SINE AND COSINE RULE

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 75

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2016] Sine and Cosine Rule **[3 Marks]**

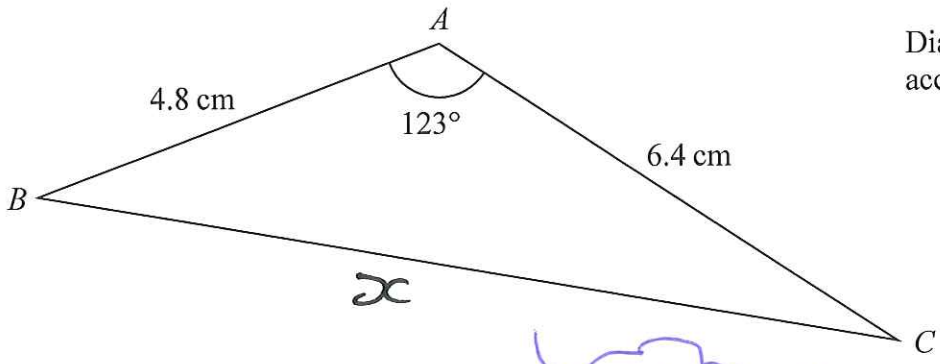


Diagram **NOT** accurately drawn

Calculate the length of *BC*.
Give your answer correct to 3 significant figures.

USE $a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 4.8^2 + 6.4^2 - 2 \times 4.8 \times 6.4 \cos 123$ (M1)

$= 97.46...$ (A1)

$x = \sqrt{97.46...}$

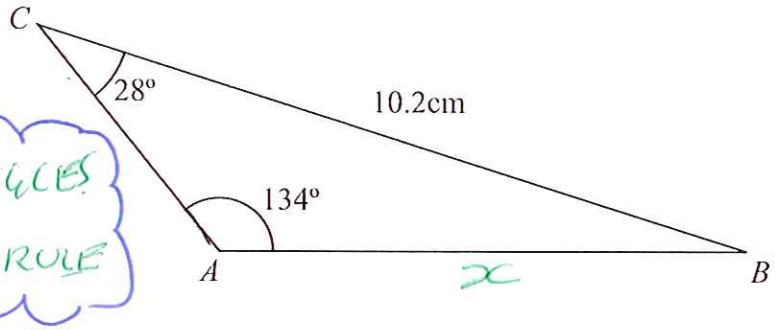
$= 9.8723...$

$= 9.87\text{cm}$ (A1)

2. [Edexcel, 2011] Sine and Cosine Rule **[3 Marks]**

The diagram shows triangle *ABC*.

Diagram **NOT** accurately drawn



TWO ANGLES → SINE RULE

Angle *BCA* = 28°
Angle *CAB* = 134°
BC =10.2 cm.

Calculate the length of *AB*.
Give your answer correct to 3 significant figures.

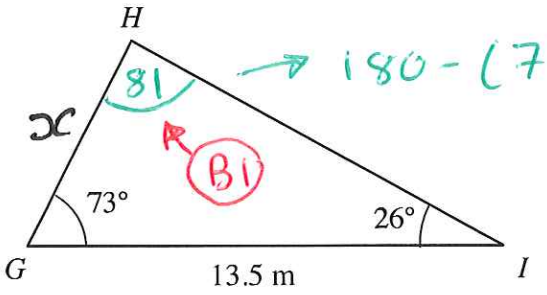
$$\frac{x}{\sin 28} = \frac{10.2}{\sin 134} \quad \text{(M1)}$$

$$x = \frac{10.2}{\sin 134} \times \sin 28$$

$$= 6.65695\ldots$$

$$= 6.66\text{cm} \quad \text{(A1)}$$

3. [New Question, by Maths4Everyone.com] Sine and Cosine Rule **[4 Marks]**



$\rightarrow 180 - (73+26)$

Diagram **NOT** accurately drawn

Calculate the length of *GH*.
Give your answer correct to 3 significant figures.

$\frac{x}{\sin 26} = \frac{13.5}{\sin 81} \Rightarrow x = \frac{13.5}{\sin 81} \times \sin 26$ M1

M1

$= 5.99177...$

5.99 m A1

.....

4. [New Question, by Maths4Everyone.com] Sine and Cosine Rule **[3 Marks]**

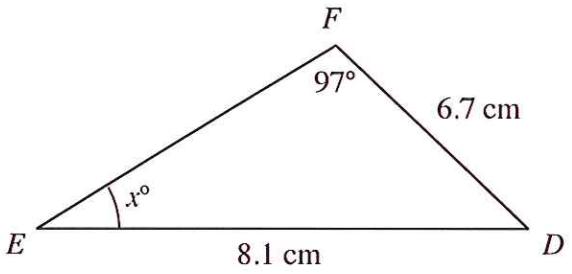


Diagram **NOT** accurately drawn

Calculate the value of x.
Give your answer correct to 1 decimal place.

$$\frac{\sin x}{6.7} = \frac{\sin 97}{8.1} \Rightarrow \sin x = 6.7 \times \frac{\sin 97}{8.1}$$

$$= 0.82099...$$

$$\Rightarrow x = \sin^{-1}(0.82099)$$

$$= 55.2^\circ$$

M1 M1 A1

(3)

5. [Edexcel, 2013] Sine and Cosine Rule **[3 Marks]**

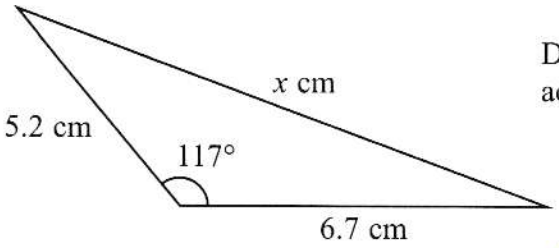


Diagram **NOT** accurately drawn

Calculate the value of x.
Give your answer correct to 3 significant figures.

COSINE RULE

$x^2 = 5.2^2 + 6.7^2 - 2 \times 5.2 \times 6.7 \cos 117$ M1

$= 103.564...$ M1

$\Rightarrow x = \sqrt{103.564}$

$= 10.1766...$

$x =$ 10.2 A1

6. [Edexcel, 2008] Sine and Cosine Rule **[3 Marks]**

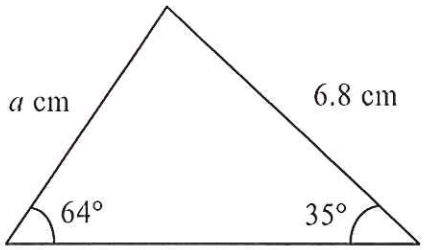


Diagram **NOT** accurately drawn

SIN RULE

Calculate the value of a.
Give your value correct to 3 significant figures.

$$\frac{a}{\sin 35} = \frac{6.8}{\sin 64}$$

$$\Rightarrow a = \frac{6.8 \sin 35}{\sin 64}$$

$$= 4.3395\ldots$$

$a =$ 4.34 cm

7. [Edexcel, 2012] Sine and Cosine Rule **[3 Marks]**

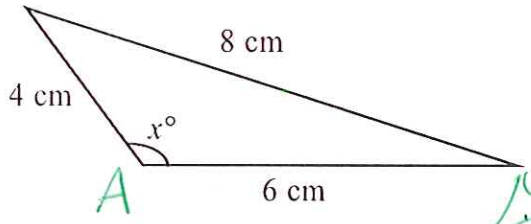


Diagram **NOT** accurately drawn

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

Calculate the value of x.
Give your answer correct to 1 decimal place.

$$\cos x = \frac{4^2+6^2-8^2}{2\times4\times6} \quad \text{(M1)}$$

$$= -0.25 \quad \text{(A1)}$$

$$\Rightarrow x = \cos^{-1}(-0.25)$$

$$= 104.477\ldots$$

(A1)

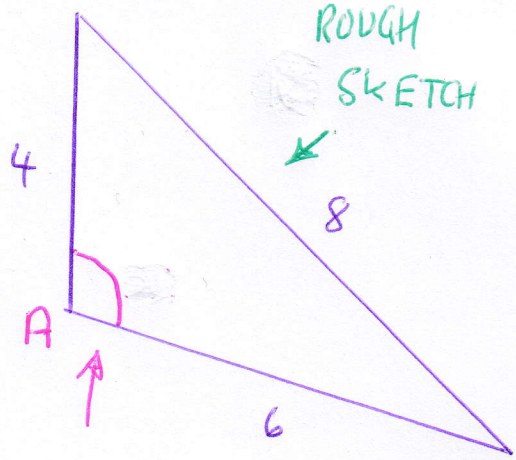
$x =$ 104.5

8. [New Question, by Maths4Everyone.com] Sine and Cosine Rule **[4 Marks]**

A triangle has sides of length 4 cm, 6 cm and 8 cm.
Calculate the size of the largest angle in this triangle.

Give your answer correct to 1 decimal place.

USE $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$



LARGEST ANGLE IS OPPOSITE THE LONGEST SIDE!

$\cos A = \dfrac{6^2+4^2-8^2}{2\times 6\times 4}$ (M1)

$\Rightarrow \cos A = -0.25$ (A1)

$\Rightarrow A = \cos^{-1}(-0.25)$ (M1)

$= 104.477...$

$= 104.5^\circ$ (A1)

9. [New Question, by Maths4Everyone.com] Sine and Cosine Rule **[4 Marks]**

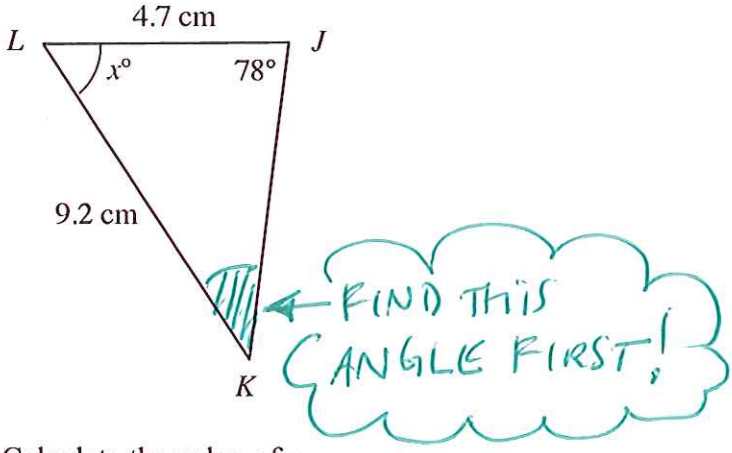


Diagram **NOT** accurately drawn

Calculate the value of x.
Give your answer correct to 1 decimal place.

$$\frac{\sin k}{4.7} = \frac{\sin 78}{9.2} \Rightarrow \sin k = 4.7 \times \frac{\sin 78}{9.2}$$

(M1)

$$= 0.4997...$$

$$\Rightarrow k = \sin^{-1}(0.4997...)$$

(M1)

$$= 29.98 \quad \text{(B1)}$$

$$\Rightarrow x = 180 - (29.98 + 78)$$

$$= 72.02$$

$$= \underline{72.0°} \quad \text{(A1)}$$

10. [New Question, by Maths4Everyone.com] Sine and Cosine Rule **[5 Marks]**

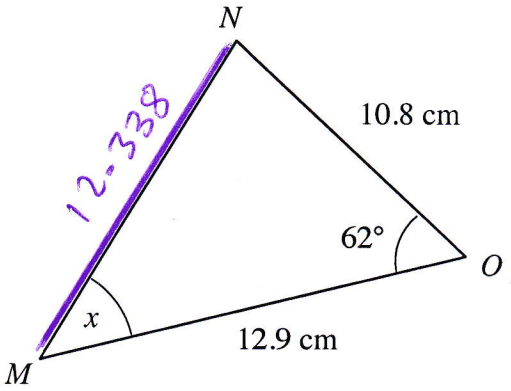


Diagram **NOT** accurately drawn

Calculate the size of angle *NMO*.
Give your answer correct to 1 decimal place.

1ST [FIND NM]

$NM^2 = 12.9^2 + 10.8^2 - 2 \times 12.9 \times 10.8 \times \cos 62$ (M1)

$= 152.236...$

$\Rightarrow NM = 12.338...$ (A1)

2ND [USE SINE RULE TO FIND x]

$\frac{\sin x}{10.8} = \frac{\sin 62}{12.338}$ (M1) $\Rightarrow \sin x = 10.8 \times \frac{\sin 62}{12.338}$

$= 0.77285...$

$\Rightarrow x = \sin^{-1}(0.77285)$ (M1)

$= 50.611...$

$= 50.6°$ (A1)

11. [Edexcel, 2014] Sine and Cosine Rule **[4 Marks]**

A circular clock face, centre O, has a minute hand OA and an hour hand OB.
$OA = 10$ cm.
$OB = 7$ cm.

Calculate the length of AB when the hands show 5 o'clock.
Give your answer correct to 3 significant figures.

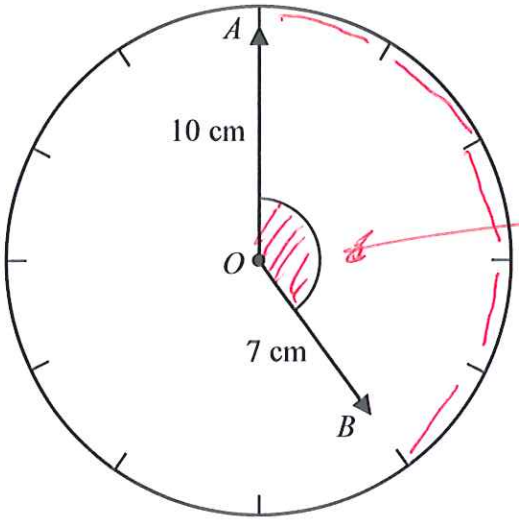
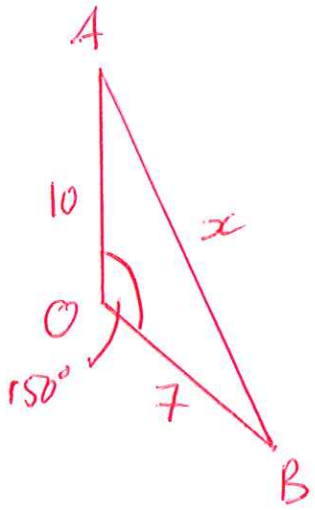


Diagram **NOT** accurately drawn

$\frac{5}{12} \times 360 = 150°$



(B1) (M1)

$x^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 150°$

$= 270.243...$

$x = \sqrt{270.24...}$ (M1)

$= 16.439...$

(A1)

$= 16.4$ cm

12. [Edexcel, 2013] Sine and Cosine Rule **[3 Marks]**

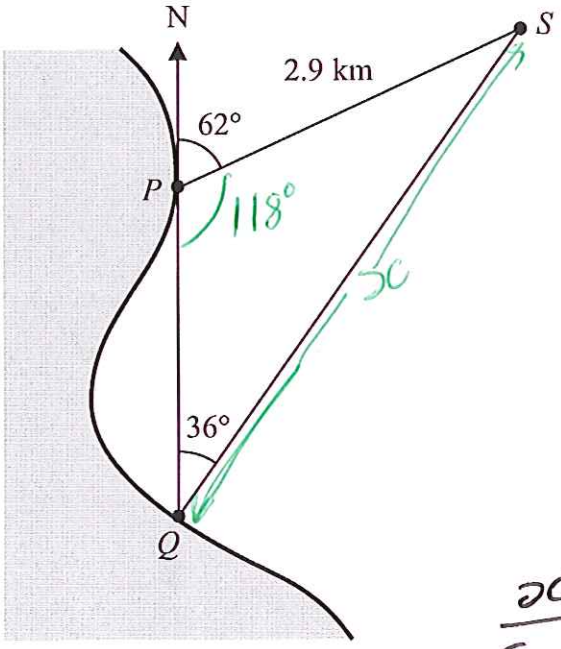


Diagram **NOT** accurately drawn

P and Q are two points on a coast.
P is due North of Q.
A ship is at the point S.
$PS = 2.9$ km.
The bearing of the ship from P is 062°
The bearing of the ship from Q is 036°

Calculate the distance QS.
Give your answer correct to 3 significant figures.

$$\frac{x}{\sin 118} = \frac{2.9}{\sin 36} \quad \text{(M1)}$$

$$\Rightarrow x = \frac{2.9}{\sin 36} \times \sin 118 \quad \text{(M1)}$$

$$= 4.3562\ldots$$

$$= 4.36 \text{ km} \quad \text{(A1)}$$

13. [Edexcel, 2013] Sine and Cosine Rule **[6 Marks]**

The sides of triangle PQR are tangents to a circle.
The tangents touch the circle at the points S, T and U.
$QS = 6$ cm. $PS = 7$ cm.

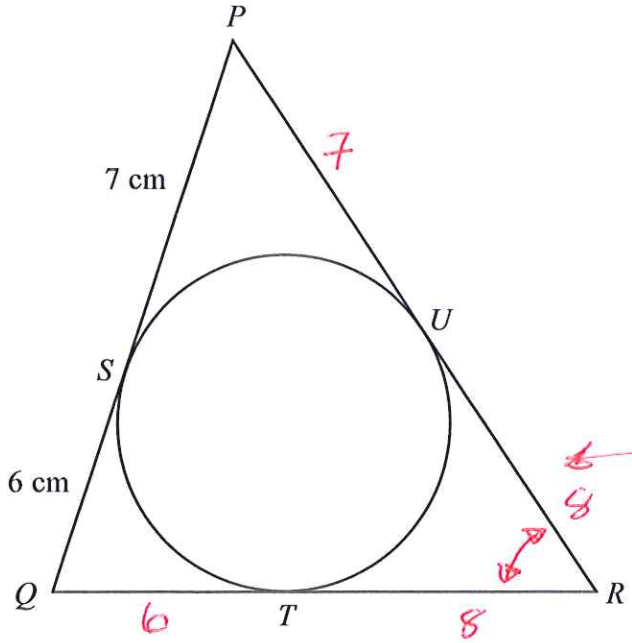


Diagram **NOT** accurately drawn

(a) (i) Write down the length of QT.

6 cm (A1)

(ii) Give a reason for your answer.

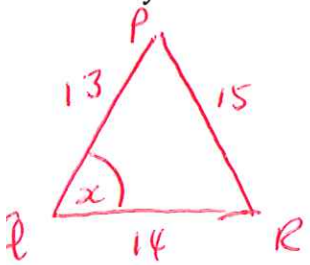
TANGENTS FROM THE SAME POINT ARE EQUAL (A1)

(2)

The perimeter of triangle PQR is 42 cm. → $42 - (2\times7 + 2\times6) = 16$

So $TR = UR = 8$ (B1)

(b) Calculate the size of angle PQR.
Give your answer correct to 1 decimal place.



$$\cos x = \frac{13^2 + 14^2 - 15^2}{2 \times 13 \times 14} \quad \text{(M1)}$$

$$= 0.3846...$$

$$x = \cos^{-1}(0.3846...) \quad \text{(M1)}$$

$$= 67.380... \quad 67.4^\circ \quad \text{(A1)}$$

(4)

14. [Edexcel, 2009] Sine and Cosine Rule **[3 Marks]**

The diagram shows the positions of two ships, A and B, and a lighthouse L.

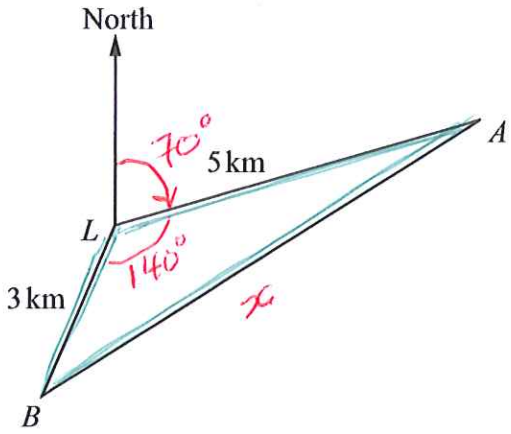


Diagram **NOT** accurately drawn

Ship A is 5 km from L on a bearing of 070° from L.
Ship B is 3 km from L on a bearing of 210° from L.
Calculate the distance between ship A and ship B.
Give your answer correct to 3 significant figures.

B1

$x^2 = 3^2 + 5^2 - 2\times3\times5\times\cos 140^\circ$ M1

$= 56.981...$

$x = \sqrt{56.981}$

$= 7.54859...$

A1

7.55 ..... km

15. [Edexcel, 2009] Sine and Cosine Rule **[5 Marks]**

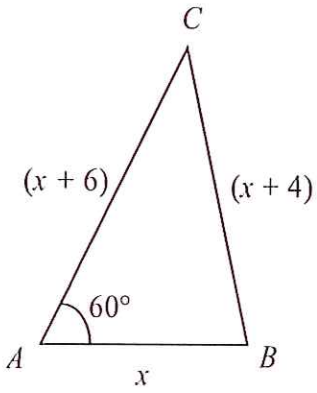


Diagram **NOT** accurately drawn

THREE SIDES: COSINE RULE!

The diagram shows the length, in centimetres, of each side of triangle ABC.
Angle $BAC = 60°$.

Find the value of x.

GENERATING AND SOLVING AN EQUATION

USING COSINE RULE (M1)

$$(x+4)^2 = (x+6)^2 + x^2 - 2x(x+6)\cos 60$$

$\cos 60 = \frac{1}{2}$ (M1)

$$(x+4)(x+4) = (x+6)(x+6) + x^2 - x(x+6)$$

$$x^2 + 8x + 16 = x^2 + 12x + 36 + x^2 - x^2 - 6x$$ (M1)

$$8x + 16 = 12x + 36 - 6x$$

$$2x = 20$$ (M1)

$$x = \frac{20}{2}$$

$$= 10$$ (A1)

16. [Edexcel, 2014] Sine and Cosine Rule **[6 Marks]**

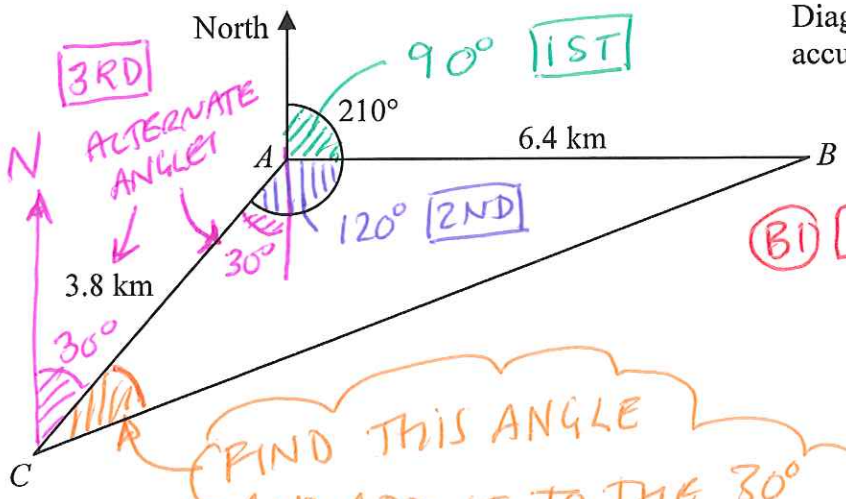


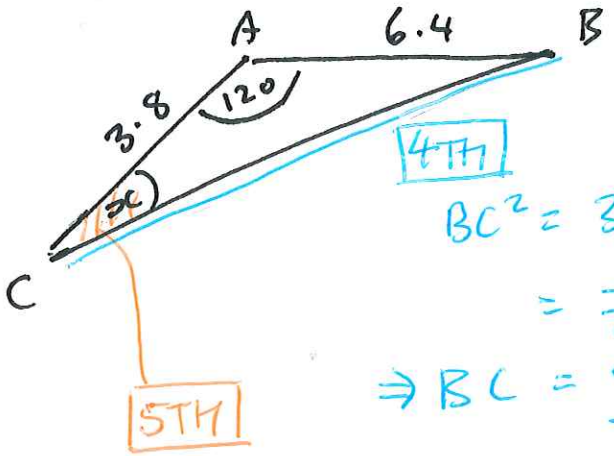
Diagram **NOT** accurately drawn

B1 [FOR EITHER 30° OR 120°]

FIND THIS ANGLE AND ADD IT TO THE 30° TO GET THE BEARING OF B FROM C

A, B and C are 3 villages.
B is 6.4 km due east of A.
C is 3.8 km from A on a bearing of 210°

Calculate the bearing of B from C.
Give your answer correct to the nearest degree.
Show your working clearly.



4TH

M1

$BC^2 = 3.8^2 + 6.4^2 - 2\times3.8\times6.4\times\cos120$

$= 79.72$

$\Rightarrow BC = 8.9286...$ A1

5TH

$\frac{\sin x}{6.4} = \frac{\sin 120}{8.9286}$ M1 $\Rightarrow \sin x = 6.4 \times \frac{\sin 120}{8.9286}$

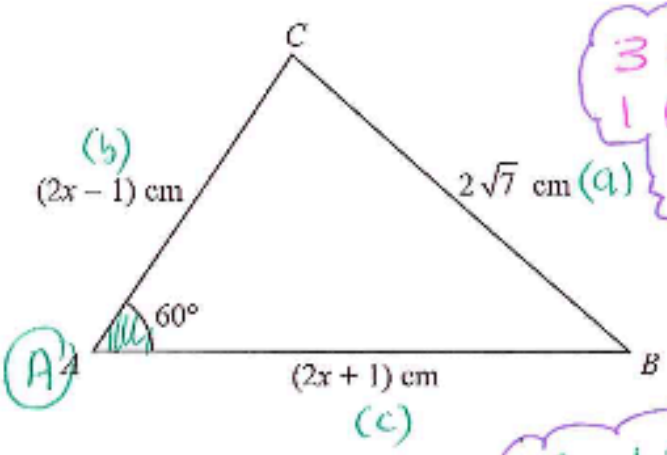
$\Rightarrow x = 38.4°$ A1

6TH!

BEARING $= 30 + 38.4$

$= 068°$ A1

17. [Edexcel, 2015] Sine and Cosine Rule **[3 Marks]**



3 SIDES AND 1 ANGLE ⇒ COSINE RULE

$a^2 = b^2 + c^2 - 2bc\cos A$

The diagram shows a triangle ABC.
$AB = (2x + 1)$ cm, $AC = (2x - 1)$ cm and $BC = 2\sqrt{7}$ cm.
Angle $BAC = 60°$

Work out the value of x.
Show clear algebraic working.

$$(2\sqrt{7})^2 = (2x-1)^2 + (2x+1)^2 - 2(2x-1)(2x+1) \times \frac{1}{2}$$ (m1)

$\cos 60 = \frac{1}{2}$

$$28 = (2x-1)(2x-1) + (2x+1)(2x+1) - (2x-1)(2x+1)$$

$$28 = 4x^2 - 4x + 1 + 4x^2 + 4x + 1 - [4x^2 - 1]$$

$$28 = 4x^2 + 3$$ (m1)

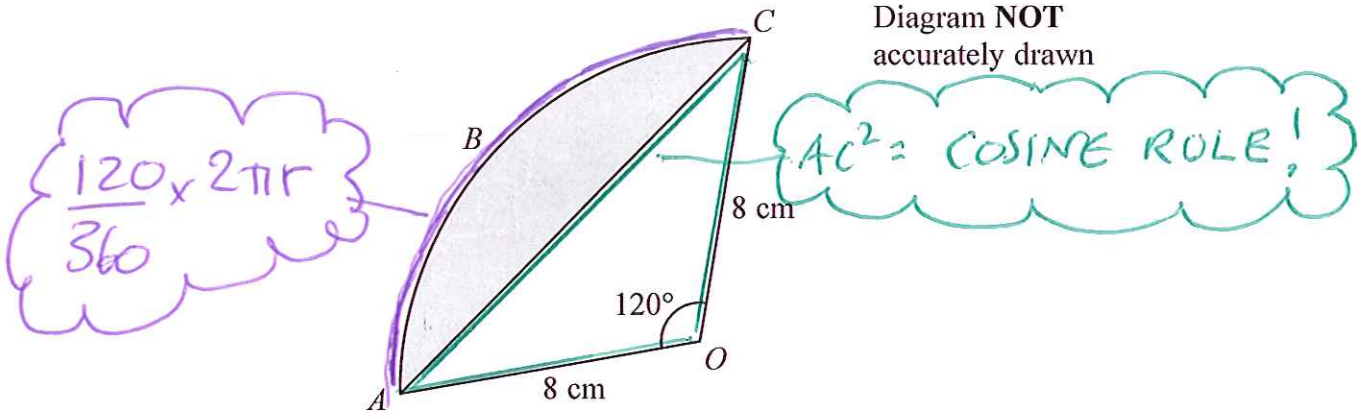
$$\therefore 4x^2 = 25$$

$$x^2 = \frac{25}{4}$$

$$x = \sqrt{\frac{25}{4}}$$

$$= \underline{2.5} \text{ cm}$$ (A1)

18. [Edexcel, 2015] Sine and Cosine Rule **[5 Marks]**



ABC is an arc of a circle with centre *O* and radius 8 cm.
AC is a chord of the circle.
Angle *AOC* = 120°

Calculate the perimeter of the shaded segment.
Give your answer correct to 3 significant figures.

1ST [ARC]

$\frac{120}{360} \times 2\pi \times 8 = 16.755...$ (M1)

2ND [LINE AC]

$AC^2 = 8^2 + 8^2 - 2 \times 8 \times 8 \times \cos 120°$

$= 192$ (M1)

$\Rightarrow AC = \sqrt{192}$

$= 13.856...$ (M1)

(M1)

TOTAL PERIMETER $= 16.755 + 13.856...$

$= 30.611...$

30.6 (A1) cm

19. [Edexcel, 2008] Sine and Cosine Rule **[5 Marks]**

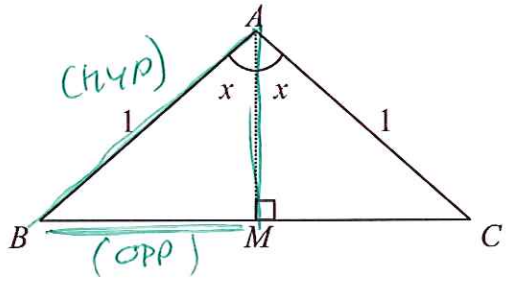


Diagram **NOT** accurately drawn

ABC is an isosceles triangle.
$AB = AC = 1$
M is the midpoint of *BC*.

(a) (i) Use trigonometry to find an expression, in terms of x, for *BM*.

$\sin x = \frac{BM}{1}$ (OPP / HYP)

$BM = \sin x$ (A1)

(ii) Hence write down an expression, in terms of x, for *BC*.

$BC = 2\sin x$ (A1)

(2)

(b) Use the cosine rule to find an expression, in terms of $\cos(2x)$, for BC^2.

$BC^2 = 1^2 + 1^2 - 2 \times 1 \times 1 \cos(2x)$ (A1)

$BC^2 = 2 - 2\cos(2x)$

(1)

(c) Hence show that $\cos(2x) = 1 - 2(\sin x)^2$

$2 - 2\cos(2x) = (2\sin x)^2$ (M1 [EQUATION])

$\Rightarrow 2 - 2\cos(2x) = 4(\sin x)^2$ } (M1 [EITHER])

$\Rightarrow 2\cos(2x) = 2 - 4(\sin x)^2$ }

$\Rightarrow \cos(2x) = 1 - 2(\sin x)^2$

(2)

Disclaimer

While reasonable endeavours have been used to verify the accuracy of these solutions, these solutions are provided on an "as is" basis and no warranties are made of any kind, whether express or implied, in relation to these solutions.

There is no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

Please report any errors or omissions that You find*. These solutions will be updated to correct errors that are discovered. It is recommended that You always check that You have the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or 'best' method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners' mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners' mark scheme.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site.

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words 'show that', 'show your working out', or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

* The best way to inform of errors or omissions is a direct Twitter message to @Maths4Everyone