

QUADRATIC EQUATIONS

DATE OF SOLUTIONS: 09/06/2018

MAXIMUM MARK: 48

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1.

Quadratic Equations [2 marks]

Solve $2x^2 = 72$

$$2x^2 = 72$$

$$x^2 = \frac{72}{2}$$

$$\Rightarrow x^2 = 36 \quad (\text{ml})$$

$$x = \sqrt{36}$$

$$= \underline{\underline{\pm 6}}$$

(A1) [BOTH]

$$\underline{\underline{x = +6 \text{ OR } x = -6}}$$

(a) Factorise $x^2 + 4x - 12$

DIFFERENT SIGNS!

1×12

2×6

3×4

-2 AND +6 GIVES 4(x)!

$$\frac{(x-2)(x+6)}{(2)}$$

(b) Hence, or otherwise, solve the equation $x^2 + 4x - 12 = 0$

$$(x-2)(x+6) = 0$$

$$x = 2$$

$$x = -6$$

(A1) [BOTH]

$$x = 2 \text{ OR } x = -6$$

(1)

Solve $3x^2 + 8x + 2 = 0$ Give your solutions correct to 3 significant figures.
Show your working clearly.

$$a = 3 \quad b = 8 \quad c = 2$$

$$\text{USE } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(3)(2)}}{2(3)} \quad \text{(M1) [SUBSTITUTION]}$$

$$= \frac{-8 \pm \sqrt{64 - 24}}{6} \quad \text{(M1) [SIMPLIFYING]}$$

$$\begin{array}{ccc} \swarrow & & \searrow \\ \frac{-8 + \sqrt{40}}{6} & & \frac{-8 - \sqrt{40}}{6} \\ = \underline{\underline{-0.279}} & & = \underline{\underline{-2.39}} \end{array}$$

(A1) [BOTH]

(a) Solve $x^2 - 8x + 15 = 0$

$$x^2 - 8x + 15 = 0$$

$$(x-3)(x-5) = 0 \quad \textcircled{B1} \text{ [FACTORISING]}$$

$$x = \underline{\underline{3}} \qquad x = \underline{\underline{5}}$$

$$\underline{\underline{x = 3 \text{ or } x = 5}} \quad \textcircled{A1} \quad \textcircled{A2}$$

(3)

(b) Hence, or otherwise, write down the solutions to $(x+2)^2 - 8(x+2) + 15 = 0$

$$(x+2) = 3 \qquad (x+2) = 5$$

$$\Rightarrow x = \underline{\underline{1}} \qquad \Rightarrow x = \underline{\underline{3}} \quad \textcircled{A1} \text{ [BOTH]}$$

$$\underline{\underline{x = 1 \text{ or } x = 3}} \quad \textcircled{A1}$$

(1)

Solve $2x^2 + 3x - 7 = 0$

Give your solutions correct to 3 significant figures.
Show your working clearly.

$a = 2$ $b = 3$ $c = -7$

USE $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-7)}}{2(2)} \quad (M1) \quad [SUBSTITUTION]$$

$$= \frac{-3 \pm \sqrt{9 + 56}}{4} \quad (M1) \quad [SIMPLIFYING]$$

$$\begin{array}{ccc} \swarrow & & \searrow \\ \frac{-3 + \sqrt{65}}{4} & & \frac{-3 - \sqrt{65}}{4} \end{array}$$

$$= \underline{\underline{1.27}}$$

$$= \underline{\underline{-2.77}}$$

(A1) [BOTH]

Mel is using the quadratic formula to solve a quadratic equation.

She substitutes values into the formula and correctly gets

$$\frac{-5 \pm \sqrt{25 - 12}}{6} \rightarrow 4ac$$

← b → b²
6 → 2a

Work out the quadratic equation that Mel is solving.

Give your answer in the form $ax^2 + bx + c = 0$, where a , b and c are integers.

1ST

$$2a = 6$$

$$\Rightarrow a = \underline{\underline{3}}$$

3RD

$$4ac = 12$$

$$\Rightarrow 4 \times 3 \times c = 12$$

$$c = \underline{\underline{1}}$$

a=3

2ND

$$b = \underline{\underline{5}}$$

$$\begin{array}{ccc} \textcircled{B1} & \textcircled{B1} & \textcircled{B1} \\ \hline 3x^2 + 5x + 1 = 0 \end{array}$$

Solve $x^2 - 7x + 3 = 0$

Give your solutions correct to 3 significant figures.

$$a = 1 \quad b = -7 \quad c = 3$$

$$\text{USE } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(3)}}{2(1)} \quad \text{(M1)}$$

[SUBSTITUTION]

$$= \frac{+7 \pm \sqrt{49 - 12}}{2} \quad \text{(M1)}$$

[SIMPLIFYING]

$$\frac{7 + \sqrt{37}}{2}$$

$$\frac{7 - \sqrt{37}}{2}$$

$$= \underline{\underline{6.54}}$$

$$= \underline{\underline{0.459}}$$

(A1)
[BOTH]

(a) Factorise $3x^2 + 7x - 6$

$ac = -18 \rightarrow -1 \times 18$
 -2×9
 -3×6

$-2 \text{ AND } 9 \text{ GIVE } 7!$

$$\underline{3x^2 - 2x + 9x - 6}$$

$$x(3x - 2) + 3(3x - 2)$$

$$= \underline{\underline{(x + 3)(3x - 2)}}$$

(b) Hence, or otherwise, solve the equation $3x^2 + 7x - 6 = 0$

$$x + 3 = 0 \qquad 3x - 2 = 0$$

$$\Rightarrow x = \underline{\underline{-3}} \qquad \Rightarrow x = \frac{2}{3}$$

(A1)
[BOTH]

(B1)

(B1)

Solve $x^2 + 5x = 12$

Give your solutions correct to 3 significant figures.

USE FORMULA!

$$x^2 + 5x = 12$$

$$x^2 + 5x - 12 = 0$$

$$[a=1, b=5, c=-12]$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-12)}}{2(1)} \quad (m)$$

[SUBSTITUTE]

$$= \frac{-5 \pm \sqrt{25 + 48}}{2} \quad (m) \quad [SIMPLIFY]$$

$$\frac{-5 + \sqrt{73}}{2}$$

$$\frac{-5 - \sqrt{73}}{2}$$

$$= \underline{\underline{1.77}}$$

$$= \underline{\underline{-6.77}}$$

(A)
[BOTH]

Solve $(2x-5)^2 = 49$

$$(2x-5)^2 = 49$$

$$2x-5 = \sqrt{49}$$

$$2x-5 = \pm 7$$

$$\left. \begin{array}{l} 2x-5 = \sqrt{49} \\ 2x-5 = \pm 7 \end{array} \right\} \textcircled{m} \text{ [EITHER]}$$

$$2x-5 = 7 \quad \leftarrow$$

$$2x = 12$$

$$x = \underline{\underline{6}} \quad \textcircled{A1}$$

$$\rightarrow 2x-5 = -7$$

$$2x = -2$$

$$x = \underline{\underline{-1}} \quad \textcircled{A1}$$

A ball is thrown vertically upwards from a point P .

The height above P of the ball t seconds after it was thrown is h metres, where $h = 11t - 5t^2$

Work out the values of t when the height of the ball above P is 5 metres.

Show your working clearly.

$$5 = 11t - 5t^2$$

$$5t^2 - 11t + 5 = 0 \quad [a=5, b=-11, c=5]$$

$$t = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(5)(5)}}{2(5)} \quad (M1)$$

[SUBSTITUTION]

$$= \frac{11 \pm \sqrt{121 - 100}}{10} \quad (M1) \quad [SIMPLIFYING]$$

$$\frac{11 + \sqrt{21}}{10} \quad \frac{11 - \sqrt{21}}{10}$$

$$= \underline{\underline{1.56}} \quad = \underline{\underline{0.642}}$$

(A1) [BOTH]

Solve $2x^2 - 8 = 3x + 5$

Give your answers correct to 3 significant figures.

$$2x^2 - 8 = 3x + 5$$

$$\Rightarrow 2x^2 - 3x - 13 = 0 \quad (B1) \quad [a=2, b=-3, c=-13]$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-13)}}{2(2)} \quad (M1)$$

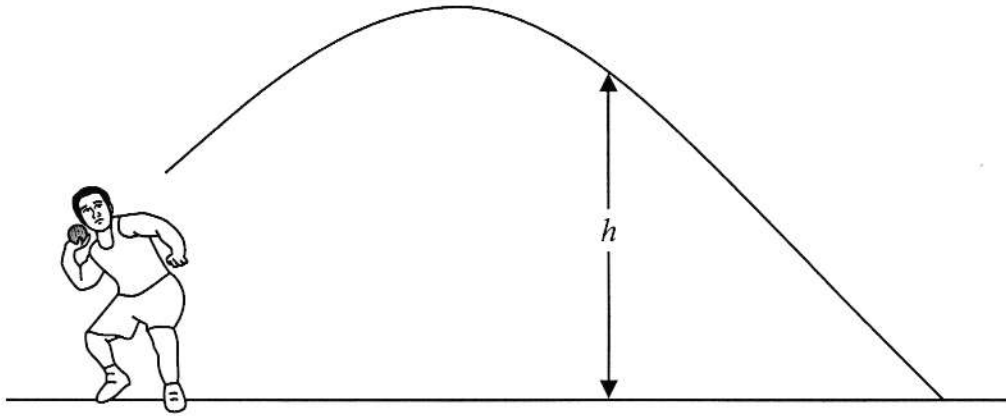
$$= \frac{+3 \pm \sqrt{9 + 104}}{4} \quad (M1) \quad [SIMPLIFYING]$$

$$\frac{3 + \sqrt{113}}{4} \quad \leftarrow \quad \frac{3 - \sqrt{113}}{4}$$

$$= \underline{\underline{3.41}}$$

$$= \underline{\underline{-1.91}}$$

(A1)
[BOTH]



Ivan is a shot putter.

The formula $h = 2 + 6t - 5t^2$ gives the height, h metres, of the shot above the ground t seconds after he has released the shot.

(i) Solve $2 + 6t - 5t^2 = 0$

Give your solutions correct to 3 significant figures.
Show your working clearly.

NOTE THAT $a = -5$, $b = 6$, $c = 2$

$$t = \frac{- (6) \pm \sqrt{(6)^2 - 4(-5)(2)}}{2(-5)} \quad \text{(M1) [SUBSTITUTION]}$$

$$= \frac{-6 \pm \sqrt{36 + 40}}{-10} \quad \text{(M1) [SIMPLIFYING]}$$

$$\begin{array}{l} \swarrow \quad \searrow \\ \frac{-6 + \sqrt{76}}{-10} \quad \quad \quad \frac{-6 - \sqrt{76}}{-10} \end{array}$$

$$= \underline{\underline{-0.272}} \quad \quad \quad = \underline{\underline{1.47}} \quad \text{(A1) [BOTH]}$$

The shot hits the ground after T seconds.

(ii) Write down the value of T .

Give your answer correct to 3 significant figures.

[THE ANSWER CANNOT
BE NEGATIVE]

$$T = \underline{\underline{1.47}} \text{ secs}$$

(A1)

Solve $3x^2 - x - 1 = 0$

Give your solutions correct to 2 decimal places.

$$a = 3 \quad b = -1 \quad c = -1$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-1)}}{2(3)} \quad (M1)$$

$$= \frac{+1 \pm \sqrt{1 + 12}}{6} \quad (M1)$$

$$\frac{1 + \sqrt{13}}{6}$$

$$\frac{1 - \sqrt{13}}{6}$$

$$= \underline{\underline{0.77}}$$

$$= \underline{\underline{-0.43}}$$

(A1)
[BOTH]

Solve $(x-3)^2 = x+5$

Give your answers correct to 3 significant figures.

$$(x-3)^2 = x+5$$

$$\Rightarrow x^2 - 6x + 9 = x + 5 \quad (M1)$$

$$\Rightarrow x^2 - 7x + 4 = 0 \quad (M1) \quad [a=1, b=-7, c=4]$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(4)}}{2(1)} \quad (M1)$$

$$= \frac{+7 \pm \sqrt{49-16}}{2}$$

$$\frac{7 + \sqrt{33}}{2}$$

$$\frac{7 - \sqrt{33}}{2}$$

$$= \underline{\underline{6.37}}$$

$$= \underline{\underline{0.628}}$$

(A1)
[BOTH]

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Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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