

SPHERES, CONES AND CYLINDERS

SOLUTIONS

DATE OF SOLUTIONS: 15/05/2018
 MAXIMUM MARK: 52

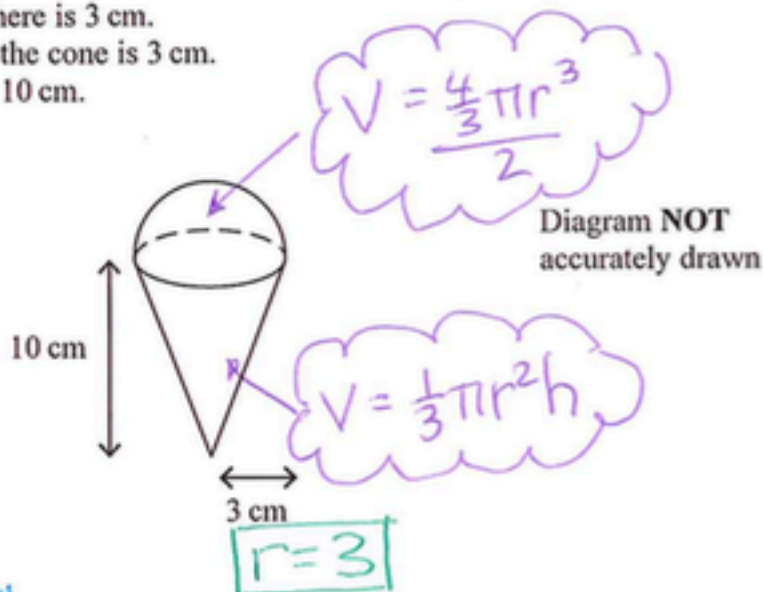
GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2006]

Spheres, Cones and Cylinders [4 Marks]



A Maxicool consists of a cone full of ice cream with a hemisphere of ice cream on top.
 The radius of the hemisphere is 3 cm.
 The radius of the base of the cone is 3 cm.
 The height of the cone is 10 cm.



Calculate the total volume of ice cream in a Maxicool.
 Give your answer correct to 3 significant figures.

HEMISPHERE $V = \frac{4}{3} \pi \times 3^3$

$= \frac{36\pi}{2}$

$= \underline{18\pi}$ (m)

CONE

$V = \frac{1}{3} \pi \times 3^2 \times 10$

$= \underline{30\pi}$ (m)

TOTAL

$18\pi + 30\pi = 48\pi$
 $= 150.79$

$= \underline{151 \text{ cm}^3}$
 (A)

A solid is made from a cylinder and a hemisphere.
 The cylinder has radius 1.5 cm and height 4 cm.
 The hemisphere has radius 1.5 cm.

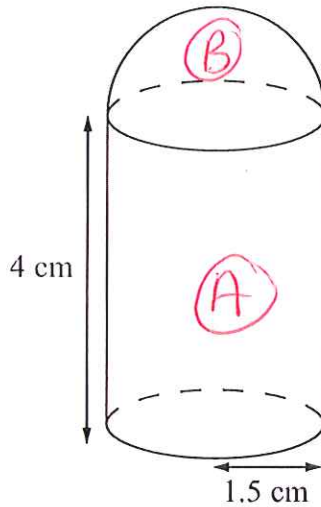


Diagram **NOT**
accurately drawn

Work out the total volume of the solid.
 Give your answer correct to 3 significant figures.

$$\textcircled{A} \quad V = \pi r^2 h \quad (r = 1.5, h = 4)$$

$$= \pi \times 1.5^2 \times 4$$

$$= 28.274\dots$$

$$\textcircled{B} \quad V = \frac{4}{3} \pi r^3 \quad (r = 1.5)$$

$$= \frac{4}{3} \times \pi \times 1.5^3$$

$$= 14.137\dots$$

WHOLE
SPHERE

$$\text{HEMISPHERE!} \rightarrow = \underline{\underline{7.0685\dots}}$$

$$\text{TOTAL} = \begin{array}{r} 28.274\dots \\ + 7.068\dots \\ \hline \end{array}$$

$$\underline{\underline{35.342\dots}}$$

$$\underline{\underline{35.3}} \text{ cm}^3$$

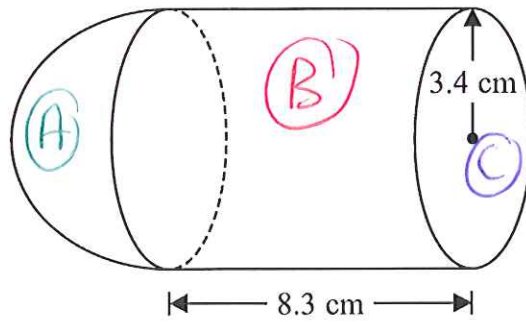


Diagram NOT
accurately drawn

The diagram shows a shape made from a solid cylinder and a solid hemisphere.
The cylinder has a radius of 3.4 cm and a length of 8.3 cm.
The hemisphere has a radius of 3.4 cm.

Calculate the total surface area of the solid shape.
Give your answer correct to 3 significant figures.

3 SURFACES

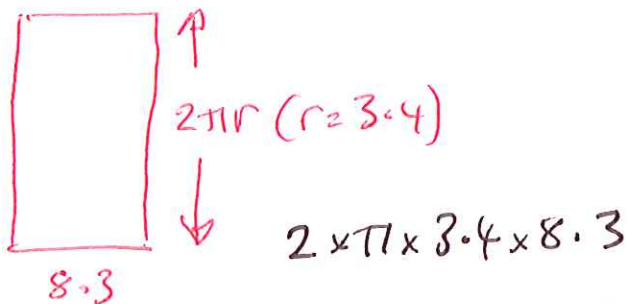
(A) [HEMISPHERE]

$$\frac{4\pi r^2}{2} \quad (r=3.4)$$

$$= \frac{4\pi \times 3.4^2}{2} = \underline{\underline{72.6336\dots}}$$

(B1)

(B) [RECTANGLE]



$$2 \times \pi \times 3.4 \times 8.3$$

$$= \underline{\underline{177.3114\dots}}$$

(B1)

(C) [CIRCLE]

$$\pi r^2 \quad (r=3.4)$$

$$= \pi \times 3.4^2$$

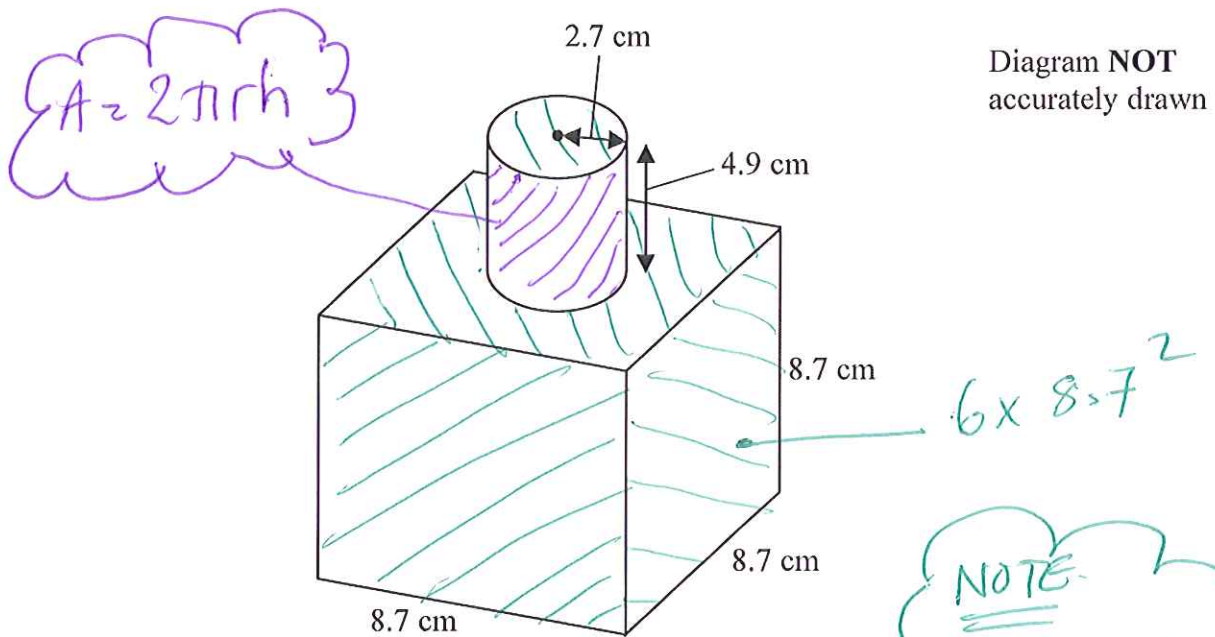
$$= \underline{\underline{36.3168\dots}}$$

(B1)

$$\underline{\underline{286}} \text{ cm}^2$$

(A1)

$$\text{TOTAL} = \underline{\underline{286.26\dots}}$$



The diagram shows a shape made from a solid cube and a solid cylinder.
The cube has sides of length 8.7 cm.
The cylinder has a radius of 2.7 cm and a height of 4.9 cm.

Calculate the total surface area of the solid shape.
Give your answer correct to 3 significant figures.

CUBE:

$$6 \times 8.7^2 = 454.14 \quad (m)$$

CYLINDER:

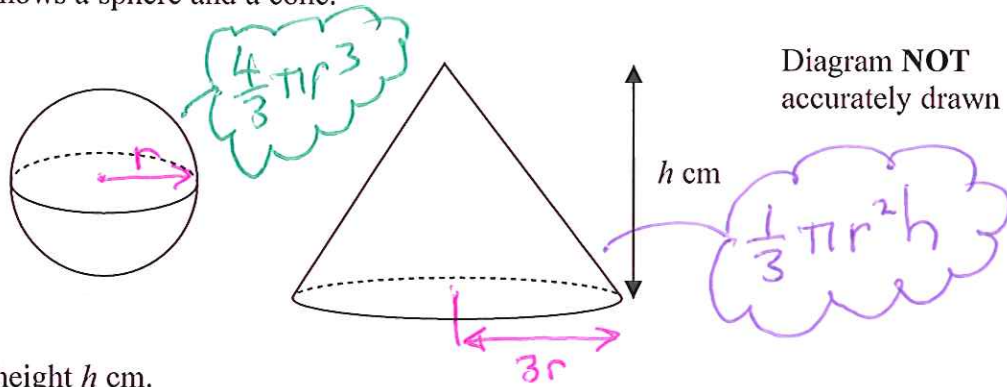
$$2 \times \pi \times 2.7 \times 4.9 = 83.126 \dots \quad (m)$$

$$\underline{\underline{TOTAL}} = 454.14 + 83.126 \dots$$

$$= 537.266 \dots$$

$$= \underline{\underline{537 \text{ cm}^3}} \quad (A)$$

The diagram shows a sphere and a cone.



The cone has height h cm.

The radius of the base of the cone is 3 times the radius of the sphere.

Given that the volume of the sphere is equal to the volume of the cone, find an expression for the radius of the sphere in terms of h .

Give your expression in its simplest form.

$$\frac{4}{3} \pi r^3 = \frac{1}{3} \pi (3r)^2 h \quad (M1)$$

$$\Rightarrow \frac{4}{3} \pi r^3 = \frac{1}{3} \pi \times 9r^2 \times h$$

$$\Rightarrow \frac{4}{3} \pi r^3 = 3 \pi r^2 h$$

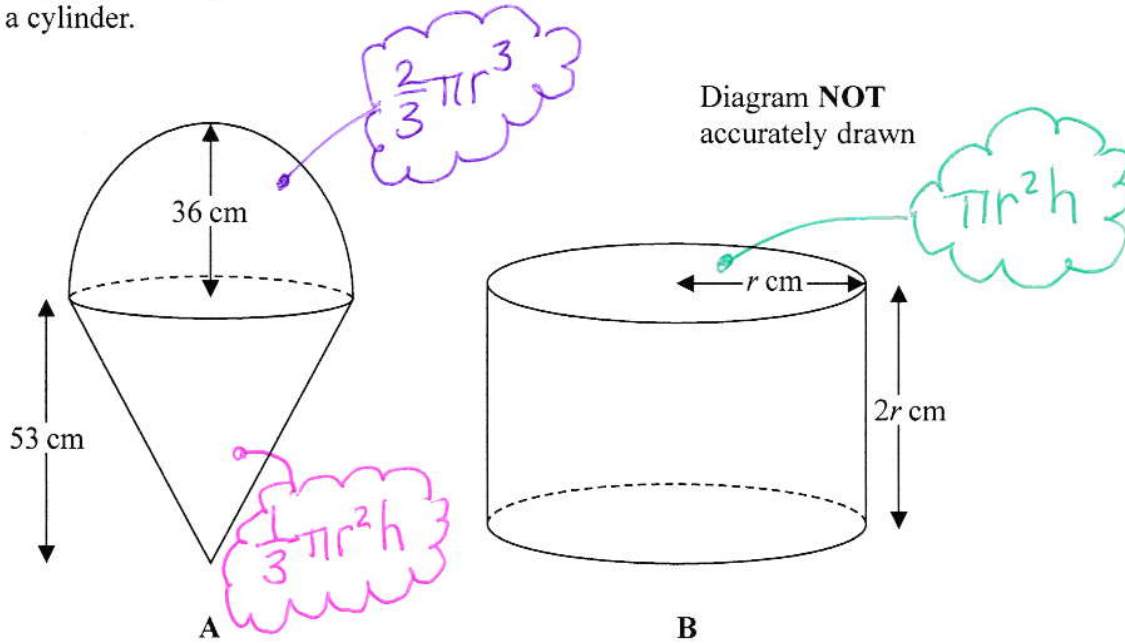
$$\Rightarrow \pi r^3 = \frac{9}{4} \pi r^2 h$$

$$\Rightarrow r^3 = \frac{9}{4} r^2 h$$

$$\Rightarrow r = \frac{9h}{4} \quad (A1)$$

=====

The diagram shows two solid shapes, shape A and shape B.
 Shape A is made of a hemisphere and a cone.
 Shape B is a cylinder.



For shape A

radius of the hemisphere is 36 cm
 radius of the base of the cone is 36 cm
 height of the cone is 53 cm

For shape B

radius of the cylinder is r cm
 height of the cylinder is $2r$ cm

The volume of shape A = the volume of shape B

Calculate the height of shape B.

$$\frac{1}{3} \pi \times 36^2 \times 53 + \frac{2}{3} \pi \times 36^3 = \pi r^2 \times 2r \quad \text{(M1) [EQUATION]}$$

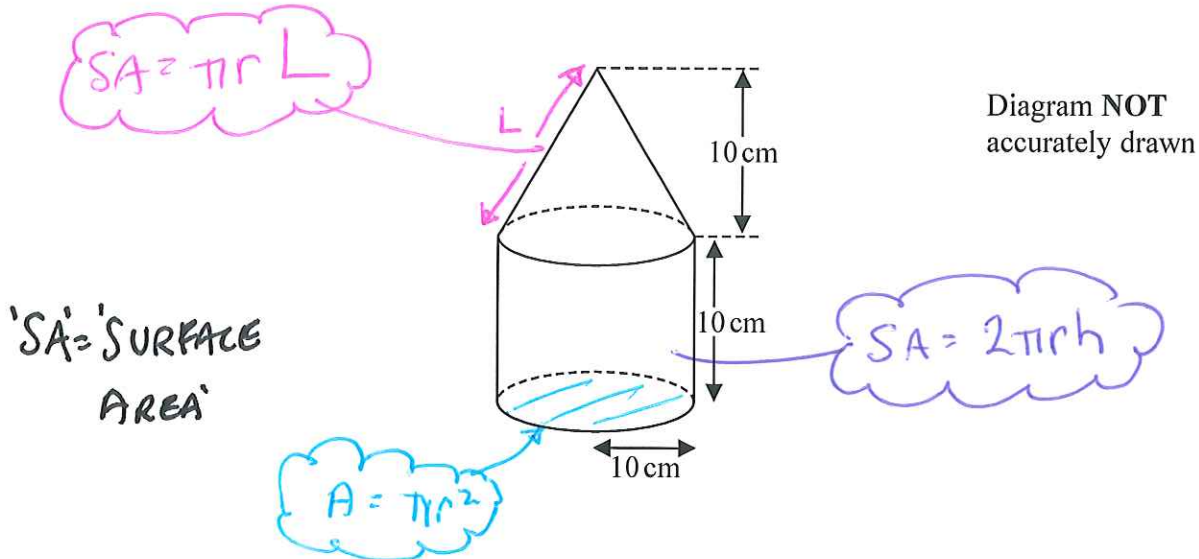
$$\Rightarrow 22896\pi + 31104\pi = 2\pi r^3$$

$$\Rightarrow 54000\pi = 2\pi r^3$$

$$\Rightarrow r^3 = \frac{54000\pi}{2\pi}$$

$$r = \sqrt[3]{27000} = \underline{\underline{30}} \quad \text{(M1)}$$

$$\Rightarrow \text{HEIGHT} = 2 \times 30 = \underline{\underline{60 \text{ cm}}} \quad \text{(A1)}$$



The diagram shows a solid shape made from a cone on top of a cylinder.

The cone has a radius of 10 cm and a height of 10 cm.

The cylinder has a radius of 10 cm and a height of 10 cm.

The centre of the base of the cone coincides with the centre of the top face of the cylinder.

The total surface area of the solid is $A \text{ cm}^2$

Show that $A = (300 + 100\sqrt{2})\pi$

1ST

$$\begin{aligned} L &= \sqrt{10^2 + 10^2} \\ &= \sqrt{200} \\ &= \underline{\underline{10\sqrt{2}}} \quad (mi) \end{aligned}$$

$$\begin{aligned} SA &= \pi \times 10 \times 10\sqrt{2} \\ &= \underline{\underline{100\pi\sqrt{2}}} \quad (mi) \end{aligned}$$

TOTAL SURFACE AREA

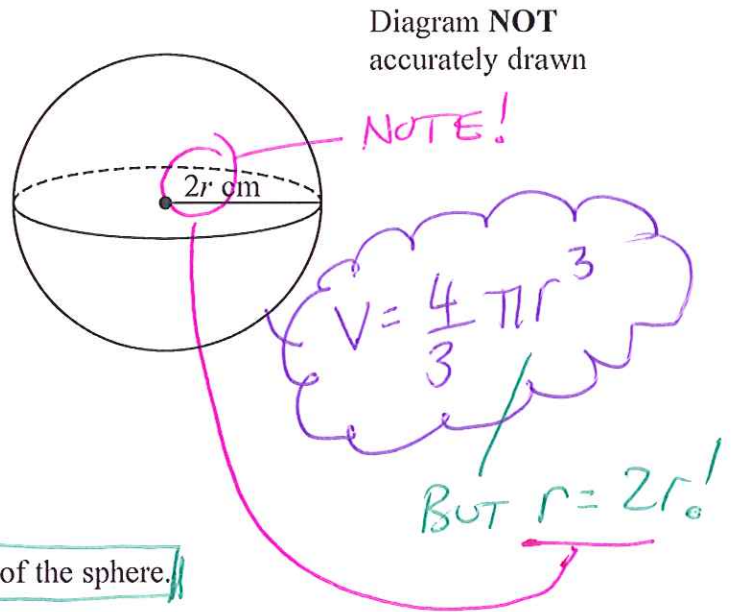
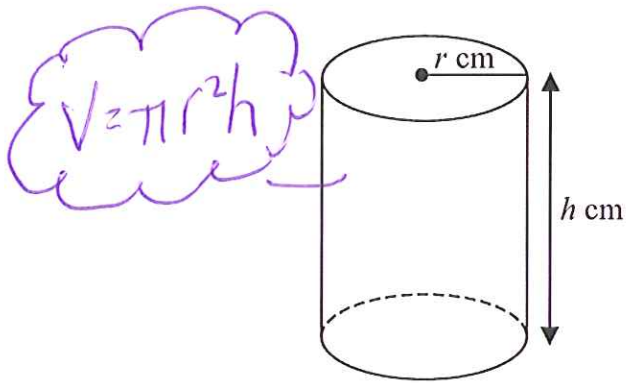
$$100\pi\sqrt{2} + \underbrace{2\pi \times 10 \times 10}_{\text{CYLINDER}} + \underbrace{\pi \times 10^2}_{\text{CIRCLE BASE}}$$

$$= 100\pi\sqrt{2} + 200\pi + 100\pi \quad (mi)$$

$$= 300\pi + 100\pi\sqrt{2}$$

$$= \underline{\underline{(300 + 100\sqrt{2})\pi}}$$

The diagram shows a cylinder and a sphere.



The cylinder has radius r cm and height h cm.
The sphere has radius $2r$ cm.

The volume of the cylinder is equal to the volume of the sphere.

Find an expression for h in terms of r .

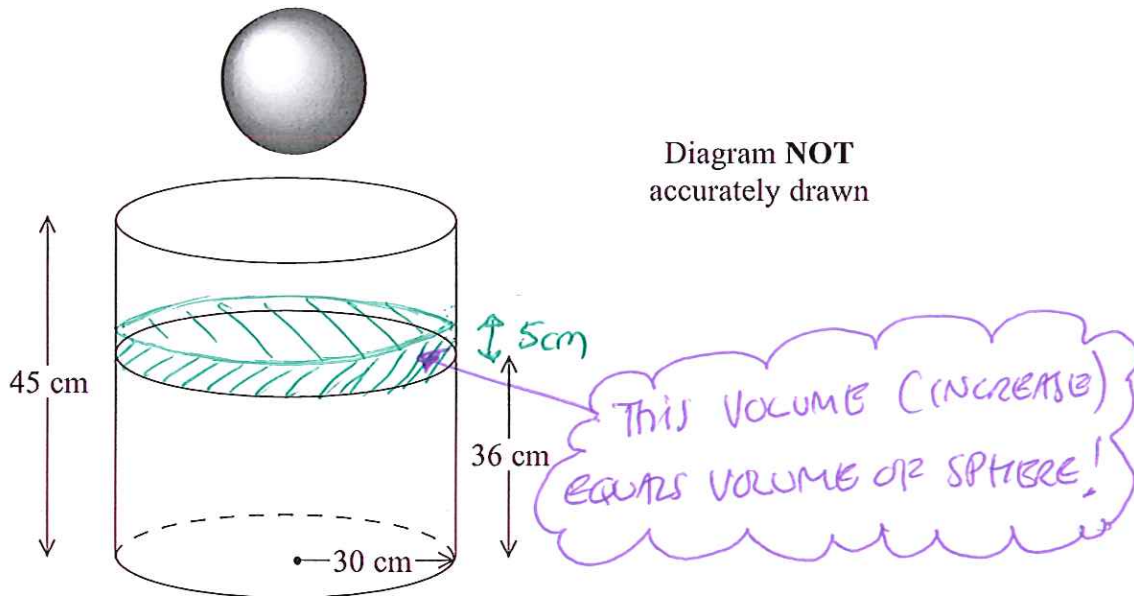
Give your answer in its simplest form.

$$\rightarrow \pi r^2 h = \frac{4}{3} \pi (2r)^3 \quad (M1)$$

$$\Rightarrow \pi r^2 h = \frac{4}{3} \pi \times 8r^3$$

$$\Rightarrow h = \frac{4}{3} \times 8r \quad (M1) \quad (\div \pi r^2)$$

$$\Rightarrow h = \frac{32}{3} r \quad (A1)$$



A cylindrical tank has a radius of 30 cm and a height of 45 cm.
The tank contains water to a depth of 36 cm.

A metal sphere is dropped into the water and is completely covered.
The water level rises by 5 cm.

Calculate the radius of the sphere.

$$\text{VOLUME OF RISE} = \pi \times 30^2 \times 5 = 14137 \text{ cm}^3$$

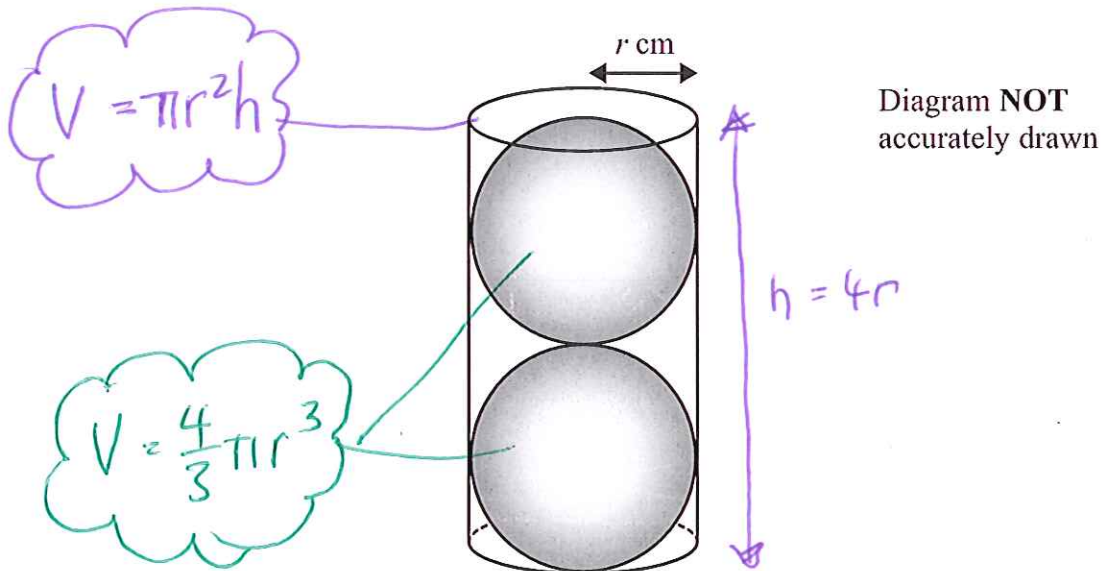
\therefore VOLUME OF SPHERE

$$\frac{4}{3} \pi r^3 = 14137$$

$$\Rightarrow r^3 = \frac{14137}{\pi} \times \frac{3}{4}$$

$$= 3375$$

$$r = \sqrt[3]{3375} = 15 \text{ cm}$$



Two solid spheres, each of radius r cm, fit exactly inside a hollow cylinder.

The radius of the cylinder is r cm.

The height of the cylinder is equal to $4r$ cm.

The volume of the space inside the cylinder, not occupied by the spheres, is $\frac{125}{6}\pi$ cm³

Calculate the value of r .

Show your working clearly.

$$\text{VOL OF CYLINDER} - 2 \times \text{VOL OF SPHERE} = \frac{125}{6}\pi$$

$$\pi r^2 \times 4r - 2 \times \frac{4}{3}\pi r^3 = \frac{125}{6}\pi$$

$$4r^3 - \frac{8}{3}r^3 = \frac{125}{6}$$

(m) [ANY CORRECT EQUATION]

$$\frac{4}{3}r^3 = \frac{125}{6}$$

$$r^3 = \frac{125}{6} \times \frac{3}{4}$$

$$r = \sqrt[3]{\frac{375}{24}} \rightarrow r = 2.5$$

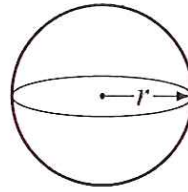
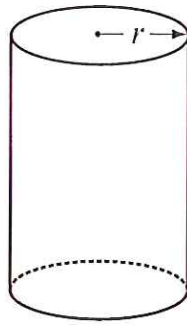


Diagram NOT
accurately drawn

The diagram shows a solid cylinder and a solid sphere.
The cylinder has radius r .
The sphere has radius r .

Given that $\frac{\text{Total surface area of cylinder}}{\text{Surface area of sphere}} = 2$

$$\Rightarrow \frac{2\pi r^2 + 2\pi rh}{4\pi r^2} = 2 \quad \text{(M1)}$$

find the value of $\frac{\text{Volume of cylinder}}{\text{Volume of sphere}}$

$$\Rightarrow 2\pi r^2 + 2\pi rh = 8\pi r^2$$

$$\Rightarrow \pi r^2 + \pi rh = 4\pi r^2$$

$$\Rightarrow r^2 + rh = 4r^2$$

$$\Rightarrow r + h = 4r$$

$$\Rightarrow \underline{\underline{h = 3r}} \quad \text{(B1)}$$

(2ND)

$$\frac{\pi r^2 h}{\frac{4}{3}\pi r^3} = \frac{r^2 h}{\frac{4}{3}r^3}$$

(M1)

$$= \frac{h}{\frac{4}{3}r}$$

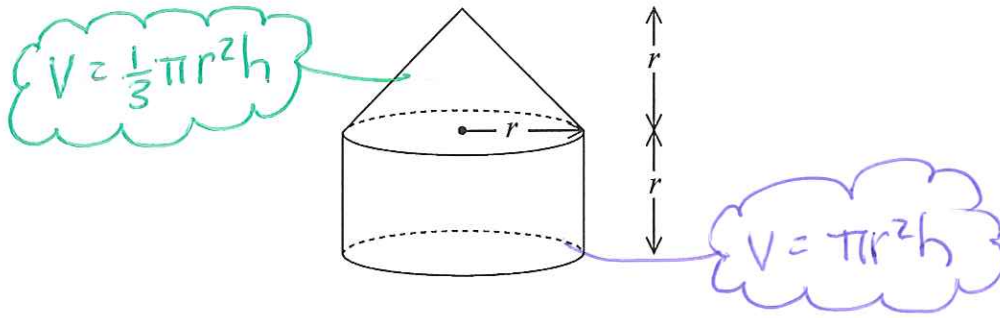
$$= \frac{3r}{\frac{4}{3}r}$$

(M1)

$$= \frac{3}{\frac{4}{3}}$$

$$= 3 \times \frac{3}{4} = \underline{\underline{\frac{9}{4}}}$$

$$\underline{\underline{\frac{9}{4}}} \quad \text{(A1)}$$



The diagram shows a solid made from a cone and a cylinder.
The cylinder has radius r and height r .
The cone has base radius r and height r .

- (a) Show that the total volume of the solid is equal to the volume of a sphere of radius r .

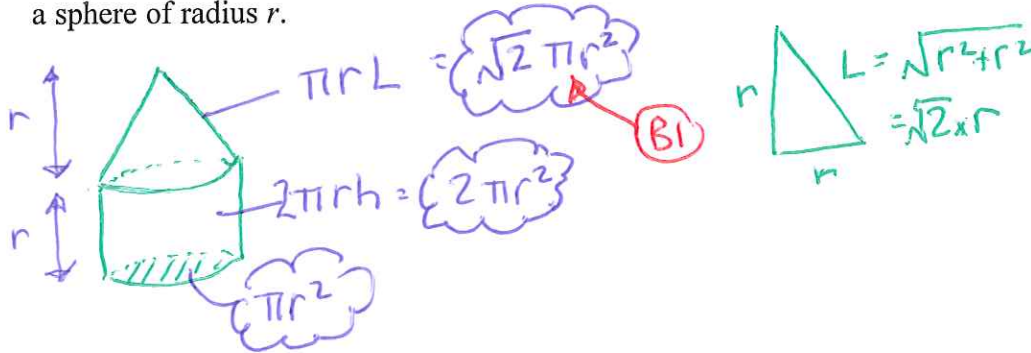
TOTAL VOLUME

$$\begin{aligned} \frac{1}{3}\pi r^2 h + \pi r^2 h &= \frac{4}{3}\pi r^2 h \quad (m1) \\ &= \frac{4}{3}\pi r^3 \quad (\text{BECAUSE } h=r) \end{aligned}$$

(2)

The curved surface area of a cylinder with base radius r and height h is $2\pi rh$.
The curved surface area of a cone with base radius r and slant height l is πrl .

- (b) Show that the **total** surface area of the above solid is greater than the surface area of a sphere of radius r .



TOTAL SURFACE AREA

(3)

$$\begin{aligned} &\sqrt{2}\pi r^2 + 2\pi r^2 + \pi r^2 \quad (m1) \quad [\text{ADDING ALL THREE}] \\ &= (\sqrt{2} + 2 + 1)\pi r^2 \\ &= 4.41\pi r^2 \quad (A1) \quad \text{WHICH IS GREATER THAN } 4\pi r^2 \end{aligned}$$

SURFACE AREA OF A SPHERE

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The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or ‘best’ method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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