# SIMULTANEOUS EQUATIONS (QUADRATIC)

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 71

# SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

**1.** [Edexcel, 2006] Simultaneous Equations (Quadratic) **[5 Marks]**

Solve the simultaneous equations

$y = x^2$ ——— (1)
$y = 2x + 15$ ——— (2)

PUT (1) = (2)

$x^2 = 2x + 15$ M1

$\Rightarrow x^2 - 2x - 15 = 0$ M1

$(x - 5)(x + 3) = 0$ M1

$x_1 = 5$

$x_2 = -3$ A1 BOTH

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SUBSTITUTE IN TO EQ. (1)

$y_1 = 5^2$

$= 25$

$y_2 = (-3)^2$

$= 9$

A1 BOTH

$x = \ldots 5 \ldots$, $y = \ldots 25 \ldots$

$x = \ldots -3 \ldots$, $y = \ldots 9 \ldots$

Solve the simultaneous equations

$y = x^2$ — (1)

$y = 7x - 10$ — (2)

$x^2 = 7x - 10$ (M1) [SINGLE EQUATION]

$\Rightarrow x^2 - 7x + 10 = 0$ (M1) [RHS = 0]

$(x - 2)(x - 5) = 0$ (M1) [FACTORISING]

$x = 2$ $x = 5$ (A1) BOTH

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SUBSTITUTE INTO (1)

$y = 2^2$ $y = 5^2$

$= 4$ $= 25$ (A1) BOTH

**3.** [Edexcel, 2012] Simultaneous Equations (Quadratic) **[5 Marks]**

Solve the simultaneous equations

BOTH ARE $y =$ { $y = 2x^2$ —— (1) ; $y = 20 - 3x$ —— (2)

Show clear algebraic working.

MATCH EQUATIONS

(1) = (2)

$2x^2 = 20 - 3x$ (M1)

$2x^2 + 3x - 20 = 0$ (A1)

$(2x - 5)(x + 4) = 0$ (M1)

$2x - 5 = 0$

$x_1 = 2.5$

$x + 4 = 0$

$x_2 = -4$ (A1) BOTH

SUBSTITUTE INTO (1)

$y_1 = 2 \times 2.5^2$

$= 12.5$

$y_2 = 2 \times (-4)^2$

$= 32$ (A1) BOTH

**4.** [Edexcel, 2007] Simultaneous Equations (Quadratic) **[6 Marks]**

Solve the simultaneous equations

$$y = 3x - 1 \quad \text{——} \quad (1)$$

$$x^2 + y^2 = 5 \quad \text{——} \quad (2)$$

SUBSTITUTE (1) INTO (2)

$$x^2 + (3x-1)^2 = 5 \quad \text{(M1)}$$

$$\Rightarrow x^2 + (3x-1)(3x-1) = 5$$

$$\Rightarrow x^2 + 9x^2 - 3x - 3x + 1 = 5$$

$$\Rightarrow 10x^2 - 6x + 1 = 5$$

$$10x^2 - 6x - 4 = 0 \quad \text{(M1)}$$

$$5x^2 - 3x - 2 = 0 \quad \leftarrow (\div 2)$$

$$(5x+2)(x-1)$$

$5x + 2 = 0$

$5x = -2$

$x = -\frac{2}{5}$ (A1)

$x_1 = -0.4$

$x - 1 = 0$

$x_2 = 1$ (A1)

$\Rightarrow y_1 = 3 \times -0.4 - 1$

$= -2.2$ (A1)

$\Rightarrow y_2 = 3 \times 1 - 1$

$= 2$ (A1)

**5.** [Edexcel, 2015] Simultaneous Equations (Quadratic) **[5 Marks]**

Solve $x^2 + y^2 = 20$ —— (1)
$y = 10 - 2x$ —— (2)

Show clear algebraic working.

$x^2 + (10-2x)^2 = 20$ (M1) [SUBSTITUTING (2) INTO (1)]

$x^2 + (10-2x)(10-2x) = 20$

$x^2 + 100 - 20x - 20x + 4x^2 = 20$ (M1) [EXPANDING BRACKETS]

$x^2 + 4x^2 - 20x - 20x + 100 - 20 = 0$

$5x^2 - 40x + 80 = 0$
$x^2 - 8x + 16 = 0$ } (M1) [EITHER]

$(x-4)(x-4) = 0$ (M1) [FACTORISING]

$x = 4$ [IDENTICAL ROOTS]

$x = 4$

SUBSTITUTE INTO (2)

$y = 10 - 2(4)$
$= 10 - 8$
$= 2$

$x = 4, \ y = 2$ (A1) [BOTH]

Solve the simultaneous equations

$$y = 3x + 2$$
$$x^2 + y^2 = 20$$

Show clear algebraic working.

$x^2 + (3x+2)^2 = 20$ (M1) [SUBSTITUTION]

$x^2 + (3x+2)(3x+2) = 20$

$x^2 + 9x^2 + 6x + 6x + 4 = 20$ } (M1) [EITHER]

$10x^2 + 12x + 4 = 20$

$10x^2 + 12x - 16 = 0$ } (M1) [EITHER]

$5x^2 + 6x - 8 = 0$

$(5x-4)(x+2) = 0$ (M1) [FACTORISING] [OR USE OF FORMULA]

$x = \frac{4}{5}$ $x = -2$ (A1) [BOTH]

FIND y

$y = 3 \times \left(\frac{4}{5}\right) + 2$ $y = 3(-2) + 2$

$= 4.4$ $= -4$ (A1) [BOTH]

Solve the simultaneous equations

$2x+y=6$ $x^2+y^2=20$

$2x+y=6 \longrightarrow y=6-2x$ (M1)

$x^2+(6-2x)^2=20$ (A1)

$x^2+(6-2x)(6-2x)=20$

$x^2+36-12x-12x+4x^2=20$

$5x^2-24x+36=20$

$5x^2-24x+16=0$ (B1)

$(5x-4)(x-4)=0$

$5x-4=0$ $x-4=0$

$5x=4$ $x_2=4$ (A1)

$x_1=\frac{4}{5}$

$=0.8$ (A1)

$y_1=6-2\times0.8$ $y_2=6-2\times4$

$=4.4$ (A1) $=-2$ (A1)

Solve the simultaneous equations

$2x - y = 7$ $\Rightarrow$ $y = 2x - 7$ — (1)

$x^2 + y^2 = 34$ — (2)

(B1)

Show clear algebraic working.

SUBSTITUTE (1) INTO (2)

$x^2 + (2x-7)^2 = 34$ (M1) [FOR EQUATION]

$x^2 + (2x-7)(2x-7) = 34$

$x^2 + 4x^2 - 14x - 14x + 49 = 34$ (M1) [EXPANDING BRACKETS]

$5x^2 - 28x + 49 - 34 = 0$

$5x^2 - 28x + 15 = 0$ (B1) [QUADRATIC]

$(5x - 3)(x - 5) = 0$ (M1) [FACTORISING]

$5x - 3 = 0$ $\quad$ $x = 5$ (A1) [BOTH x-VALUES]

$x = 0.6$

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$y = 2 \times 0.6 - 7$ $\quad$ $y = 2 \times 5 - 7$

$= -5.8$ $\quad$ $= 3$

(A1) [BOTH y-VALUES]

**9.** [New Question, by Maths4Everyone.com] Simultaneous Equations (Quadratic) **[6 Marks]**

Solve the simultaneous equations

$y = 5x - 1$

$y = (x + 1)^2$

Show clear algebraic working

$(x+1)^2 = 5x - 1$ M1 [EQUATION]

$(x+1)(x+1) = 5x - 1$

$x^2 + 2x + 1 = 5x - 1$ M1 [EXPANDING BRACKETS]

$\Rightarrow x^2 - 3x + 2 = 0$ M1 [QUADRATIC]

$(x - 2)(x - 1) = 0$ M1 [FACTORISING]

$x = \underline{\underline{2}}$

$x = \underline{\underline{1}}$ A1 [BOTH]

$\Rightarrow y = 5 \times (2) - 1$

$= \underline{\underline{9}}$

$y = 5 \times (1) - 1$

$= \underline{\underline{4}}$ A1 [BOTH]

Solve the simultaneous equations

$$xy = 12 \quad \text{——} \ (1)$$
$$y - 3x = -9$$

M1 [REARRANGING]

$y = 3x - 9$ — (2)

Show clear algebraic working.

SUBSTITUTE EQ(2) INTO (1)

$x(3x-9) = 12$ M1 [SUBSTITUTION]

$\Rightarrow 3x^2 - 9x = 12$ M1 [EXPANDING BRACKETS]

$\Rightarrow 3x^2 - 9x - 12 = 0$

$\Rightarrow x^2 - 3x - 4 = 0$ } M1 [RHS = 0]

$(x-4)(x+1) = 0$ M1 [FACTORISING]

$x = 4$ $\quad$ $x = -1$ A1 [BOTH]

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SUBTITUTE INTO EQUATION (2)

$y = 3(4) - 9$

$= 12 - 9$

$= 3$

$y = 3(-1) - 9$

$= -3 - 9$

$= -12$ A1 [BOTH]

**11.** [New Question, by Maths4Everyone.com] Simultaneous Equations (Quadratic) **[5 Marks]**

Solve the simultaneous equations

$$y = 3x^2 + 7x + 9 \quad \text{—— ①}$$
$$y = 4x + 15 \quad \text{—— ②}$$

Show clear algebraic working.

PUT ① = ②

$3x^2 + 7x + 9 = 4x + 15$ (M1) [COMBINE EQUATIONS]

$\Rightarrow 3x^2 + 3x - 6 = 0$

$\Rightarrow x^2 + x - 2 = 0$ } (M1) [RHS = 0]

$(x+2)(x-1) = 0$ (M1) FACTORISE

$x = -2$ $\quad$ $x = 1$ (A1) [BOTH]

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SUBSTITUTE INTO EQUATION ②

$y = 4(-2) + 15$ $\quad$ $y = 4(1) + 15$

$= -8 + 15$ $\quad$ $= 4 + 15$

$= 7$ $\quad$ $= 19$ (A1) [BOTH]

Solve the simultaneous equations

$3y^2 + 4x^2 = 16$ —— ①

$y - 2x = -4$ ⟶ $y = 2x - 4$ —— ②

m1 [REARRANGING]

Show clear algebraic working.

SUBSTITUTE EQ② INTO EQUATION ①

$3(2x-4)^2 + 4x^2 = 16$ m1 [SUBSTITUTION

$\Rightarrow 3(2x-4)(2x-4) + 4x^2 - 16 = 0$

$\Rightarrow 3(4x^2 - 8x - 8x + 16) + 4x^2 - 16 = 0$ m1 [EXPANDING BRACKETS]

$\Rightarrow 3(4x^2 - 16x + 16) + 4x^2 - 16 = 0$

$\Rightarrow 12x^2 - 48x + 48 + 4x^2 - 16 = 0$

$\Rightarrow 16x^2 - 48x + 32 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$ } m1 [EITHER]

$(x-2)(x-1) = 0$ m1 [FACTORISING]

$x = 2$ $\quad$ $x = 1$ A1 [BOTH]

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SUBSTITUTE INTO EQUATION ②

$y = 2(2) - 4$ $\quad$ $y = 2(1) - 4$

$= 4 - 4$ $\quad$ $= 2 - 4$

$= 0$ $\quad$ $= -2$ A1 [BOTH]

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