RECURRING DECIMALS

DATE OF SOLUTIONS: 08/06/2018
MAXIMUM MARK: 27

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1 Recurring Decimals (Converting to Fractions) **[1 mark]**

Circle the fractions which can be written as recurring decimals.

$\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{5}{6}$ $\frac{5}{7}$ $\frac{7}{8}$ $\frac{5}{12}$

[0.75] [0.8] [0.875]

B1

Write these numbers in order of size.

Start with the smallest number.

$0.57\dot{3}7373...$ $0.5\dot{7}\dot{3}5735...$ 0.573 $0.57\dot{3}3333...$

(4) (3) (1) (2)

$0.573,\ 0.57\dot{3},\ 0.5\dot{7}\dot{3},\ 0.5\dot{7}\dot{3}$ B1

Prove algebraically that the recurring decimal $0.\dot{3}4$ has the value $\frac{31}{90}$

ONE RECURRING DIGIT [MULTIPLY BY 10!]

M1

$10x = 3.44444\ldots$

$x = 0.34444\ldots$

$9x = 3.1$ [SUBTRACTING]

$\Rightarrow x = \frac{3.1}{9}$ M1 [EITHER]

$= \frac{31}{90}$

Use algebra to show that the recurring decimal $0.\dot{3}\dot{8} = \frac{7}{18}$

ONE RECURRING DIGIT [MULTIPLY BY 10!]

M1

$10x = 3.88888...$

$x = 0.38888...$

[SUBTRACTING]

$9x = 3.5$

$\Rightarrow x = \frac{3.5}{9}$

$= \frac{35}{90}$

M1 [EITHER]

$= \frac{7}{18}$

Use algebra to show that the recurring decimal $0.\dot{4}1\dot{7} = \frac{139}{333}$

THREE RECURRING DIGITS

M1

$1000x = 417.417417\ldots$

$x = 0.417417\ldots$

[MULTIPLY BY 1000!]

$999x = 417$ [SUBTRACTING]

$\Rightarrow x = \frac{417}{999}$ M1 [EITHER]

$= \frac{139}{333}$

Prove algebraically that the recurring decimal $0.\dot{3}\dot{1}\dot{8}$ can be written as $\frac{7}{22}$

TWO RECURRING DIGITS

(M1) $100x = 31.818181\ldots$

$x = 0.318181\ldots$

[MULTIPLY BY 100!]

$99x = 31.5$ [SUBTRACTING]

$\Rightarrow x = \frac{31.5}{99}$ (M1) [EITHER]

$= \frac{315}{990}$

$= \frac{7}{22}$

Using algebra, prove that $0.3\dot{2}\dot{7} \times 0.\dot{5}$ is equal in value to $\frac{2}{11}$

1ST

$100x = 32.7272\ldots$

$x = 0.3272\ldots$

$99x = 32.4$

$x = \frac{32.4}{99}$

$= \frac{324}{990}$

$= \frac{18}{55}$ B1

2ND

$10x = 5.5555\ldots$

$x = 0.5555\ldots$

$9x = 5$

$x = \frac{5}{9}$ B1

3RD

So, $0.3\dot{2}\dot{7} \times 0.\dot{5} = \frac{18}{55} \times \frac{5}{9}$ M1

$= \frac{2}{11}$

x is an integer such that $1 \leq x \leq 9$

Show that:

(a) $0.\dot{x} = \frac{x}{9}$

M1 $0.\dot{x} = 0.xxxx\ldots$

$10 \times 0.\dot{x} = x.xxxx\ldots$ } SUBTRACT

$9 \times 0.\dot{x} = x$ M1

$\Rightarrow 0.\dot{x} = \frac{x}{9}$

(2)

(b) $0.\dot{0}\dot{x} = \frac{x}{99}$

M1 $0.\dot{0}\dot{x} = 0.0x0x0x\ldots$

$100 \times 0.\dot{0}\dot{x} = x.0x0x0x\ldots$ } SUBTRACT

$99 \times 0.\dot{0}\dot{x} = x$ M1

$\Rightarrow 0.\dot{0}\dot{x} = \frac{x}{99}$

(2)

y is a whole number such that $1 \leq y \leq 9$

Show that $0.\dot{3}\dot{y} = \frac{y}{33}$

M1

M1 $0.\dot{3}\dot{y} = 0.3y3y3y...$

$100 \times 0.\dot{3}\dot{y} = 3y.3y3y3y...$ } SUBTRACT

$99 \times 0.\dot{3}\dot{y} = 3y$

$\Rightarrow 0.\dot{3}\dot{y} = \frac{3y}{99}$ } M1 [EITHER]

$= \frac{y}{33}$

Rita says:

"I can tell from the denominators that $\frac{17}{40}$ will convert into a terminating decimal but $\frac{17}{70}$ will be recurring."

Explain how Rita can tell from the denominators, whether a fraction will convert into a terminating decimal or a recurring decimal.

A1

THE ONLY PRIME FACTORS OF 40 ARE 2 AND 5 WHICH BOTH DIVIDE INTO TEN, SO $\frac{17}{40}$ WILL TERMINATE. 70 HAS A PRIME FACTOR THAT WILL NOT DIVIDE INTO TEN, SO $\frac{17}{70}$ WILL BE A RECURRING DECIMAL

A1

(a) Convert the recurring decimal $0.\dot{7}$ to a fraction.

M1 $10x = 7.7777\ldots$
$x = 0.7777\ldots$ } SUBTRACT
$9x = 7$ → $x = \frac{7}{9}$ A1

(2)

$0.0\dot{y}$ is a recurring decimal.
y is a whole number such that $1 \leqslant y \leqslant 9$

(b) (i) Write the recurring decimal $0.0\dot{y}$ as a fraction.

$0.\dot{7} = \frac{7}{9} \Rightarrow 0.\dot{y} = \frac{y}{9}$
$\Rightarrow 0.0\dot{y} = \frac{y}{9} \div 10$ → $\frac{y}{90}$ A1

(ii) $0.1\dot{y}$ is also a recurring decimal.
Using your answer to part (i), or otherwise, convert the recurring decimal $0.1\dot{y}$ to a fraction.
Give your answer as simply as possible.

$\frac{y}{90} + \frac{1}{10} = \frac{y}{90} + \frac{9}{90} = \frac{y+9}{90}$ A1

M1

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Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners' mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners' mark scheme.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site.

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words 'show that', 'show your working out', or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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