

SECTORS

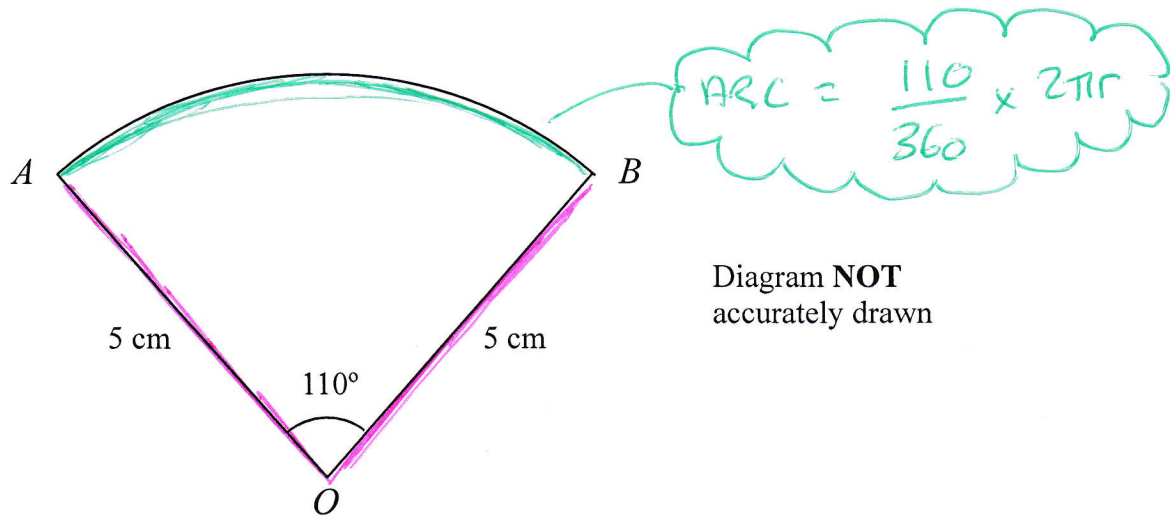
DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 59

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [New Question, by Maths4Everyone.com]

Sectors (Area and Perimeter of Part-Circles) [3 Marks]



The diagram shows a sector of a circle, centre O .
The radius of the circle is 5 cm.
Angle $AOB = 110^\circ$.

Work out the **perimeter** of the sector.
Give your answer correct to 3 significant figures.

$$\begin{aligned} \text{PERIMETER} &= \frac{110}{360} \times 2\pi \times 5 + 10 \\ &= 19.599 \dots \end{aligned}$$

..... 19.6 cm

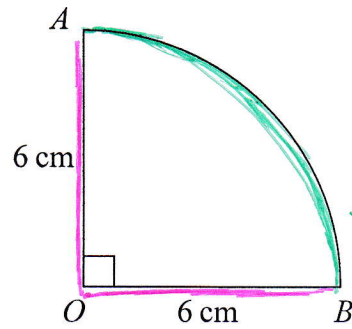


Diagram NOT
accurately drawn

$$\text{ARC} = \frac{1}{4} \times 2\pi r$$

The diagram shows a shape.

AB is an arc of a circle, centre O .

Angle $AOB = 90^\circ$.

$OA = OB = 6$ cm.

Calculate the perimeter of the shape.

Give your answer correct to 3 significant figures.

$$\begin{aligned} \text{PERIMETER} &= \frac{1}{4} \times 2\pi r + 12 \quad \leftarrow \text{(m)} \\ &= 21.424\dots \end{aligned}$$

$$\dots 21.4 \text{ (A)} \text{ cm}$$

The diagram shows a sector of a circle, radius 45 cm, with angle 84° .

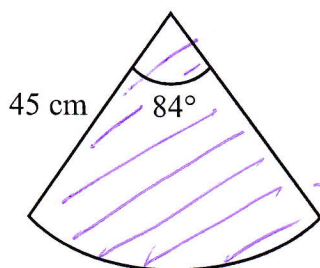


Diagram **NOT** accurately drawn

$$A = \frac{84}{360} \times \pi r^2$$

Calculate the area of the sector.

Give your answer correct to 3 significant figures.

$$A = \frac{84}{360} \times \pi \times 45^2 = 1484.402\dots$$

\rightarrow 1480 cm^2

Work out the area of the shaded sector of the circle.
Give your answer correct to 3 significant figures.

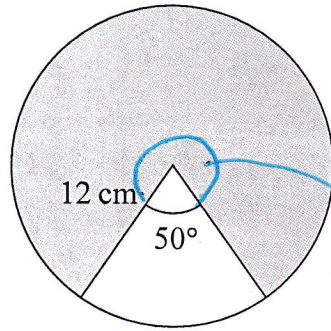


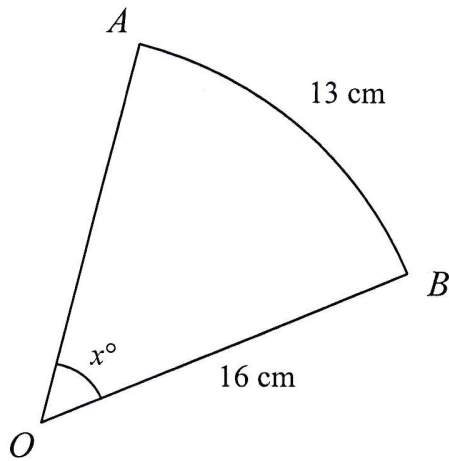
Diagram NOT
accurately drawn

$$A = \frac{310}{360} \times \pi \times 12^2$$

$$= 124\pi$$

$$= 389.557\dots$$

$$\dots\dots\dots 390 \text{ cm}^2$$



USE $L = \frac{x}{360} \times 2\pi r$

Diagram **NOT**
accurately drawn

AB is an arc of length 13 cm of a circle centre O . The radius of the circle is 16 cm.
Calculate the value of x .

$$\frac{x}{360} \times 2 \times \pi \times 16 = 13$$

(m)

$$\Rightarrow x = \frac{13 \times 360}{2 \times \pi \times 16}$$

(m)
[REARRANGING]

$$= 46.55\dots$$

$$x = 46.6^\circ$$

(A)

A fan is shaped as a sector of a circle, radius 12 cm, with angle 110° at the centre.

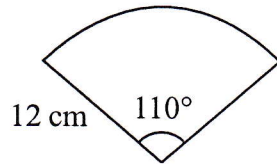


Diagram NOT accurately drawn

$$A = \frac{110}{360} \times \pi r^2$$

(a) Calculate the area of the fan.

$$A = \frac{110}{360} \times \pi \times 12^2 \quad (M1)$$

$$= 138.23\dots$$

$$\dots 138 \dots \text{cm}^2 \quad (A1) \quad (2)$$

Another fan is shaped as a sector of a circle, radius r cm, with angle 120° at the centre.

$$L = \frac{120}{360} \times 2\pi r$$

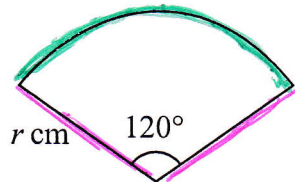


Diagram NOT accurately drawn

(b) Show that the total perimeter of this fan is $\frac{2}{3}r(3 + \pi)$ cm.

$$\text{PERIMETER} = \frac{120}{360} \times 2\pi r + 2r \quad (M1) \quad [\text{STATEMENT}]$$

$$= \frac{1}{3} \times 2\pi r + 2r \quad (M1) \quad [\text{SIMPLIFYING}]$$

$$= 2r \left(\frac{1}{3}\pi + 1 \right) \quad (M1) \quad [\text{FACTORISING}]$$

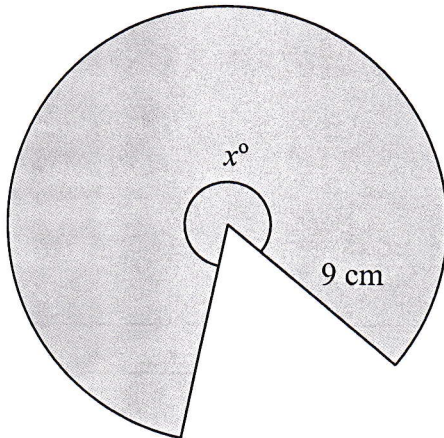
$$= \frac{2}{3}r (\pi + 3)$$

$$\left[= \frac{2}{3}r (3 + \pi) \right]$$

In the diagram, a sector of a circle of radius 9 cm is shaded.

The area of the sector is $72\pi \text{ cm}^2$.

Calculate the value of x .



$$A = \frac{x}{360} \times \pi r^2$$

Diagram NOT
accurately drawn

$$\frac{x}{360} \times \pi \times 9^2 = 72\pi$$

$$\Rightarrow x = \frac{72\pi \times 360}{\pi \times 9^2} \quad (M1)$$

$$= \frac{2592}{81}$$

$$x = \underline{320} \quad (A1)$$

(M1) [STATEMENT]

The diagram shows sector OAB of a circle, centre O .

$$A = \frac{50}{360} \times \pi r^2$$

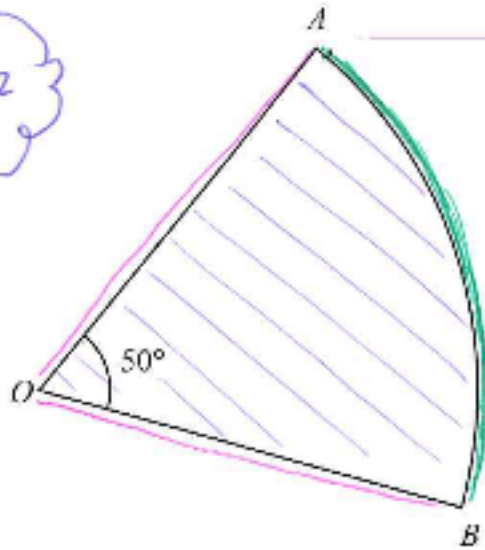


Diagram NOT
accurately drawn

Angle $AOB = 50^\circ$
Sector OAB has area $20\pi \text{ cm}^2$

Calculate the perimeter of sector OAB .
Give your answer correct to 3 significant figures.

$$\text{ARC} = \frac{50}{360} \times 2\pi r$$

1ST

$$\text{AREA} = 20\pi$$

$$\text{So } \frac{50}{360} \times \pi r^2 = 20\pi \quad (\text{mf})$$

$$\Rightarrow \frac{50}{360} \times r^2 = 20$$

$$\Rightarrow r^2 = \frac{20 \times 360}{50}$$

$$r = \sqrt{\frac{20 \times 360}{50}}$$

$$= \underline{\underline{12 \text{ cm}}} \quad (\text{AI})$$

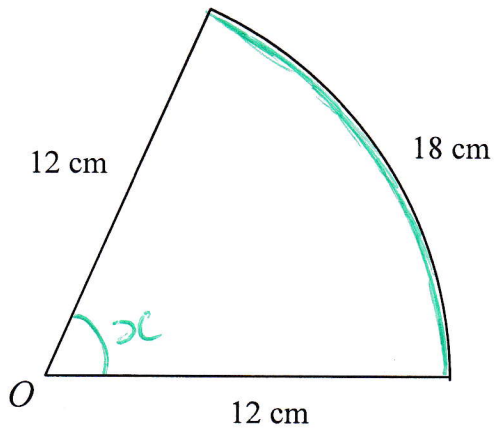
2ND

$$\text{PERIMETER} = \frac{50}{360} \times 2\pi r + 2r$$

$$= \frac{50}{360} \times 2\pi \times 12 + \frac{2 \times 12}{\uparrow} \quad (\text{mf}) \quad (\text{BI})$$

$$= 34.4719\dots$$

$$= \underline{\underline{34.5 \text{ cm}}} \quad (\text{AI})$$



$$L = \frac{x}{360} \times 2\pi \times 12$$

Diagram **NOT**
accurately drawn

The diagram shows a sector of a circle, centre O , radius 12 cm.
The arc length of the sector is 18 cm.

Calculate the area of the sector.

[1ST] $\frac{x}{360} \times 2\pi \times 12 = 18$ (m) [STATEMENT]

[FIND x]

$$\Rightarrow x = \frac{18 \times 360}{2\pi \times 12}$$

(m) [REARRANGE]

$$= 85.94\dots$$

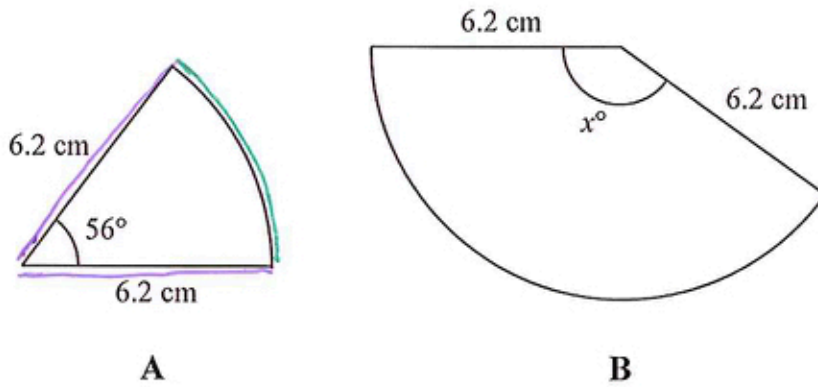
(A)

$$A = \frac{85.94}{360} \times \pi \times 12^2$$

(m) [USE '85.94' IN AREA FORMULA]

$$= \underline{\underline{108}}$$

..... 108 (A) cm²



Diagrams NOT
accurately Drawn

The diagram shows two different sectors from a circle of radius 6.2 cm

(a) Calculate the perimeter of sector A.

$$\frac{56}{360} \times 2\pi \times 6.2 \quad + \quad 12.4$$

(m1) (m1)

[NOTE THAT '6.1' SCORES
JUST ONE MARK]

$$\dots\dots\dots 18.5 \text{ (A1) [AWRT]} \text{ cm}$$

(3)

(b) The area of sector B is 48 cm^2 .
Calculate the value of x .

$$\frac{x}{360} \times \pi \times 6.2^2 = 48 \quad \text{(m1) [CORRECT EQUATION]}$$

$$x = \frac{48}{\pi \times 6.2^2} \times 360 \quad \text{(m1) [ISOLATING 'x']}$$

$$= 143.09\dots$$

$$\dots\dots\dots 143^\circ \text{ (A1) [AWRT]}$$

(3)

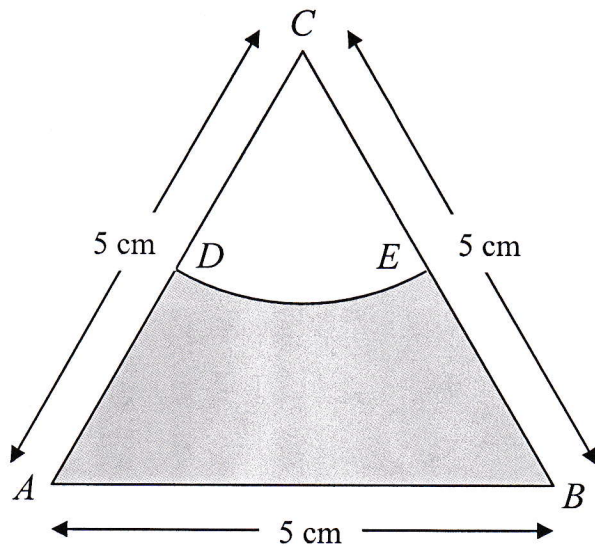


Diagram NOT
accurately drawn

The diagram shows an equilateral triangle ABC with sides of length 5 cm.

D is the midpoint of AC .

E is the midpoint of BC .

CDE is a sector of a circle, centre C .

Calculate the area of the shaded region.

Give your answer correct to 3 significant figures.

1ST

TRIANGLE ABC

$$A = \frac{1}{2} \times 5 \times 5 \times \sin 60 \quad (\text{M1})$$

$$= \underline{10.825 \dots} \quad (\text{A1})$$

2ND

SECTOR

$$A = \frac{60}{360} \times \pi \times 2.5^2 \quad (\text{M1})$$

$$= \underline{3.272 \dots} \quad (\text{A1})$$

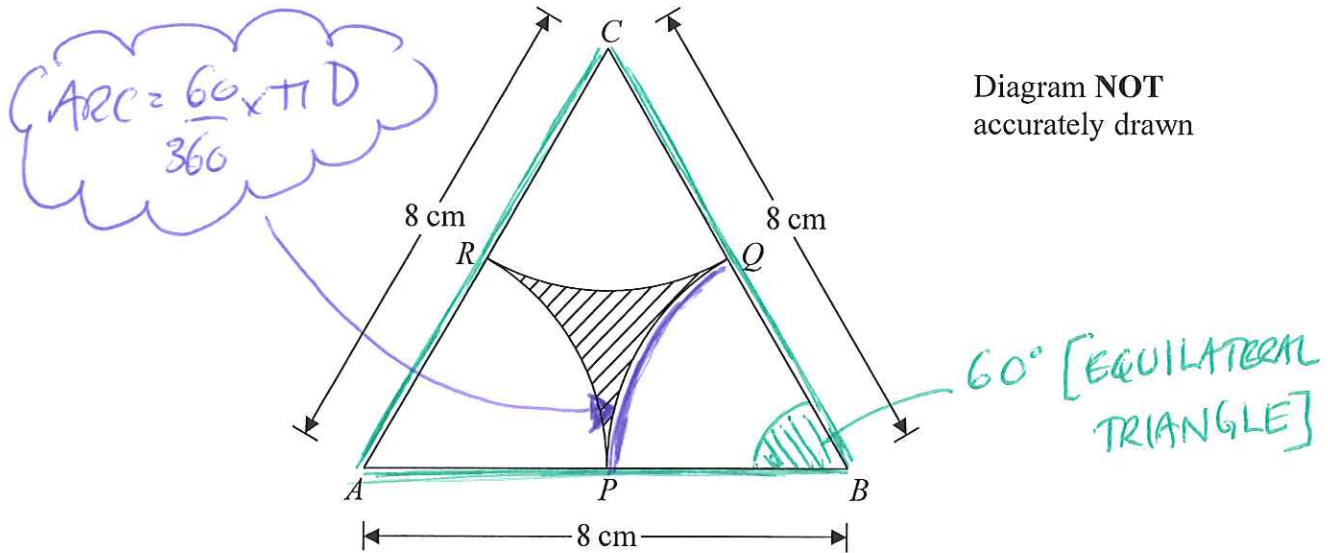
3RD

SHADED REGION

$$10.825 - 3.272 = 7.552 \dots$$



$$\underline{7.55} \quad (\text{A1}) \text{ cm}^2$$



ABC is an equilateral triangle of side 8 cm.

With the vertices A , B and C as centres, arcs of radius 4 cm are drawn to cut the sides of the triangle at P , Q and R .

The shape formed by the arcs is shaded.

- (a) Calculate the perimeter of the shaded shape.
Give your answer correct to 1 decimal place.

$$\text{SINGLE ARC, } PQ = \frac{60}{360} \times \pi \times 8$$

$$= 4.1887... \text{ (m)}$$

$$\therefore \text{WHOLE PERIMETER} = 3 \times 4.1887...$$

$$= 12.56637...$$

$$= \underline{\underline{12.6 \text{ cm}}} \text{ (A)}$$

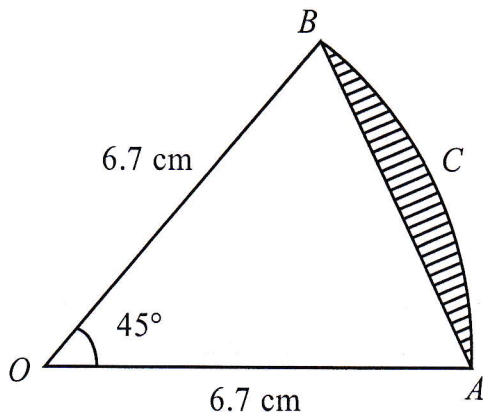


Diagram **NOT**
accurately drawn

AB is a chord of a circle, centre O .

ACB is an arc of the circle.

$OA = OB = 6.7$ cm.

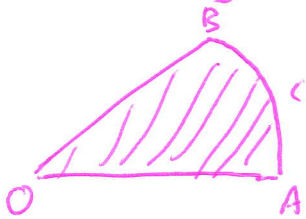
Angle $AOB = 45^\circ$.

Calculate the area of the shaded segment.

Give your answer correct to 3 significant figures.

1ST

[FIND WHOLE
SECTOR]

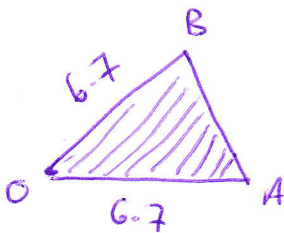


$$A = \frac{45}{360} \times \pi \times 6.7^2 \quad (M1)$$

$$= \underline{\underline{17.628\dots}} \quad (A1)$$

2ND

[TRIANGLE]



$$A = \frac{1}{2} \times 6.7 \times 6.7 \times \sin 45 \quad (M1)$$

$$= \underline{\underline{15.871\dots}} \quad (A1)$$

$$\dots\dots\dots 1.76 \quad (A1) \text{ cm}^2$$

3RD

SHADED REGION

$$= 17.628 - 15.871$$

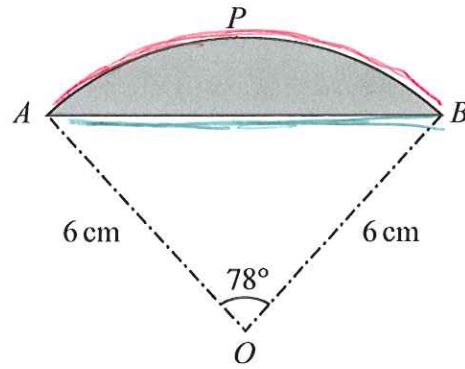


Diagram NOT
accurately drawn

The diagram shows a sector $OAPB$ of a circle, centre O .
 AB is a chord of the circle.
 The radius of the circle is 6 cm.
 Angle $AOB = 78^\circ$.

Calculate the perimeter of the shaded **segment** APB .
 Give your answer correct to 3 significant figures.

$$\begin{aligned} \underline{\text{ARC } APB} &= \frac{78}{360} \times 2\pi \times 6 \quad (\text{mi}) \\ &= \underline{8.1681} \quad (\text{AI}) \end{aligned}$$

LINE AB:

$$AB^2 = 6^2 + 6^2 - 2 \times 6 \times 6 \times \cos 78 \quad (\text{mi})$$

$$= 57.03$$

$$AB = \sqrt{57.03} \quad (\text{AI})$$

$$= \underline{7.5518} \dots$$

$$\underline{15.7} \dots \text{ cm} \quad (\text{AI})$$

$$\text{TOTAL} = \underline{8.1681} + \underline{7.5518} \quad (\text{mi})$$

$$= \underline{\underline{15.719}} \dots$$

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There is no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

Please report any errors or omissions that You find*. These solutions will be updated to correct errors that are discovered. It is recommended that You always check that You have the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or ‘best’ method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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