

STANDARD FORM

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 87

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2005]

Standard Form [4 Marks]

The table shows the populations of five countries.

Country	Population
The Gambia	1.4×10^6
Kenya	3.2×10^7
Mali	1.2×10^7
Nigeria	1.4×10^8
Swaziland	1.2×10^6

LARGEST POWER OF 10

(a) Which of these countries has the largest population?

NIGERIA (A1)
(1)

(b) Calculate the difference between the population of Kenya and the population of Nigeria.

Give your answer in standard form.

$$\begin{array}{r} 1.4 \times 10^8 \\ - 3.2 \times 10^7 \\ \hline \Rightarrow 1.4 \times 10^8 \\ - 0.32 \times 10^8 \\ \hline \underline{\underline{1.08 \times 10^8}} \end{array}$$

1.08×10^8 (A2)
(2)

(c) The population of South Africa is 30 times the population of The Gambia.

Calculate the population of South Africa.

Give your answer in standard form.

$$\begin{aligned} & 30 \times 1.4 \times 10^6 \\ & = 42 \times 10^6 \\ & = \underline{\underline{4.2 \times 10^7}} \end{aligned}$$

4.2×10^7 (A1)
(1)

The table shows the diameters, in kilometres, of five planets.

Planet	Diameter (km)
Venus	1.2×10^4
Jupiter	1.4×10^5
Neptune	5.0×10^4
Mars	6.8×10^3
Saturn	1.2×10^5

(a) Which of these planets has the smallest diameter?

MARS (B1)
(1)

(b) Calculate the difference, in kilometres, between the diameter of Saturn and the diameter of Neptune.

Give your answer in standard form.

$$\begin{array}{r} 1.2 \times 10^5 \\ - 5.0 \times 10^4 \end{array}$$

$$\begin{array}{r} 12 \times 10^4 \\ - 5 \times 10^4 \\ \hline 7 \times 10^4 \end{array}$$

(M1) FOR SUBTRACTING

7×10^4 km
(2)

The diameter of the Moon is 3.5×10^3 km.

The diameter of the Sun is 1.4×10^6 km.

(c) Calculate the ratio of the diameter of the Moon to the diameter of the Sun.

Give your answer in the form $1 : n$

$$\begin{array}{l} 3.5 \times 10^3 : 1.4 \times 10^6 \\ 3500 : 1400000 \\ 35 : 14000 \\ 5 : 2000 \\ \underline{\underline{1 : 400}} \end{array}$$

(M1) FOR ANY DIVISION

1:400 (A1)

The table shows some information about the five Great Lakes in North America.

Name	Surface area (m ²)	Volume of water (m ³)
Lake Erie	2.57×10^{10}	4.80×10^{11}
Lake Huron	6.01×10^{10}	3.52×10^{12}
Lake Michigan	5.80×10^{10}	4.87×10^{12}
Lake Ontario	1.91×10^{10}	1.64×10^{12}
Lake Superior	8.21×10^{10}	1.22×10^{13}

- (a) Work out the total surface area of the five Great Lakes.
Give your answer in standard form.

$$2.57 \times 10^{10} + 6.01 \times 10^{10} + \dots \quad (\text{mi})$$

$$\underline{2.45 \times 10^{11}} \quad (\text{AI}) \quad \text{m}^2$$

(2)

Loch Ness is the largest lake in Scotland.
The lake has a volume of water of $7.45 \times 10^9 \text{ m}^3$

The volume of water in Lake Superior is k times the volume of water in Loch Ness.

- (b) Work out the value of k .
Give your answer correct to 3 significant figures.

$$k = \frac{\text{SUPERIOR}}{\text{NESS}}$$

$$= \frac{1.22 \times 10^{13}}{7.45 \times 10^9} \quad (\text{mi}) = 1637.58\dots$$

$$k = \underline{1640} \quad (\text{AI})$$

(2)

The table gives the surface areas, in square kilometres, of five seas.

Sea	Surface area in square kilometres
Mediterranean Sea	2.97×10^6
East China Sea	1.25×10^6
Baltic Sea	4.22×10^5
Red Sea	4.38×10^5
Okhotsk Sea	1.59×10^6

(a) Write 1.59×10^6 as an ordinary number.

MILLION!

$$1\ 590\ 000 \quad \text{(B1)}$$

(1)

(b) Work out the difference, in square kilometres, between the largest surface area and the smallest surface area for these five seas.
Give your answer in standard form.

$$2.97 \times 10^6 - 4.22 \times 10^5$$

$$= 2\ 548\ 000$$

(M1) [EITHER]

$$2.548 \times 10^6 \text{ km}^2 \quad \text{(A1)}$$

(2)

The surface area of the East China Sea is k times the surface area of the Baltic Sea.

(c) Work out the value of k .

Give your answer to the nearest whole number.

$$1.25 \times 10^6 = k \times (4.22 \times 10^5)$$

$$\Rightarrow k = \frac{1.25 \times 10^6}{4.22 \times 10^5} \quad \text{(M1)}$$

$$= 2.962\dots$$

$$k = 3 \quad \text{(A1)}$$

(2)

The table shows the annual world production of four foods.

Food	Annual world production, in tonnes
Cocoa	1.75×10^6
Coffee	1.85×10^6
Sugar	9.72×10^7
Wheat	4.98×10^8

- (a) Calculate the total annual world production of coffee and sugar.

ADD

$$1.85 \times 10^6 + 9.72 \times 10^7 = 9.91 \times 10^7 \text{ tonnes}$$

(2)

- (b) Brazil produces 9.7% of the world's sugar.
Calculate the annual production of sugar from Brazil.

$$0.097 \times 9.72 \times 10^7 = 9.43 \times 10^6 \text{ tonnes}$$

(2)

- (c) Express the world production of wheat as a percentage of the total production of all four foods.

$$\frac{4.98 \times 10^8}{1.75 \times 10^6 + \dots + 4.98 \times 10^8} \times 100 = 83.2\%$$

(3)

Write as ordinary numbers

(i) 3.6×10^5

360 000 (A1)

(ii) 2.9×10^{-3}

0.0029 (A1)

The distance between the Earth and the Sun is 150 000 000 km.

- (a) Write the number 150 000 000 in standard form.

$$\frac{1.5 \times 10^8}{\dots\dots\dots} \quad \text{(AI)}$$

(1)

The distance between Neptune and the Sun is 30 times greater than the distance between the Earth and the Sun.

- (b) Calculate the distance between Neptune and the Sun.
Give your answer in standard form.

$$\begin{aligned} 30 \times 1.5 \times 10^8 &= 45 \times 10^8 \\ &= \underline{4.5 \times 10^9} \quad \text{(AI)} \end{aligned}$$

(AI) \nearrow

(a) Write the number 78 000 000 in standard form.

$$\underline{7.8 \times 10^9} \quad \text{(1)}$$

(b) Write 4×10^{-3} as an ordinary number.

$$\underline{0.004} \quad \text{(1)}$$

(c) Work out the value of $\frac{3 \times 10^{-2}}{8 \times 10^9}$

Give your answer in standard form.

$$\underline{3.75 \times 10^{-12}} \quad \text{(1)}$$

The table shows the population of each of three countries in 2012.

Country	Population
India	1.21×10^9
Turkey	7.48×10^7
Singapore	5.2×10^6

- (a) Find the total population of India, Turkey and Singapore in 2012.
Give your answer in standard form.

$$\frac{1.29 \times 10^9}{(2)}$$

(A2)

[A1] IF ANSWER IS CORRECT

Population density is calculated by the formula

BUT NOT IN STANDARD FORM]

$$\text{Population density} = \text{Population} \div \text{Land area}$$

The land area of India is $3.29 \times 10^6 \text{ km}^2$

- (b) Calculate the population density of India in 2012.
Give your answer correct to 3 significant figures.

$$(m) \quad \frac{1.21 \times 10^9}{3.29 \times 10^6} = 367.78\dots$$

$$\frac{368}{(2)} \text{ people/km}^2$$

(A1)

The table gives the populations of each of five countries in 2014

Country	Population
China	1.4×10^9
India	1.3×10^9
USA	3.2×10^8
Ethiopia	9.7×10^7
Mexico	1.2×10^8

(a) Write 9.7×10^7 as an ordinary number.

$$\underline{97\ 000\ 000} \quad \text{(1)} \quad \text{(B1)}$$

The population of Russia in 2014 was 140 000 000

(b) Write 140 000 000 in standard form.

$$\underline{1.4 \times 10^8} \quad \text{(1)} \quad \text{(B1)}$$

In 2014, there were more people living in China than were living in the USA.

(c) How many more?

Give your answer in standard form.

$$1.4 \times 10^9 - 3.2 \times 10^8 = 1\ 080\ 000\ 000$$

(M1) [SUBTRACTION]

$$= \underline{1.08 \times 10^9} \quad \text{(2)} \quad \text{(A6)}$$

In 2014, the population of India was k times the population of Mexico.

(d) Work out the value of k .

Give your answer to the nearest whole number.

$$\frac{1.3 \times 10^9}{1.2 \times 10^8} = 10.8$$

(M1) [DIVISION]

11 (A1)

(a) Write as an ordinary number

(i) 4.2×10^6

4 200 000 (A1)

(ii) 3.82×10^{-4}

0.000382 (A1)
(2)

(b) Here are three numbers written in standard form.
Arrange these numbers in order of size.
Start with the smallest number.

5.6×10^{-7}

8.6×10^{-9}

5.64×10^{-8}

8.6×10^{-9} , 5.64×10^{-8} , 5.6×10^{-7} (A2)
(2)

1 astronomical unit = 150 million kilometres.

- (a) Write the number 150 million in standard form.

$$\frac{1.5 \times 10^8}{(2)}$$

(BI) (BI)

The distance from Venus to the Sun is 108 million kilometres.

- (b) Express 108 million kilometres in astronomical units.
Give your answer in standard form.

$$\frac{1.08 \times 10^8}{1.5 \times 10^8} = 0.72 \text{ (M)} = 7.2 \times 10^{-1} \text{ (A)}$$

[ACCEPT 7.2×10^{-1}]

(a) a , b and c are positive numbers such that $1 \leq ab < 10$ and $1 \leq c < 10$

$$(a \times 10^4) \times (b \times 10^7) = c \times 10^m$$

(i) Write down the value of m .

$$10^4 \times 10^7 = 10^{11}$$

$$m = \dots\dots\dots 11 \quad \text{(A0)}$$

(ii) Find an expression for c in terms of a and b .

$$a \times b = c$$

$$c = \dots\dots\dots ab \quad \text{(A1)} \\ \text{(2)}$$

(b) $N = (3.2 \times 10^p) \times (4.5 \times 10^q)$, where p and q are integers.

Express N in terms of p and q .

Give your answer in standard form.

$$N = 3.2 \times 4.5 \times 10^p \times 10^q$$

$$= 14.4 \times 10^{p+q} \quad \text{(M1)}$$

$$= \underline{\underline{1.44 \times 10^{p+q+1}}} \quad \text{(A1)}$$

(a) Write 0.000076 in standard form.

0.000076
 5 PLACES

$$\frac{7.6 \times 10^{-5}}{\dots\dots\dots}$$

(1)

The area covered by the Pacific Ocean is $1.6 \times 10^8 \text{ km}^2$

The area covered by the Arctic Ocean is $1.4 \times 10^7 \text{ km}^2$

(b) Write 1.6×10^8 as an ordinary number.

$$\frac{160\,000\,000}{\dots\dots\dots}$$

(1)

The area covered by the Pacific Ocean is k times the area covered by the Arctic Ocean.

(c) Find, correct to the nearest integer, the value of k .

$$\frac{\text{PACIFIC}}{\text{ARCTIC}} = \frac{1.6 \times 10^8}{1.4 \times 10^7} \quad \text{(mi)}$$

$$= 11.428\dots$$

$$k = \frac{11}{\dots\dots\dots}$$

(2)

- (a) Work out $(9 \times 10^8) \times (4 \times 10^6)$
Give your answer in standard form.

$$36 \times 10^{14}$$

$$\underline{3.6 \times 10^{15}} \quad \text{(A1)}$$

(1)

- (b) $x = 7 \times 10^m$ and $y = 5 \times 10^n$, where m and n are integers.

- (i) It is given that $xy = 3.5 \times 10^{12}$
Show that $m + n = 11$

$$7 \times 10^m \times 5 \times 10^n = 35 \times 10^{m+n}$$

$$= 3.5 \times 10^{m+n+1}$$

COMPARE INDICES

$$m+n+1 = 12$$

$$\Rightarrow \underline{m+n = 11}$$

BUT $xy = 3.5 \times 10^{12}$

$$\Rightarrow 3.5 \times 10^{m+n+1} = 3.5 \times 10^{12}$$

(B1)

- (ii) It is also given that $\frac{x}{y} = 1.4 \times 10^{27}$

Find the value of m and the value of n .

$$\frac{x}{y} = \frac{7 \times 10^m}{5 \times 10^n} = 1.4 \times 10^{m-n}$$

$$m-n = 27$$

BUT $\frac{x}{y} = 1.4 \times 10^{27}$

(B1)

$$\begin{array}{l} m+n = 11 \quad \text{--- (1)} \\ m-n = 27 \quad \text{--- (2)} \end{array} \quad \left. \vphantom{\begin{array}{l} m+n = 11 \\ m-n = 27 \end{array}} \right\} \text{ADD (M1)}$$

$$2m = 38$$

$$m = \underline{\underline{19}}$$

SUBSTITUTE $m = 19$ INTO EQ. (1)

$$19 + n = 11$$

$$\Rightarrow \underline{\underline{n = -8}} \quad \text{(A1)}$$

(a) $x = 9 \times 10^{2m}$ where m is an integer.

Find, in standard form, an expression for \sqrt{x}

$$\begin{aligned}\sqrt{x} &= \sqrt{9 \times 10^{2m}} \\ &= \sqrt{9} \times \sqrt{10^{2m}} \\ &= 3 \times (10^{2m})^{\frac{1}{2}} \\ &= \underline{3 \times 10^m} \quad \text{(AI)} \quad \text{(AI)}\end{aligned}$$

(b) $y = 9 \times 10^{2n}$ where n is an integer.

Find, in standard form, an expression for $y^{\frac{3}{2}}$

Give your answer as simply as possible.

$$\begin{aligned}y &= 9 \times 10^{2n} \\ y^{\frac{1}{2}} &= (9 \times 10^{2n})^{\frac{1}{2}} \\ &= 9^{\frac{1}{2}} \times 10^{2n \times \frac{1}{2}} \\ &= \underline{3 \times 10^n} \\ \therefore y^{\frac{3}{2}} &= (3 \times 10^n)^3 \quad \text{(MI)} \\ &= 3^3 \times 10^{n \times 3} \\ &= 27 \times 10^{3n} \\ &= \underline{2.7 \times 10^{3n+1}} \quad \text{(AI)}\end{aligned}$$

$x = a \times 10^n$ where n is an integer and $\sqrt{10} \leq a < 10$

Find, in standard form, an expression for x^2 .
Give your expression as simply as possible.

$$\begin{aligned} x^2 &= (a \times 10^n)^2 \\ &= a^2 \times 10^{2n} \quad \text{(ml)} \end{aligned}$$

BUT IF $\sqrt{10} \leq a < 10$

THEN $10 \leq a^2 < 100$

STANDARD FORM REQUIRES NUMBER-PART TO BE BETWEEN 1 AND 9.99... SO WE NEED TO DIVIDE NUMBER-PART BY 10 [AND THEREFORE MULTIPLY POWER OF TEN PART].

I.E.

$$x^2 = a^2 \times 10^{2n} \quad \text{[NOT STANDARD FORM]}$$

$$\text{SO } x^2 = 0.1a^2 \times 10^{2n+1}$$

- (a) Evaluate $5 \times 10^{12} + 9 \times 10^{12}$
Give your answer in standard form.

$$\begin{array}{r} 5 \times 10^{12} \\ + 9 \times 10^{12} \\ \hline 14 \times 10^{12} \end{array} \quad \text{(M1)} \quad \rightarrow \quad \frac{1.4 \times 10^{13}}{(2)} \quad \text{(A1)}$$

- (b) Each of the numbers p , q and r is greater than 1 and less than 10

$$\begin{aligned} p \times 10^{15} + q \times 10^{15} &= r \times 10^n \\ p + q &> 10 \end{aligned}$$

- (i) Find the value of n .

$$n = \frac{16}{(2)} \quad \text{(A1)}$$

- (ii) Find an expression for r in terms of p and q .

$$\begin{array}{r} p \times 10^{15} \\ + q \times 10^{15} \\ \hline (p+q) \times 10^{15} \end{array} \quad \text{(M1)} \quad \rightarrow \quad 0.1(p+q) \times 10^{16}$$

BUT $p+q > 10$

$$r = \frac{0.1(p+q)}{(3)} \quad \text{(A1)}$$

- (a) Work out $5.2 \times 10^2 + 2.3 \times 10^4$
Give your answer in standard form.

$$0.052 \times 10^4 + 2.3 \times 10^4 \quad \text{(M1)} \rightarrow \underline{2.352 \times 10^4} \quad \text{(A1) (2)}$$

- (b) $a \times 10^2 + b \times 10^4 = c \times 10^4$

Express c in terms of a and b .

$$\begin{aligned} a \times 10^2 + b \times 10^4 &= \frac{a}{100} \times 10^4 + b \times 10^4 \\ &= \left(\frac{a}{100} + b \right) \times 10^4 \end{aligned}$$

$$c = \underline{\frac{a}{100} + b} \quad \text{(2)}$$

- (a) Each of the numbers x , y and z is greater than 1 and less than 10

$$x \times 10^5 + y \times 10^4 = z \times 10^5$$

Find an expression for z in terms of x and y .
Give your answer as simply as possible.

$$x \times 10^5 + 0.1y \times 10^5$$

SAME POWERS

$$z = \frac{x + 0.1y}{1}$$

(2)

- (b) Each of the numbers 3×10^n , 4×10^m and $a \times 10^p$ is in standard form.

$$\frac{3 \times 10^n}{4 \times 10^m} = a \times 10^p \longrightarrow \frac{3}{4} \times 10^{n-m} = 0.75 \times 10^{n-m}$$

- (i) Find the value of a .

MUST BE
IN STANDARD
FORM

$$= 7.5 \times 10^{n-m-1}$$

$$a = 7.5$$

- (ii) Find an expression for p in terms of n and m .

$$\underline{\underline{p = n - m - 1}}$$

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Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

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B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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