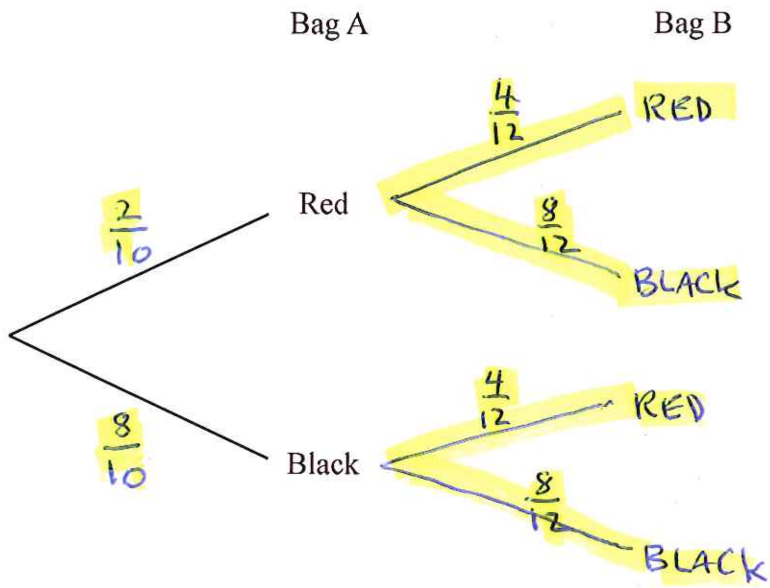
$= p$

7. Bag A contains 10 marbles of which 2 are red and 8 are black.

Bag B contains 12 marbles of which 4 are red and 8 are black.

Imogen chooses a marble at random from each bag.

(a) Complete the probability tree.



(b) Find the probability that Imogen chooses:

(i) two red marbles

$$P(R,R) = \frac{2}{10} \times \frac{4}{12} = \frac{8}{120} \quad \left(\frac{1}{15}\right)$$

(ii) two black marbles

$$P(B,B) = \frac{8}{10} \times \frac{8}{12} = \frac{64}{120} \quad \left(\frac{8}{15}\right)$$

(iii) one black and one red marble

$$P(B,R) = \frac{8}{10} \times \frac{4}{12} = \frac{32}{120}$$

$$P(R,B) = \frac{2}{10} \times \frac{8}{12} = \frac{16}{120}$$

$$\text{TOTAL} = \frac{48}{120} \quad \left(\frac{2}{5}\right)$$

(iv) at least one red marble.

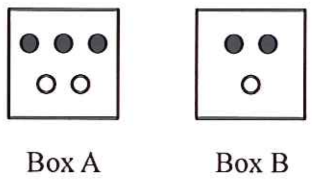
$$1 - P(B,B) = 1 - \frac{8}{10} \times \frac{8}{12}$$

$$= 1 - \frac{64}{120}$$

$$= \frac{56}{120} \quad \left(\frac{7}{15}\right)$$

8. In box A, there are 3 black counters and 2 white counters.

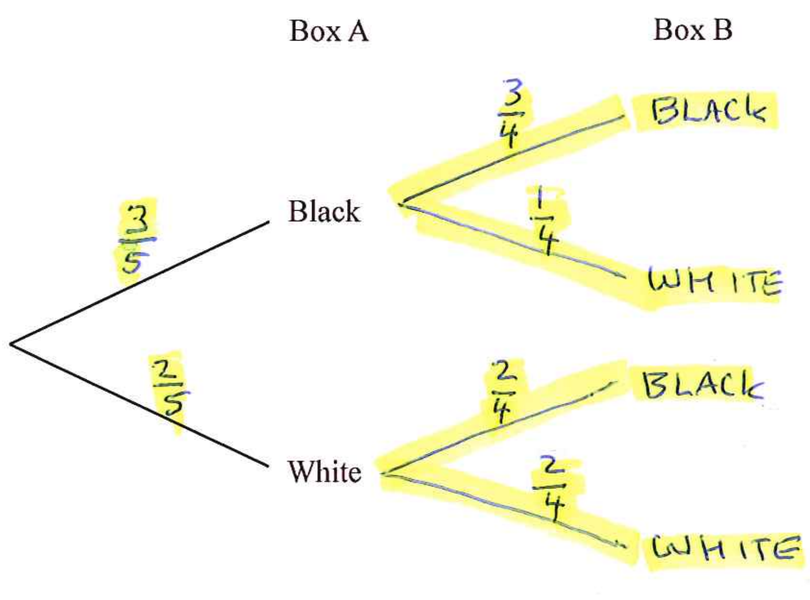
In box B there are 2 black counters and 1 white counter.



Alec takes at random a counter from Box A and puts it into Box B.

He then takes at Random a counter from Box B.

(a) Complete the probability tree.



(b) Work out the probability that the counter that he takes from Box B will be a black counter.

$$P(B,B) = \frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$$

$$P(W,B) = \frac{2}{5} \times \frac{2}{4} = \frac{4}{20}$$

TOTAL $= \frac{13}{20}$