SOLUTIONS

1. [Edexcel, 2004] Surds **[2 Marks]**

Express $\sqrt{98}$ in the form $a\sqrt{b}$ where a and b are integers and $a > 1$.

$$\sqrt{98} = \sqrt{49 \times 2} = \sqrt{49} \times \sqrt{2}$$

$$= \underline{\underline{7\sqrt{2}}} \quad \text{A1}$$

B1

[RELEVANT WORKING MUST BE SEEN]

2. [Edexcel, 2011] Surds **[3 Marks]**

Express $\sqrt{48}+\sqrt{108}$ in the form $k\sqrt{6}$ where k is a surd.

$\sqrt{4\times 12} = 2\sqrt{12}$ $\quad$ $\sqrt{9\times 12} = 3\sqrt{12}$

$\therefore \sqrt{48}+\sqrt{108} = 2\sqrt{12} + 3\sqrt{12}$ (M1)

$= 5\sqrt{12}$

$= 5\sqrt{2\times 6}$ (M1)

$= 5\sqrt{2}\times\sqrt{6}$.

$(k = 5\sqrt{2})$

(B1)

$5\sqrt{2}\times\sqrt{6}$

3. [New Question, by Maths4Everyone.com] Surds **[2 Marks]**

Show that $\sqrt{27} + \sqrt{147}$ can be expressed in the form $a\sqrt{b}$, where a and b are integers.

$$\sqrt{27} + \sqrt{147} = \sqrt{9 \times 3} + \sqrt{49 \times 3}$$

$$= 3\sqrt{3} + 7\sqrt{3}$$

M1 EITHER

$$= \underline{\underline{10\sqrt{3}}} \quad (a = 10,\ b = 3)$$

A1

Simplify $(7 + 2\sqrt{50})(5 - 2\sqrt{2})$

18!

Give your answer in the form $a + b\sqrt{18}$ where a and b are integers.
Show your working clearly.

M1

$$(7+2\sqrt{50})(5-2\sqrt{2}) = 35 - 14\sqrt{2} + 10\sqrt{50} - 4\sqrt{100}$$

$$= 35 - 14\sqrt{2} + 10\sqrt{50} - 4\times 10$$

$$= -5 - 14\sqrt{2} + 10\sqrt{50}$$

A1

$$= -5 - 14\sqrt{2} + 10\sqrt{25\times 2}$$

$$= -5 - 14\sqrt{2} + 50\sqrt{2}$$

$$= -5 + 36\sqrt{2}$$

$$= -5 + 12\times 3\sqrt{2}$$

$$= -5 + 12\times\sqrt{9\times 2}$$

$$= -5 + 12\sqrt{18}$$

A1

5. [Edexcel, 2012] Surds **[3 Marks]**

Show that $(6-\sqrt{8})^2 = 44 - 24\sqrt{2}$

Show each stage of your working clearly.

$(6-\sqrt{8})(6-\sqrt{8})$

$36 - 6\sqrt{8} - 6\sqrt{8} + 8 = 44 - 12\sqrt{8}$ B1

M1

$= 44 - 12\sqrt{4\times 2}$ B1 (EITHER)

$= 44 - 12\times 2\sqrt{2}$

$= \underline{\underline{44 - 24\sqrt{2}}}$

6. [New Question, by Maths4Everyone.com] Surds **[4 Marks]**

(a) Show that $\sqrt{48}+\sqrt{108}$ can be expressed in the form $a\sqrt{b}$, where a and b are integers.

$$\left.\begin{aligned}\sqrt{48}+\sqrt{108} &= \sqrt{16}\sqrt{3}+\sqrt{36}\sqrt{3}\\ &= 4\sqrt{3}+6\sqrt{3}\end{aligned}\right\} \text{M1 [EITHER]}$$

$$= \underline{\underline{10\sqrt{3}}} \quad \text{A1}$$

(b) Show that $(5-\sqrt{12})(6-\sqrt{3}) = 36-17\sqrt{3}$
Show each stage of your working.

F O L L

$$\underbrace{(5-\sqrt{12})(6-\sqrt{3})}_{\text{LHS}} = 30-5\sqrt{3}-6\sqrt{12}+\sqrt{3}\sqrt{12}$$

$$= 30-5\sqrt{3}-6\times 2\sqrt{3}+\sqrt{36}$$

B1

$$= 30-5\sqrt{3}-12\sqrt{3}+6$$

B1

$$= \underbrace{36-17\sqrt{3}}_{\text{RHS}} \quad \underline{\underline{\text{QED}}}$$

7. [Edexcel, 2012] Surds **[3 Marks]**

Show that $\frac{\sqrt{3}+\sqrt{27}}{\sqrt{2}}$ can be expressed in the form $\sqrt{k}$ where k is an integer.

State the value of k.

$$\frac{\sqrt{3}+\sqrt{27}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{3}\sqrt{2}+\sqrt{27}\sqrt{2}}{\sqrt{2}\sqrt{2}}$$

$$= \frac{\sqrt{6}+\sqrt{54}}{2}$$

$$= \frac{\sqrt{6}+3\sqrt{6}}{2}$$

$$= \frac{4\sqrt{6}}{2} = 2\sqrt{6} = \sqrt{4\times 6} = \sqrt{24}$$

$k =$ 24

8. [Edexcel, 2015] Surds **[4 Marks]**

(a) Show that $(3 + 2\sqrt{2})(4 - \sqrt{2}) = 8 + 5\sqrt{2}$

Show your working clearly.

F O M1 I L

$$(3+2\sqrt{2})(4-\sqrt{2}) = 12 - 3\sqrt{2} + 8\sqrt{2} - 2\times 2 \quad \text{M1}$$

$$= 12 - 4 - 3\sqrt{2} + 8\sqrt{2} \quad [\sqrt{2}\times\sqrt{2} = 2]$$

$$= 8 + 5\sqrt{2}$$

(2)

(b) Rationalise the denominator and simplify fully $\frac{10 + 3\sqrt{2}}{\sqrt{2}}$

Show your working clearly.

$$\frac{10+3\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2} + 3\times 2}{2}$$

M1

$$= \frac{10\sqrt{2}}{2} + \frac{6}{2}$$

$$= 5\sqrt{2} + 3 \quad \text{A1}$$

[I.E $3 + 5\sqrt{2}$]

9. [Edexcel, 2008] Surds **[2 Marks]**

Show that $\frac{12}{\sqrt{8}} = 3\sqrt{2}$

$$\frac{12}{\sqrt{8}} = \frac{12}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}}$$

$$= \frac{12\sqrt{8}}{8} \quad \text{B1}$$

$$= \frac{12\sqrt{4 \times 2}}{8}$$

$$= \frac{12 \times 2\sqrt{2}}{8} \quad \text{B1}$$

$$= \frac{24\sqrt{2}}{8}$$

$$= \underline{\underline{3\sqrt{2}}}$$

(a) Expand $(5 + 3\sqrt{2})^2$

Give your answer in the form $(a + b\sqrt{2})$, where a and b are integers.
Show your working clearly.

$(5+3\sqrt{2})(5+3\sqrt{2}) = 25 + 15\sqrt{2} + 15\sqrt{2} + 9 \times 2$

$= 43 + 30\sqrt{2}$

A1 A1

(b) $(5 + 3\sqrt{2})^2 = p + \frac{q}{\sqrt{8}}$, where p and q are integers.

Find the value of q.

COMPARING SURD PARTS:-

$\frac{q}{\sqrt{8}} = 30\sqrt{2}$ M1 $\Rightarrow q = 30\sqrt{2} \times \sqrt{8}$ M1

$= 30 \times \sqrt{16}$

$= 120$ A1

11. [Edexcel, 2014] Surds **[3 Marks]**

Given that $(5-\sqrt{x})^2 = \boxed{y} - \boxed{20\sqrt{2}}$ where x and y are positive integers, find the value of x and the value of y.

$$(5-\sqrt{x})(5-\sqrt{x}) = 25 - 5\sqrt{x} - 5\sqrt{x} + x$$

$$= 25 - 10\sqrt{x} + x$$

$$= \underline{(25+x)} - \underline{10\sqrt{x}} \quad \text{M1}$$

$$y = 25 + x$$

$$10\sqrt{x} = 20\sqrt{2}$$

$$= 10 \times 2\sqrt{2}$$

$$= 10 \times \sqrt{8}$$

$$\therefore x = \underline{\underline{8}} \quad \text{A1}$$

$$y = 25 + 8$$

$$= \underline{\underline{23}} \quad \text{A1}$$

12. [Edexcel, 2013] Surds **[3 Marks]**

$(3+\sqrt{a})(4+\sqrt{a}) = 17 + k\sqrt{a}$ where a and k are positive integers.

Find the value of a and the value of k.

$$(3+\sqrt{a})(4+\sqrt{a}) = 12 + 3\sqrt{a} + 4\sqrt{a} + a \quad \text{M1}$$

$$= \underbrace{12 + a}_{17} + \underbrace{7\sqrt{a}}_{k\sqrt{a}}$$

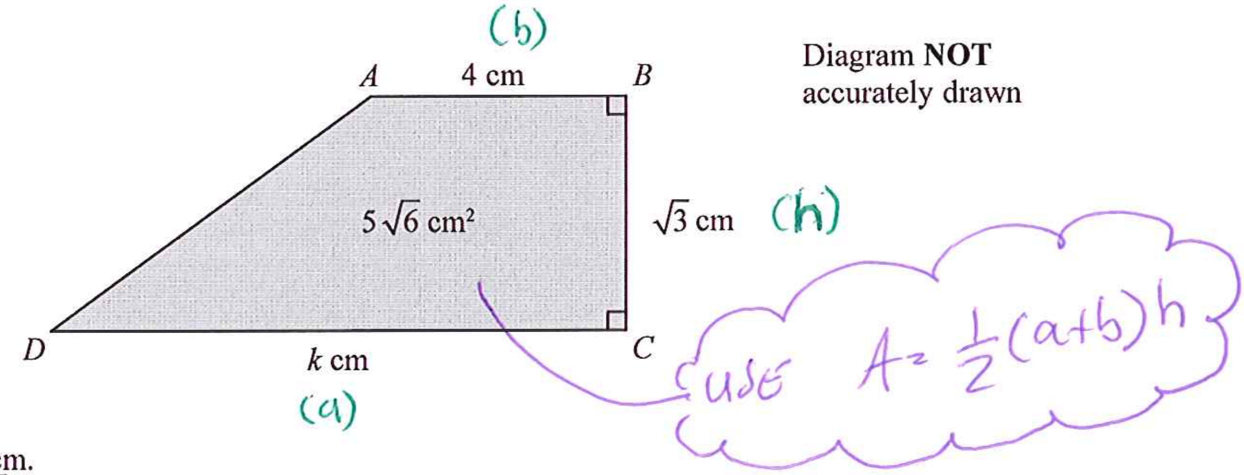
$$12 + a = 17$$

$$a = \underline{\underline{5}}$$

$a =$ 5 A1

$k =$ 7 B1

A trapezium $ABCD$ has an area of $5\sqrt{6}$ cm².



$AB = 4$ cm.
$BC = \sqrt{3}$ cm.
$DC = k$ cm.

Calculate the value of k, giving your answer in the form $a\sqrt{b} - c$ where a, b and c are positive integers.
Show each step in your working.

$$\frac{1}{2}(k+4) \times \sqrt{3} = 5\sqrt{6}$$ M1 [EQUATION]

$$\Rightarrow (k+4) \times \sqrt{3} = 10\sqrt{6}$$

$$\Rightarrow k+4 = \frac{10\sqrt{6}}{\sqrt{3}}$$

$$\Rightarrow k+4 = 10\sqrt{2}$$

$$\Rightarrow k = 10\sqrt{2} - 4$$

A1 [INTEGER PART]

A1 [SURD PART]

(a) Show that $(5-\sqrt{8})(7+\sqrt{2}) = 31 - 9\sqrt{2}$

Show each stage of your working.

F O I L

$(5-\sqrt{8})(7+\sqrt{2}) = 35 + 5\sqrt{2} - 7\sqrt{8} - \sqrt{8}\sqrt{2}$ (M1)

$= 35 + 5\sqrt{2} - 7\times 2\sqrt{2} - \sqrt{16}$

(M1)

$= 35 - 9\sqrt{2} - 4$

(M1)

$= \underline{\underline{31 - 9\sqrt{2}}}$

(3)

Given that c is a prime number,

(b) rationalise the denominator of $\dfrac{3c-\sqrt{c}}{\sqrt{c}}$

Simplify your answer.

$$\frac{3c-\sqrt{c}}{\sqrt{c}} \times \frac{\sqrt{c}}{\sqrt{c}} = \frac{3c\sqrt{c} - \sqrt{c}\sqrt{c}}{\sqrt{c}\sqrt{c}}$$

(M1)

$$= \frac{3c\sqrt{c} - c}{c}$$

$$= \frac{c(3\sqrt{c}-1)}{c}$$

$$= \underline{\underline{3\sqrt{c} - 1}}$$ (A1)

$$\left(\sqrt{a}+\sqrt{8a}\right)^2 = 54 + b\sqrt{2}$$

a and b are positive integers.
Find the value of a and the value of b.
Show your working clearly.

$(\sqrt{a}+\sqrt{8a})(\sqrt{a}+\sqrt{8a})$

$= a + \sqrt{a}\sqrt{8a} + \sqrt{8a}\sqrt{a} + 8a$

$= a + 2\sqrt{8a \times a} + 8a$

M1 ANY CORRECT EXPANSION

$= a + 2a\sqrt{8} + 8a$

$= 9a + 2a\sqrt{8}$

$= 9a + 2a \times 2\sqrt{2}$

$= 9a + 4a\sqrt{2}$ A1

COMPARING

$9a + 4a\sqrt{2}$

$54 + b\sqrt{2}$

$9a = 54$

$a = \underline{\underline{6}}$

$b = 4a$

$= 4 \times 6$

$= \underline{\underline{24}}$

A1 FOR BOTH

16. [Edexcel, 2016] Surds **[3 Marks]**

$(a + \sqrt{b})^2 = 49 + 12\sqrt{b}$ where a and b are integers, and b is prime.

Find the value of a and the value of b

$$(a+\sqrt{b})(a+\sqrt{b}) = a^2 + a\sqrt{b} + a\sqrt{b} + \sqrt{b}\sqrt{b}$$
$$= a^2 + 2a\sqrt{b} + b \quad \text{M1}$$
$$= (a^2+b) + 2a\sqrt{b}$$

$$2a\sqrt{b} = 12\sqrt{b}$$
$$\Rightarrow 2a = 12$$
$$\Rightarrow a = \underline{\underline{6}} \quad \text{A1}$$

$$a^2 + b = 49$$
$$b = 49 - a^2$$
$$= 49 - 36$$
$$= \underline{\underline{13}} \quad \text{A1}$$

Simplify fully $\dfrac{(6-\sqrt{5})(6+\sqrt{5})}{\sqrt{31}}$

You must show your working.

$$\frac{(6-\sqrt{5})(6+\sqrt{5})}{\sqrt{31}} = \frac{6^2-(\sqrt{5})^2}{\sqrt{31}}$$

M1 [FOR MULTIPLYING THE BRACKETS, OR SPOTTING THE 'DIFFERENCE OF TWO SQUARES']

$$= \frac{36-5}{\sqrt{31}}$$

$$= \frac{31}{\sqrt{31}} \quad \text{M1}$$

$$= \underline{\underline{\sqrt{31}}} \quad \text{A1}$$

Express $\frac{\sqrt{18}+10}{\sqrt{2}}$ in the form $p+q\sqrt{2}$, where p and q are integers.

Show clear working out.

$$\frac{\sqrt{18}+10}{\sqrt{2}} = \frac{\sqrt{18}+10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$ M1

$$= \frac{\sqrt{36}+10\sqrt{2}}{2}$$ M1

$$= \frac{6+10\sqrt{2}}{2}$$

$3+5\sqrt{2}$ A1

(3)

19. [New Question, by Maths4Everyone.com] Surds **[4 Marks]**

Rationalise the denominator and simplify fully $\frac{33}{4+\sqrt{5}}$

Show clear working out.

$$\frac{33}{4+\sqrt{5}} = \frac{33}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4-\sqrt{5}}$$ M1

$$= \frac{132-33\sqrt{5}}{16-5}$$ M1 [NUMERATOR] M1 [DENOMINATOR]

$$= \frac{132-33\sqrt{5}}{11}$$

$12-3\sqrt{5}$ A1

(4)

20. [New Question, by Maths4Everyone.com] Surds **[4 Marks]**

Express $\frac{39}{4-\sqrt{3}}$ in the form $a+b\sqrt{3}$, where a and b are integers

Show clear working out.

$$\frac{39}{4-\sqrt{3}} = \frac{39}{4-\sqrt{3}} \times \frac{4+\sqrt{3}}{4+\sqrt{3}}$$ M1

$$= \frac{156+39\sqrt{3}}{16-3}$$ M1 [NUMERATOR] M1 [DENOMINATOR]

$$= \frac{156+39\sqrt{3}}{13}$$

$12-3\sqrt{3}$ A1

(4)

21. [New Question, by Maths4Everyone.com] Surds **[4 Marks]**

Simplify $\frac{7-\sqrt{5}}{2+\sqrt{5}}$, giving your answer in the form $a+b\sqrt{5}$, where a and b are integers.

Show clear working out.

$$\frac{7-\sqrt{5}}{2+\sqrt{5}} = \frac{7-\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$ M1

$$= \frac{(7-\sqrt{5})(2-\sqrt{5})}{4-5}$$ M1 [DENOMINATOR]

$$= \frac{14-7\sqrt{5}-2\sqrt{5}+5}{-1}$$ M1 [NUMERATOR]

$$= \frac{19-9\sqrt{5}}{-1}$$

$-19+9\sqrt{5}$ A1

(4)

22. [New Question, by Maths4Everyone.com] Surds **[4 Marks]**

Show that $\dfrac{3}{\sqrt{27}-\sqrt{18}}$ can be written in the form $\sqrt{m}+\sqrt{n}$, where m and n are integers.

$$\frac{3}{\sqrt{27}-\sqrt{18}} = \frac{3}{\sqrt{27}-\sqrt{18}} \times \frac{\sqrt{27}+\sqrt{18}}{\sqrt{27}+\sqrt{18}}$$ M1

$$= \frac{3\sqrt{27}+3\sqrt{18}}{27-18}$$ M1 [NUMERATOR] M1 DENOMINATOR

$$= \frac{3\times3\sqrt{3}+3\times3\sqrt{2}}{9}$$

$$= \frac{9\sqrt{3}+9\sqrt{2}}{9}$$

$\sqrt{3}+\sqrt{2}$ A1

(4)

23. [New Question, by Maths4Everyone.com] Surds **[4 Marks]**

Show that $\frac{16}{\sqrt{2}} - \sqrt{8} = 6\sqrt{2}$

$$\frac{16}{\sqrt{2}} - \sqrt{8} = \frac{16}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} - \sqrt{8}$$

M1

$$= \frac{16\sqrt{2}}{2} - \sqrt{8}$$

B1

$$= \underbrace{8\sqrt{2}}_{\text{M1}} - \sqrt{8}$$

$$= 8\sqrt{2} - \underbrace{2\sqrt{2}}_{\text{M1}}$$

$$= \underline{\underline{6\sqrt{2}}}$$

(4)

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M1 – This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

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Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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