

SIMILAR SHAPES

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 80

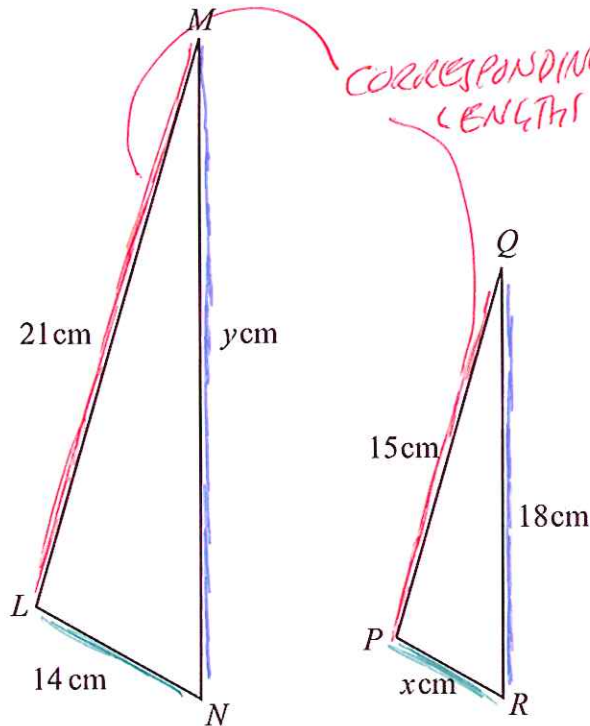
SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2010]

Similar Shapes [4 Marks]

Here are two similar triangles.



CORRESPONDING LENGTHS Diagrams NOT accurately drawn

$$SF = \frac{21}{15} = 1.4$$

LM corresponds to PQ .
 MN corresponds to QR .

(a) Find the value of x .

$$\frac{14}{1.4} \quad (M)$$

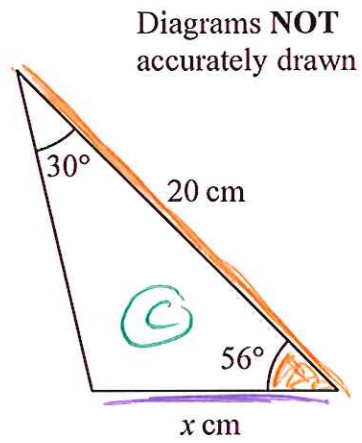
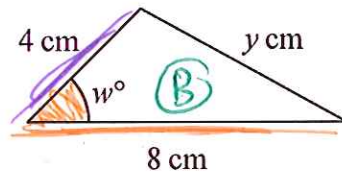
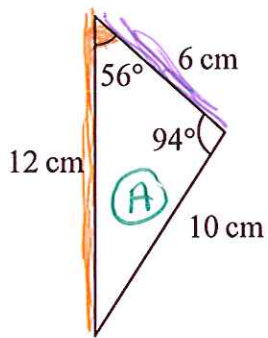
$$x = \frac{10}{(2)} \quad (A)$$

(b) Find the value of y .

$$18 \times 1.4 \quad (M)$$

$$y = 25.2 \quad (A)$$

Here are three similar triangles.



Find the value of

(a) w ,

$$w = \frac{56^\circ}{(1)} \quad \text{(B1)}$$

(b) x , [USE Δ S (A) AND (C)]

$$\begin{aligned} SF &= \frac{20}{12} \\ &= \underline{\underline{1.6}} \quad \text{(M1)} \end{aligned} \quad \rightarrow \quad x = 6 \times 1.6$$

$$x = \frac{10}{(2)} \quad \text{(A1)}$$

(c) y , [USE Δ S (A) AND (B)]

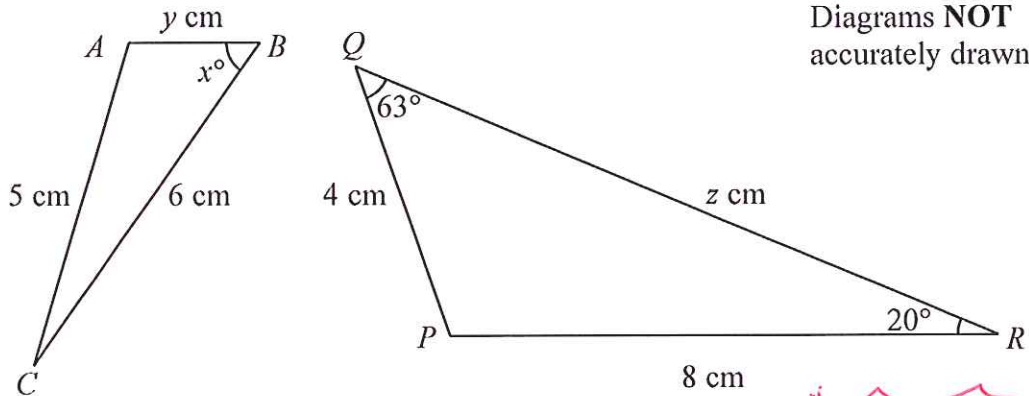
$$\begin{aligned} SF &= \frac{12}{8} \\ &= \underline{\underline{1.5}} \quad \text{(M1)} \end{aligned} \quad \rightarrow \quad y = \frac{10}{1.5}$$

$$y = \frac{6.6}{(2)} \quad \text{(A1)}$$

Here are two similar triangles.

AB corresponds to PQ .

BC corresponds to QR .



Diagrams NOT
accurately drawn

Find the value of

(a) x

$$SF = \frac{8}{5} = \underline{\underline{1.6}}$$

$$x = \dots 63^\circ \quad \text{(AI)}$$

(1)

(b) y

$$\frac{4}{1.6} \quad \text{mi}$$

$$y = \dots 2.5 \quad \text{(AO)}$$

(2)

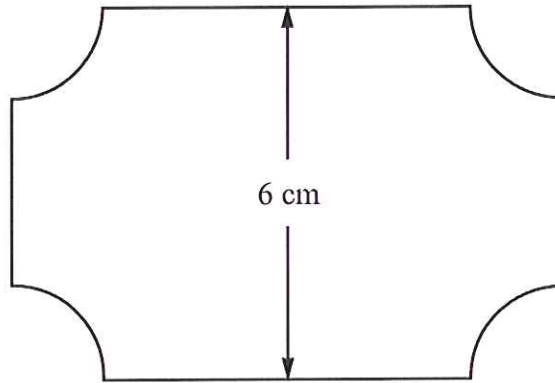
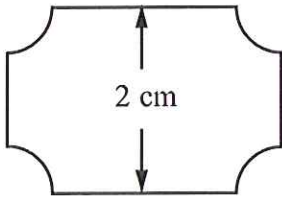
(c) z

$$\underbrace{6 \times 1.6}_{\text{mi}}$$

$$z = \dots 9.6 \quad \text{(AI)}$$

(2)

Here are two supermarket price tickets.



Diagrams NOT accurately drawn

$$SF = \frac{6}{2} = 3$$

The two supermarket price tickets are mathematically similar.

The area of the smaller ticket is 7 cm^2 .

Calculate the area of the larger ticket.

$$7 \times 3^2$$

(B1)

..... 63 cm^2 (A1)

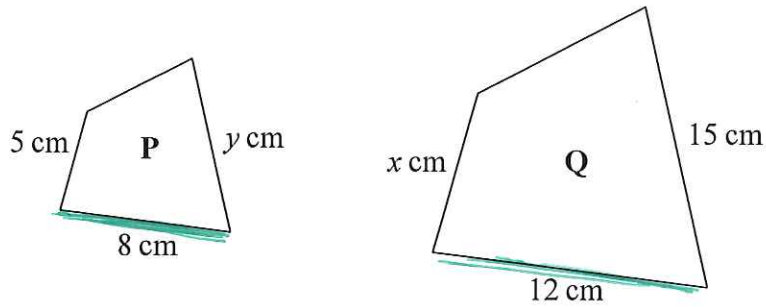


Diagram **NOT**
accurately drawn

$$\text{SCALE FACTOR} = \frac{12}{8} = 1.5$$

Quadrilateral **P** is mathematically similar to quadrilateral **Q**.

(a) Calculate the value of x .

$$x = \frac{5 \times 1.5}{(1)}$$

$$x = \frac{7.5 \text{ cm}}{(2)} \quad (A1)$$

(b) Calculate the value of y .

$$y = \frac{15}{1.5} \quad (1)$$

$$y = \frac{10 \text{ cm}}{(2)} \quad (A1)$$

The area of quadrilateral **P** is 60 cm^2 .

(c) Calculate the area of quadrilateral **Q**.

$$\frac{60 \times 1.5^2}{(1)}$$

$$\frac{135 \text{ cm}^2}{(2)} \quad (A1)$$

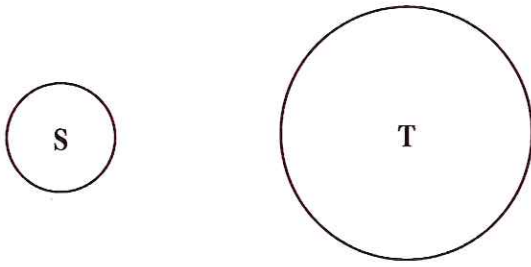


Diagram **NOT**
accurately drawn

The area of circle S is 4 cm^2 .
The radius of circle T is 3 times the radius of circle S.

Work out the area of circle T.

$$SF = 3$$

4 x 3
 (m) ² — COMPARING AREAS

..... 36 (A) ² cm²

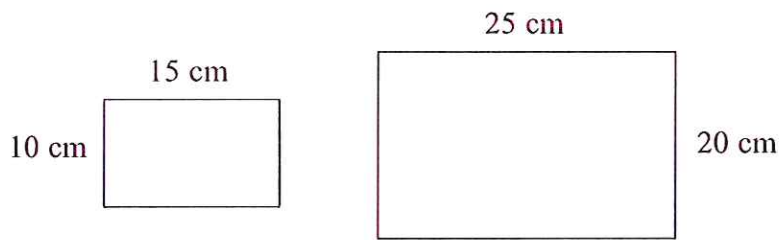


Diagram **NOT**
accurately drawn

Are the two rectangles mathematically similar?
Tick (✓) the appropriate box.
You must show working to justify your answer.

Yes

No

$$\frac{25}{15} = 1.6 \quad (B1)$$

$$\frac{20}{10} = 2 \quad (B1)$$

[ONLY IF AT
LEAST ONE (B1)]

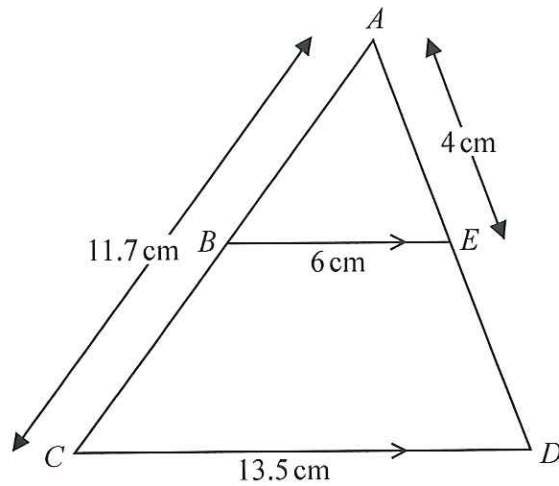


Diagram NOT
accurately drawn

SCALE FACTOR

$$\frac{13.5}{6} = \underline{\underline{2.25}}$$

(B1) [ANYWHERE]

The diagram shows triangle ACD .

B is a point on AC and E is a point on AD so that BE is parallel to CD .

$$AE = 4 \text{ cm}$$

$$AC = 11.7 \text{ cm}$$

$$BE = 6 \text{ cm}$$

$$CD = 13.5 \text{ cm}$$

(a) Calculate the length of AB .

$$AB = \frac{11.7}{2.25}$$

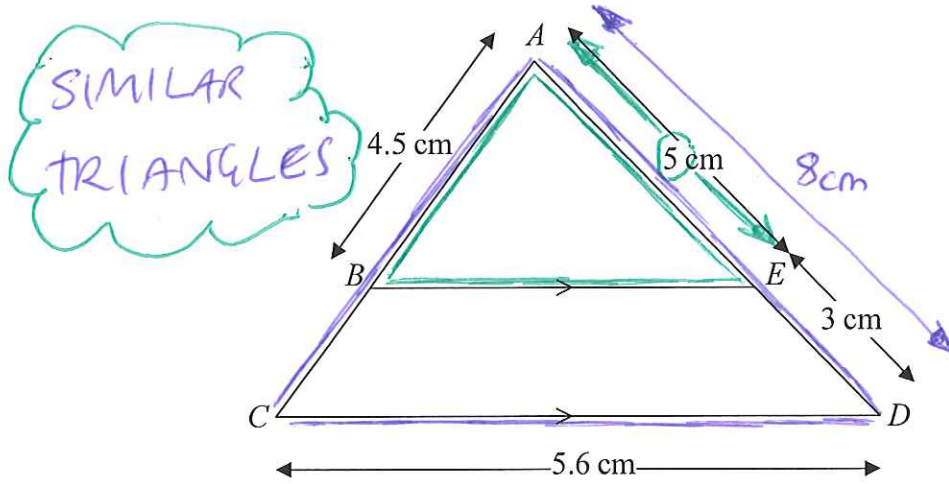
$$\begin{array}{r} 5.2 \text{ cm} \\ \hline (2) \end{array} \quad \text{(A1)}$$

(b) Calculate the length of ED .

IST FIND AD

$$\begin{aligned} AD &= 4 \times 2.25 \\ &= 9 \end{aligned} \quad \text{(M1)}$$

$$\begin{aligned} \therefore ED &= 9 - 4 \\ &= \underline{\underline{5 \text{ cm}}} \end{aligned} \quad \text{(A1)}$$



BE is parallel to CD .

$AB = 4.5$ cm, $AE = 5$ cm, $ED = 3$ cm, $CD = 5.6$ cm.

SCALE FACTOR = $\frac{8}{5} = 1.6$

(a) Calculate the length of BE .

$$\frac{5.6}{1.6} \text{ (M1)}$$

$$\underline{\underline{3.5}} \text{ cm (2)}$$

(b) Calculate the length of BC .

$$AC = 4.5 \times 1.6$$

$$= \underline{\underline{7.2}} \text{ (M1)}$$

$$\therefore BC = 7.2 - 4.5$$

$$= \underline{\underline{2.7}}$$

$$\underline{\underline{2.7}} \text{ cm (2)}$$

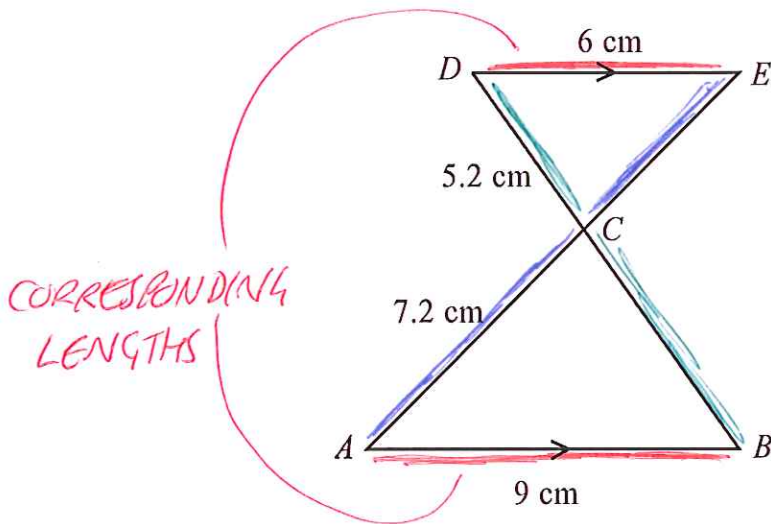


Diagram NOT
accurately drawn

$$SF = \frac{9}{6} = 1.5$$

AB is parallel to DE .
 ACE and BCD are straight lines.
 $AB = 9$ cm.
 $AC = 7.2$ cm.
 $CD = 5.2$ cm.
 $DE = 6$ cm.

(a) Calculate the length of BC .

$$BC = 5.2 \times 1.5$$

(m)

(A)

$$\begin{array}{r} 7.8 \\ \hline (2) \end{array} \text{ cm}$$

(b) Calculate the length of CE .

$$CE = \frac{7.2}{1.5} \text{ (m)}$$

(A)

$$\begin{array}{r} 4.8 \\ \hline (2) \end{array} \text{ cm}$$

$ABCD$ and $APQR$ are two similar quadrilaterals.

- $PQ = 9$ cm.
 $BC = 6$ cm.
 $AD = 5$ cm.
 $QR = 12$ cm.

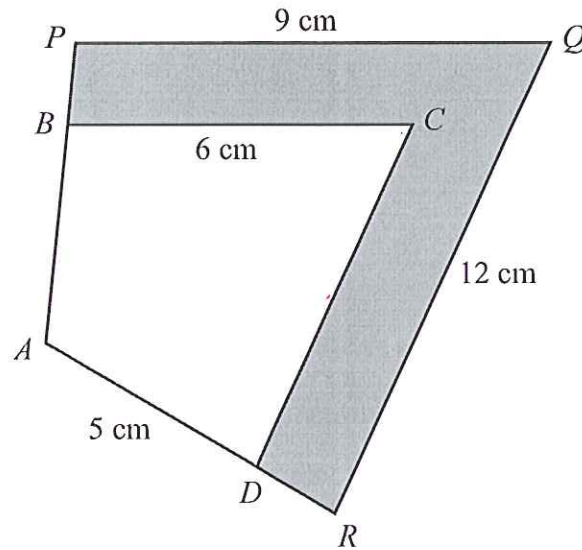


Diagram NOT accurately drawn

$$SF = \frac{9}{6} = 1.5$$

(a) Find the length of DC .

$$\frac{12}{1.5} \text{ (M1)}$$

$$\frac{8}{(2)} \text{ (A1) cm}$$

(b) Find the length of AR .

$$5 \times 1.5 \text{ (M1)}$$

$$\frac{7.5}{(2)} \text{ (A1) cm}$$

The area of the quadrilateral $ABCD$ is 32 cm^2 .

(c) Calculate the area of the shaded region.

$$\begin{aligned}
 \text{AREA OF } APQR &= 32 \times 1.5^2 \text{ (M1)} \\
 &= 72
 \end{aligned}$$

$$\begin{aligned}
 \text{SHADED REGION} &= 72 - 32 \text{ (M1)} \\
 &= \underline{\underline{40 \text{ cm}^2}} \text{ (A1)}
 \end{aligned}$$

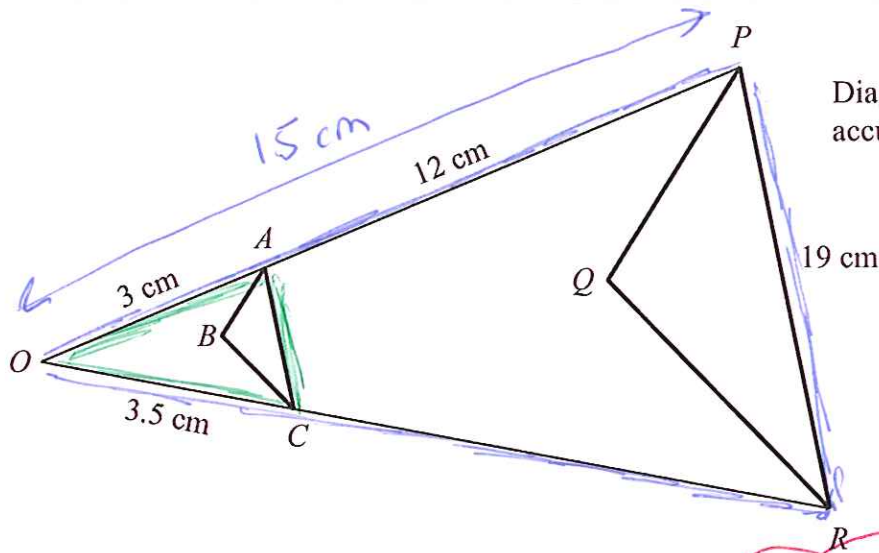


Diagram NOT accurately drawn

GREEN AND BLUE TRIANGLES ARE SIMILAR!

SCALE FACTOR = $\frac{15}{3} = 5$

Triangle PQR is an enlargement, centre O, of triangle ABC. OAP and OCR are straight lines.

- OA = 3 cm.
- AP = 12 cm.
- OC = 3.5 cm.
- PR = 19 cm.

(a) Work out the length of CR.

$OC = 3.5 \times 5 = 17.5$ (mi)
 $CR = 17.5 - 3.5 = 14$ (A1) cm (2)

(b) Work out the length of AC.

$AC = \frac{19}{5} = 3.8$ (mi) (A1) cm (3)

The area of triangle ABC is 2 cm²

(c) Work out the area of triangle PQR.

$2 \times 5^2 = 50$ (mi) (A1) cm² (3)

AREAS!

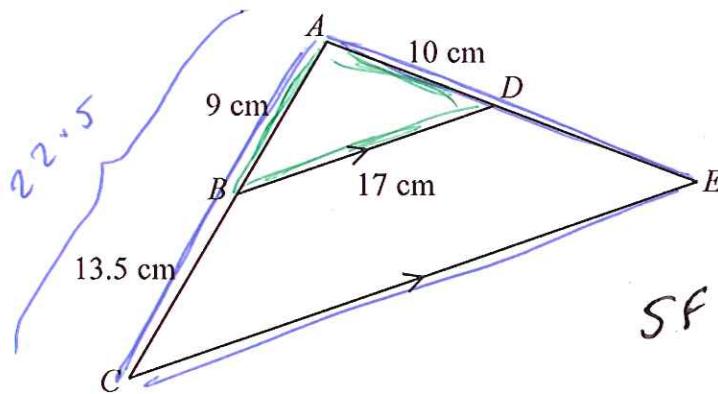


Diagram NOT
accurately drawn

$$SF = \frac{22.5}{9}$$

$$= 2.5$$

In the diagram ABC and ADE are straight lines.
 BD is parallel to CE .

$AB = 9$ cm, $BC = 13.5$ cm, $AD = 10$ cm, $BD = 17$ cm

(a) Calculate the length of CE .

$$CE = 17 \times 2.5$$

$$\begin{array}{r} 42.5 \\ \hline \end{array} \text{ cm}$$

(2)

(b) Calculate the length of DE .

$$\begin{aligned} AE &= 10 \times 2.5 \\ &= 25 \end{aligned} \quad \rightarrow \quad \begin{aligned} DE &= 25 - 10 \\ &= 15 \end{aligned}$$

$$\begin{array}{r} 15 \\ \hline \end{array} \text{ cm}$$

(2)

The area of triangle ABD is 36 cm²

(c) Calculate the area of quadrilateral $BDEC$.

$$\begin{aligned} \text{TRIANGLE } ACE &= 36 \times 2.5^2 \\ &= 225 \end{aligned}$$

$$\begin{aligned} \therefore \text{TRAPEZIUM} &= 225 - 36 \\ &= 189 \end{aligned}$$

$$\begin{array}{r} 189 \\ \hline \end{array} \text{ cm}^2$$

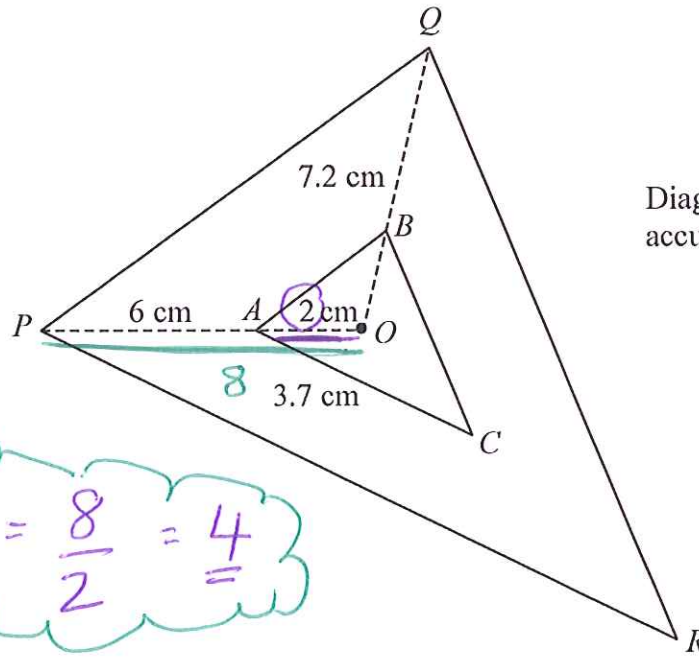


Diagram NOT
accurately drawn

SCALE
FACTOR = $\frac{8}{2} = 4$

Triangle PQR is an enlargement, centre O , of triangle ABC .

OAP and OBQ are straight lines.

$OA = 2$ cm.

$AP = 6$ cm.

$BQ = 7.2$ cm.

$AC = 3.7$ cm.

SIMILAR SHAPES!

(a) Work out the length of OB .

BECAUSE
CENTRE OF
ENLARGEMENT
IS KNOWN

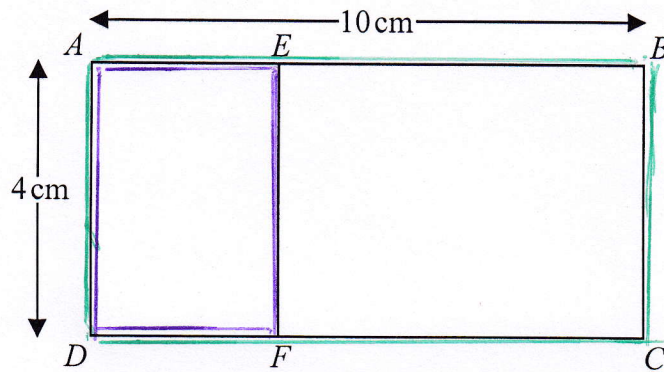
$$\frac{BQ}{AP} = \frac{7.2}{6} = 1.2 \quad \therefore \frac{OB}{OA} = 1.2$$

(b) Work out the length of PR .

$$\begin{aligned} PR &= 3.7 \times 4 \\ &= 14.8 \text{ cm} \end{aligned}$$

$$\begin{aligned} \Rightarrow OB &= 1.2 \times OA \\ &= 1.2 \times 2 \\ &= 2.4 \end{aligned}$$

Rectangle $ABCD$ is mathematically similar to rectangle $DAEF$.



$$AB = 10 \text{ cm.}$$

$$AD = 4 \text{ cm.}$$

Work out the area of rectangle $DAEF$.

$$\begin{aligned} \text{AREA OF } ABCD &= 10 \times 4 \\ &= 40 \end{aligned}$$

$$\therefore \text{ AREA OF } DAEF = \frac{40}{2.5^2}$$

(M1)

[DIVIDING BY
SQUARE OF SCALE FACTOR]

$$6.4 \text{ cm}^2$$

(A1)

SCALE FACTOR

$$= \frac{10}{4}$$

$$= \underline{\underline{2.5}}$$

(B1)

[FINDING THE SCALE
FACTOR]

[NOTE THAT THERE ARE MANY WAYS TO GET TO THE ANSWER AND MARKS WOULD BE AWARDED DIFFERENTLY IF A DIFFERENT METHOD IS USED. FULL MARKS FOR CORRECT ANSWER, AS LONG AS METHOD IS CLEARLY SHOWN]

The diagram shows two regular hexagons, $OABCDE$ and $OFGHIJ$.

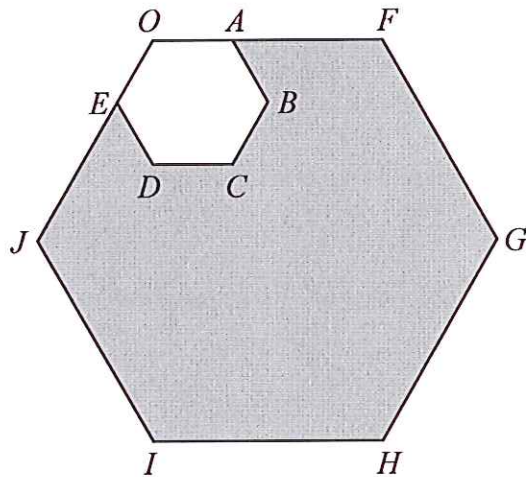


Diagram **NOT** accurately drawn

OAF and OEJ are straight lines.

$OF = 3 OA$.

The area of $OABCDE$ is 4 cm^2 .

Calculate the area of the shaded region.

SCALE FACTOR = 3
SMALL HEXAGON

$$\text{AREA OF LARGE HEXAGON} = 4 \times \underbrace{3^2}_{(m)} = \underline{\underline{36}} \quad (B)$$

$$\begin{aligned} \text{SHADED AREA} &= 36 - 4 \\ &= \underline{\underline{32 \text{ cm}^2}} \quad (A) \end{aligned}$$

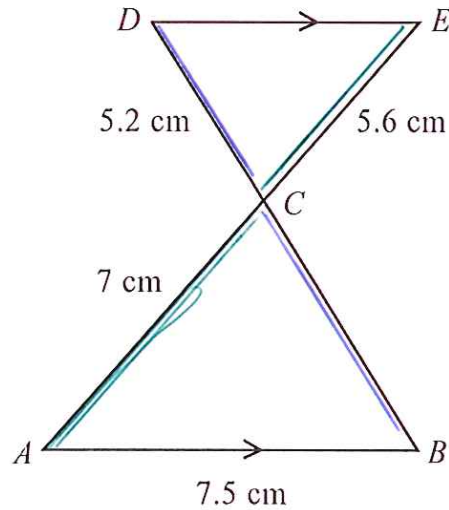


Diagram NOT accurately drawn

$$SF = \frac{7}{5.6} = 1.25$$

AB is parallel to DE .
 The lines AE and BD intersect at C .
 $AB = 7.5$ cm, $AC = 7$ cm, $CD = 5.2$ cm, $CE = 5.6$ cm.

(a) Calculate the length of BC .

$$5.2 \times 1.25 \quad (B1)$$

$$\begin{array}{r} 6.5 \\ \hline (2) \end{array} \text{ cm} \quad (A1)$$

(b) Calculate the length of DE .

$$\frac{7.5}{1.25} \quad (M1) \text{ DIVIDE}$$

$$\begin{array}{r} 6 \\ \hline (2) \end{array} \text{ cm} \quad (A1)$$

(c) The area of triangle ABC is 21 cm^2 .
 Calculate the area of triangle EDC .

$$\frac{21}{1.25^2} = 13.44 \quad (M1) \text{ DIVIDE}$$

$$13.44 \text{ cm}^2 \quad (A1)$$

Disclaimer

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There is no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

Please report any errors or omissions that You find*. These solutions will be updated to correct errors that are discovered. It is recommended that You always check that You have the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or ‘best’ method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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