SPHERES, CONES AND CYLINDERS

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 53

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2014] Spheres, Cones and Cylinders **[3 Marks]**

A cylinder has diameter 12 cm and length 30 cm.

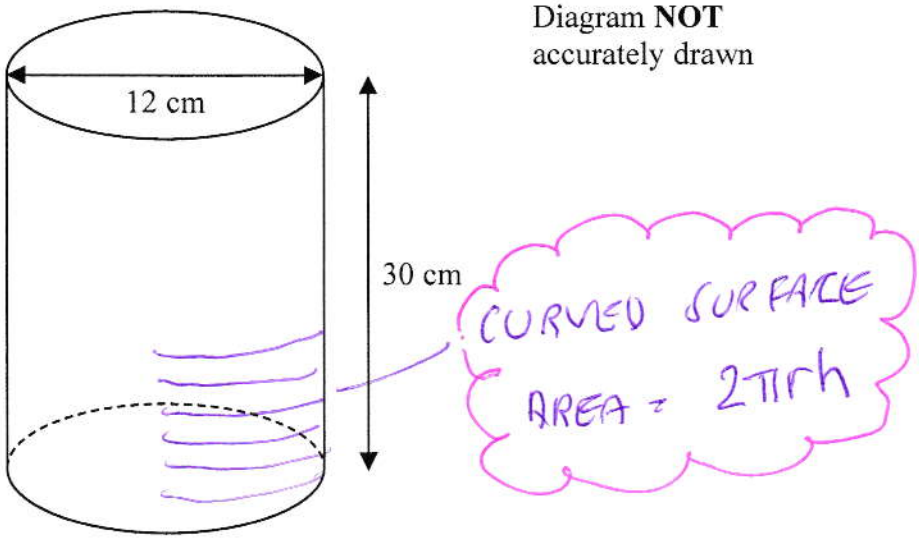


Diagram **NOT** accurately drawn

Work out the curved surface area of the cylinder.
Give your answer correct to 3 significant figures.

CURVED SURFACE AREA = $2\pi rh$ (B1)

$= 2 \times \pi \times 6 \times 30$ (M1)

$= 1130.97...$

$= 1130\ cm^2$ (A1)

2. [Edexcel, 2015] Spheres, Cones and Cylinders **[3 Marks]**

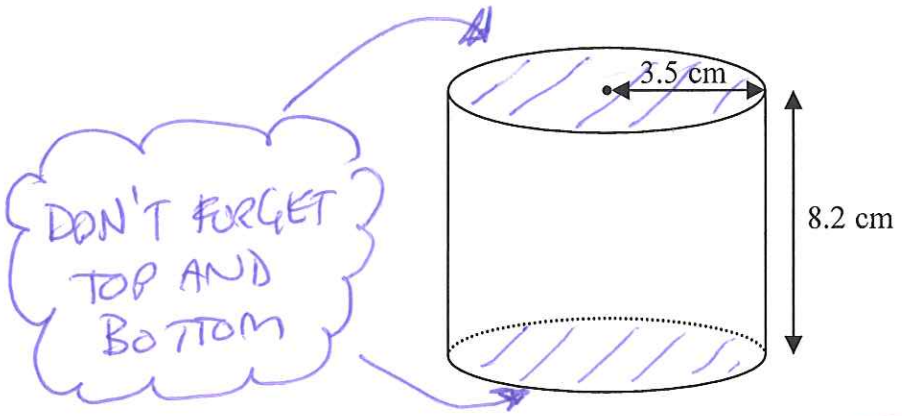


Diagram **NOT** accurately drawn

A solid cylinder has radius 3.5 cm and height 8.2 cm.

Work out the **total** surface area of the cylinder.
Give your answer correct to 3 significant figures.

USE $2\pi rh$

CURVED SURFACE AREA $= 2\pi \times 3.5 \times 8.2$

$= 180.327...$ B1

TOP + BOTTOM $= 2 \times [\pi \times 3.5^2]$

$= 76.969...$ B1

TOTAL $= 180.32... + 76.969$

$= 257.29...$

A1

257 cm^2

3. [Edexcel, 2006] Spheres, Cones and Cylinders **[3 Marks]**

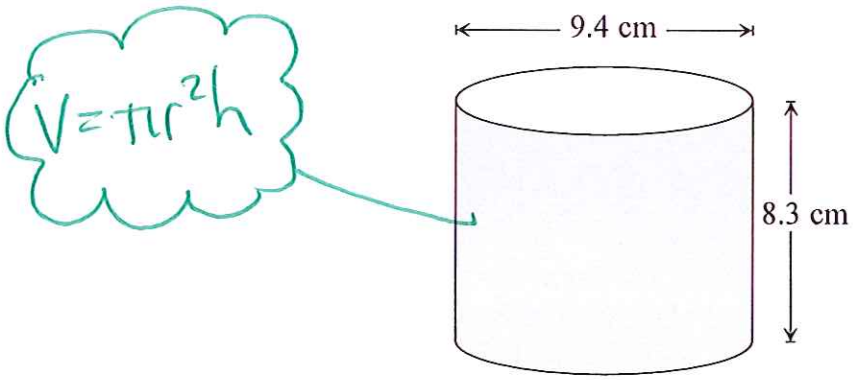


Diagram **NOT** accurately drawn

A solid cylinder has a diameter of 9.4 cm and a height of 8.3 cm.

Work out the volume of the cylinder.
Give your answer correct to 3 significant figures.

$V = \pi \times 4.7^2 \times 8.3$ (M1)

(B1)

$= 576.001$

$= 576 \text{ cm}^3$ (A1)

4. [Edexcel, 2015] Spheres, Cones and Cylinders **[3 Marks]**

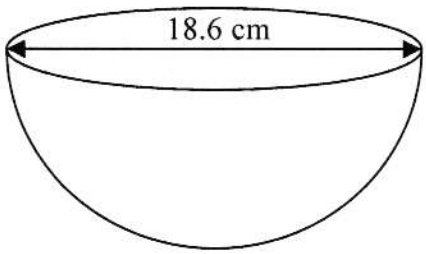


Diagram **NOT** accurately drawn

FOR A SPHERE

$V = \frac{4}{3}\pi r^3$

The diagram shows a hemisphere with a diameter of 18.6 cm.

Work out the volume of the hemisphere.
Give your answer correct to 3 significant figures.

$V = \left[\frac{4}{3}\pi r^3\right] \div 2$

$= \left[\frac{4}{3} \times \pi \times 9.3^3\right] \div 2$ (M1)

(B1)

$= 1684.64\ldots$

(A1)

1680 ..... cm^3

5. [Edexcel, 2004] Spheres, Cones and Cylinders **[6 Marks]**

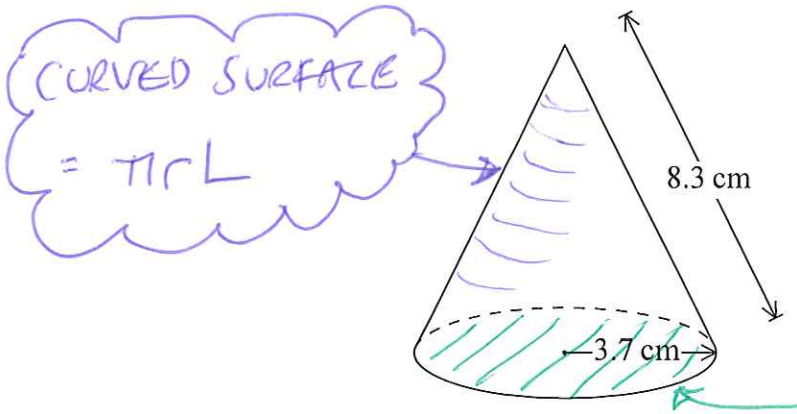


Diagram **NOT** accurately drawn

CURVED SURFACE = πrL

AREA OF BASE = πr^2

The diagram shows a solid cone.
The radius of its base is 3.7 cm and the slant height is 8.3 cm.

(a) Calculate the total surface area of the cone.
Give your answer correct to 3 significant figures.

CIRCULAR B $= \pi \times 3.7^2$

$= 43.008...$

CURVED SURFACE $= \pi \times 3.7 \times 8.3$

$= 96.478...$ A1

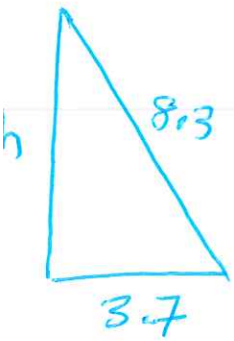
TOTAL $= 139.486$

$= 139\ cm^2$ A1

(b) Calculate the volume of the cone.
Give your answer correct to 3 significant figures.

NEED TO KNOW THE VERTICAL HEIGHT, h

$V = \frac{1}{3}\pi r^2 h$



$h = \sqrt{8.3^2 - 3.7^2}$ M1

$= 7.4296...$ A1

$\Rightarrow V = \frac{1}{3}\pi \times 3.7^2 \times 7.4296$ M1

$= 106.512...$

$= 107\ cm^3$ A1

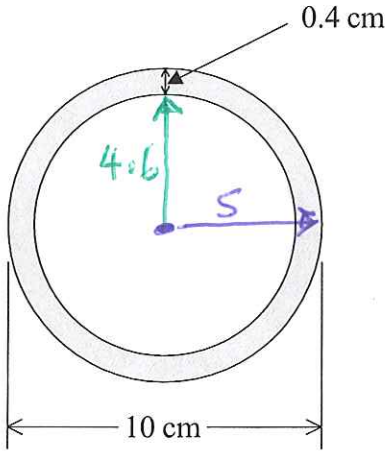


Diagram **NOT** accurately drawn

$V = \frac{4}{3}\pi r^3$

The outer diameter of a hollow spherical ball is 10 cm.
The ball is made from rubber which is 0.4 cm thick.

Calculate the volume of rubber needed to make the ball.
Give your answer correct to 3 significant figures.

VOLUME OF SOLID BALL, DIAMETER 10 cm

$= \frac{4}{3}\pi \times 5^3$

$= 523.598...$ (M1)

VOLUME OF 'SPACE' INSIDE BALL (DIAMETER 9.2!)

$= \frac{4}{3}\pi \times 4.6^3$ (B1)

$= 407.72$

VOLUME OF RUBBER

$= 523.598 - 407.72$ (M1)

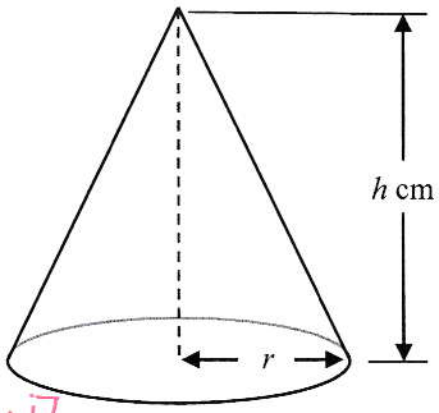
$= 115.877...$

116 (A1) ..... cm^3

7. [Edexcel, 2004] Spheres, Cones and Cylinders **[3 Marks]**

A cone has a base radius r cm and vertical height h cm.

$V = \frac{1}{3}\pi r^2 h$



The volume of the cone is 12π cm^3.
Find an expression for r in terms of h.

$\frac{1}{3}\pi r^2 h = 12\pi$ (M1) [EQUATION]

$r^2 h = 12 \times 3$

$r^2 = \frac{36}{h}$ (M1) [EITHER]

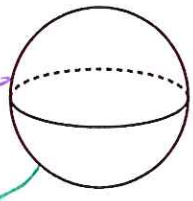
$r = \sqrt{\frac{36}{h}}$ (A1)

8. [Edexcel, 2014] Spheres, Cones and Cylinders **[4 Marks]**

A sphere has a surface area of 81π cm^2.

Work out the volume of the sphere.
Give your answer correct to 3 significant figures.

$V = \frac{4}{3}\pi r^3$



$SA = 4\pi r^2$

1ST

$4\pi r^2 = 81\pi$ (M1)

$\Rightarrow r^2 = \frac{81}{4}$

$r = \sqrt{\frac{81}{4}}$

$= \frac{9}{2}$

$= 4.5$ (A1)

2ND

$V = \frac{4}{3} \times \pi \times 4.5^3$ (M1)

$= 381.703...$

$= 382\ cm^3$ (A1)

9. [Edexcel, 2010] Spheres, Cones and Cylinders **[4 Marks]**

A cone has slant height 4 cm and base radius r cm.

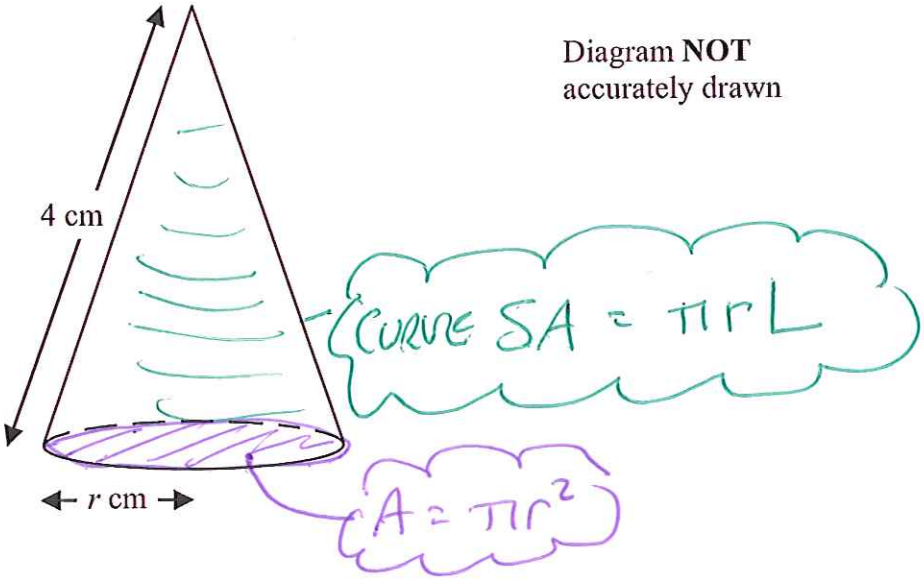


Diagram **NOT** accurately drawn

The **total** surface area of the cone is $\frac{33}{4}\pi$ cm².

Calculate the value of r.

$$\pi rL + \pi r^2 = \frac{33}{4}\pi$$

$$rL + r^2 = \frac{33}{4}$$

$$4r + r^2 = \frac{33}{4}$$

M1 [ANY EQUATION]

$$4r^2 + 16r - 33 = 0$$ M1 [QUADRATIC]

$$(2r + 11)(2r - 3) = 0$$ M1 [FACTORISING]

$2r + 11 = 0$

$r = -\frac{11}{2}$

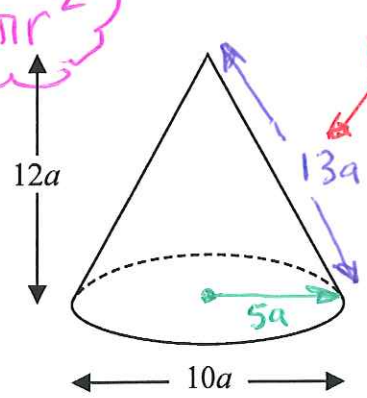
$2r - 3 = 0$

$r = \frac{3}{2}$ A1

[-VE IS NOT POSSIBLE]

The diagram shows a solid cone.

$SA = \pi r L + \pi r^2$



B1

$[\sqrt{12^2+5^2}]$

$V = \frac{1}{3}\pi r^2 h$

Diagram **NOT** accurately drawn

The diameter of the base of the cone is $10a$ cm.
The height of the cone is $12a$ cm.

The total surface area of the cone is 360π cm^2 — 1ST
The volume of the cone is $k\pi$ cm^3, where k is an integer. — 2ND

Find the value of k.

1ST

$$\pi \times 5a \times 13a + \pi \times (5a)^2 = 360\pi \quad \text{M1 [EQUATION]}$$

$$\Rightarrow 65a^2\pi + 25a^2\pi = 360\pi$$

$$90a^2\pi = 360\pi$$

$$a^2 = \frac{360}{90}$$

$$a = \underline{\underline{2}} \quad \text{A1}$$

$$\frac{1}{3}\pi \times (5a)^2 \times 12a = k\pi \quad \text{M1 [EQUATION]}$$

$$\Rightarrow \frac{1}{3}\pi \times 25a^2 \times 12a = k\pi$$

$$\Rightarrow 100a^3\pi = k\pi$$

$$100a^3 = k$$

M1 [SUBSTITUTION]

$$k = 100 \times 2^3$$

$$= \underline{\underline{800}} \quad \text{A1}$$

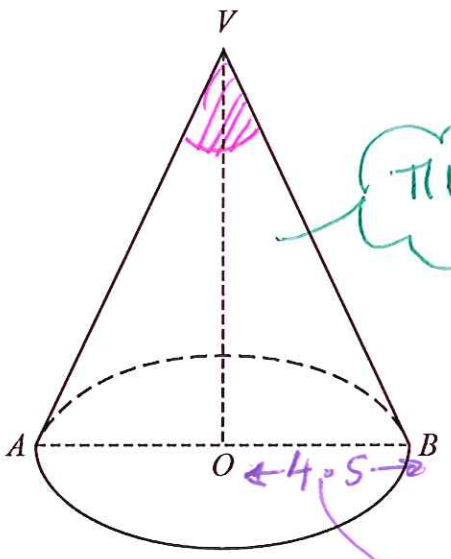


Diagram **NOT** accurately drawn

$\pi r L = 130$

The diagram shows a solid cone.
The base of the cone is a horizontal circle, centre O, with radius 4.5 cm.
AB is a diameter of the base and OV is the vertical height of the cone.
The curved surface area of the cone is 130 cm^2

Calculate the size of the angle AVB.

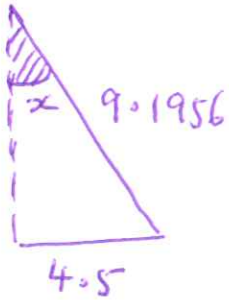
Give your answer correct to 1 decimal place.

1ST

$$\pi r L = 130 \Rightarrow L = \frac{130}{\pi r}$$

$$= \frac{130}{\pi \times 4.5} = 9.1956\ldots \quad \text{(B1)}$$

2ND



$$\sin x = \frac{4.5}{9.1956}$$

$$\Rightarrow x = \sin^{-1}\left(\frac{4.5}{9.1957}\right) \quad \text{(M1)}$$

$$= 29.298\ldots$$

$$\therefore AVB = 2 \times 29.298\ldots \quad \text{(M1)} \qquad = 58.6^\circ \quad \text{(A1)}$$

USE $V = \frac{1}{3}\pi r^2 h$

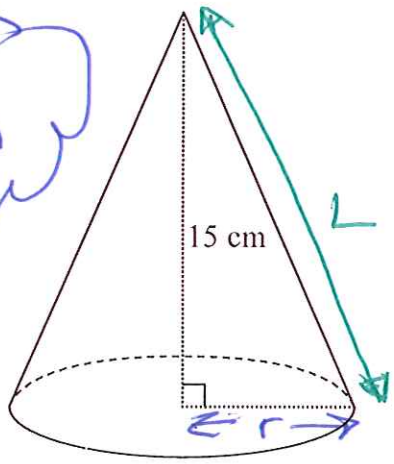


Diagram **NOT** accurately drawn

[$L^2 = 15^2 + r^2$!]

A solid cone has a height of 15 cm.
The volume of the cone is 320π cm^3

Work out the curved surface area of the cone.
Give your answer correct to 3 significant figures.

USE $A = \pi r L$

STEP 1 [FIND r]

$\frac{1}{3}\pi r^2 h = 320\pi$ (M1)

$\pi r^2 h = 960\pi$ $(\times 3)$

$r^2 h = 960$ $(\div \pi)$

$\Rightarrow r^2 = \frac{960}{h}$

$\Rightarrow r = \sqrt{\frac{960}{15}}$ $(h = 15)$

$= 8$ (A1)

STEP 2 [FIND L]

$L^2 = 15^2 + 8^2$ (M1)

$= 289$

$\Rightarrow L = 17$ (B1)

STEP 3

$A = \pi \times 8 \times 17$

$= 427.25\ldots$

$= 427$ cm^2 (A1)

13. [Edexcel, 2016] Spheres, Cones and Cylinders **[5 Marks]**

The diagram shows a solid cylinder.

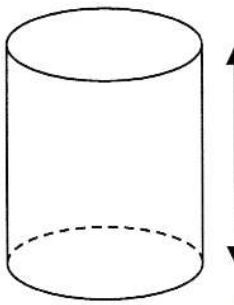


Diagram **NOT** accurately drawn

The cylinder has radius $4\sqrt{3}$ cm and height h cm.
The total surface area of the cylinder is $56\pi\sqrt{6}$ cm^2

$2\pi rh + 2\pi r^2$

Find the exact value of h.
Give your answer in the form $a\sqrt{2} + b\sqrt{3}$, where a and b are integers.
Show your working clearly.

$2\pi rh + 2\pi r^2 = 56\pi\sqrt{6}$ M1 [EQUATION]

M1

$2rh + 2r^2 = 56\sqrt{6}$

$rh + r^2 = 28\sqrt{6}$

$4\sqrt{3}h + (4\sqrt{3})^2 = 28\sqrt{6}$ M1 [SUBSTITUTING 'r']

$4\sqrt{3}h + 48 = 28\sqrt{6}$

$\sqrt{3}h + 12 = 7\sqrt{6}$

$h = \dfrac{7\sqrt{6} - 12}{\sqrt{3}}$ M1 [ISOLATING 'h']

$= \dfrac{7\sqrt{6} - 12}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{7\sqrt{18} - 12\sqrt{3}}{3}$

$= \dfrac{21\sqrt{2} - 12\sqrt{3}}{3}$

$= 7\sqrt{2} - 4\sqrt{3}$ A1

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The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or 'best' method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners' mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners' mark scheme.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site.

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words 'show that', 'show your working out', or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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