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WORKBOOK

TRANSFORMATIONS

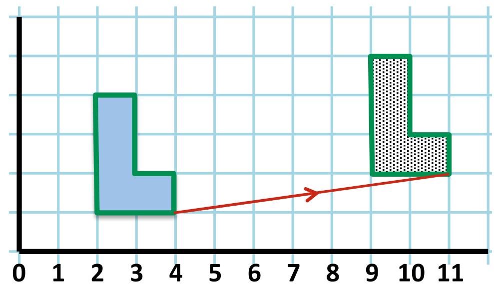
A transformation is a change in the size, location or orientation of an object/shape.

There are **four** types of transformation to consider at GCSE:

TRANSLATION

Described by:

the vector

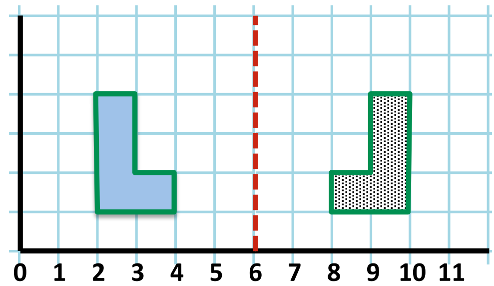


Vector is $\begin{pmatrix}7\\1\end{pmatrix}$

REFLECTION

Described by:

the mirror line

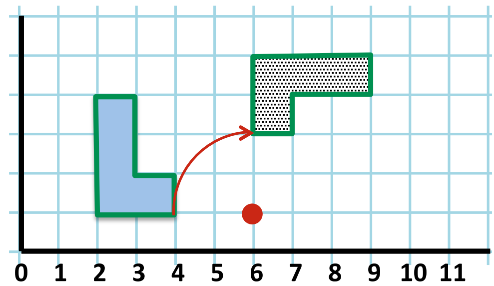


Mirror line is $x = 6$

ROTATION

Described by:

the angle and the centre



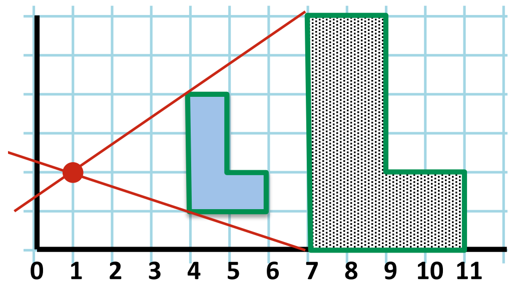
90º clockwise

Centre at (6, 1)

ENLARGEMENT

Described by:

the scale factor and the centre



Scale factor = 2

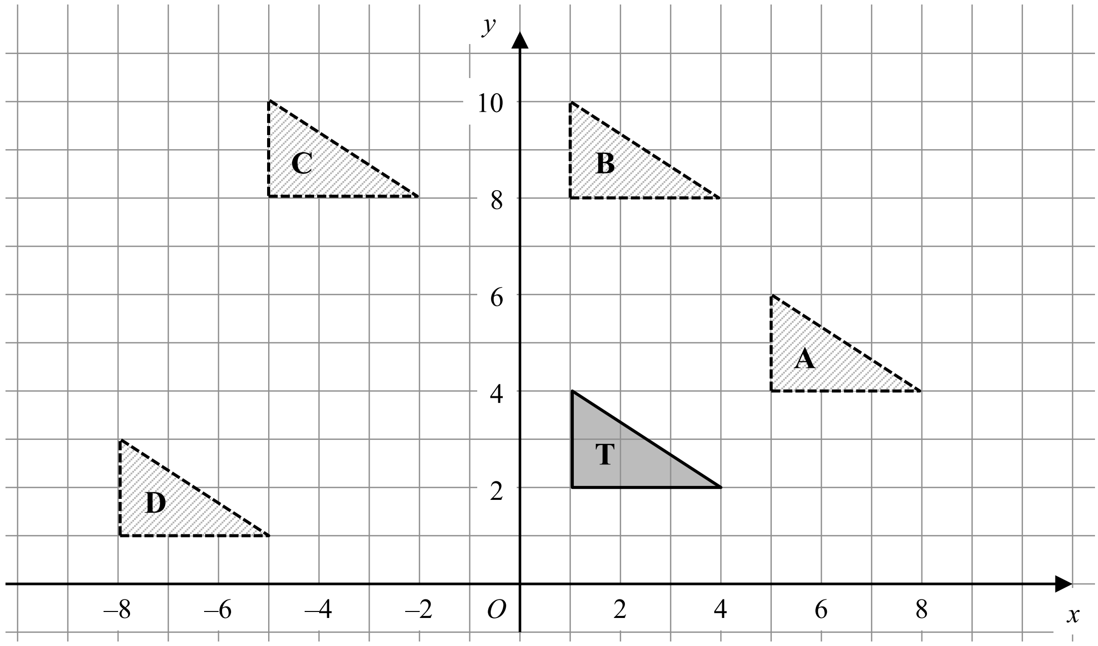
Centre at (1, 2)

****Note that the 'new' shape, which is the result of the transformation, is called the 'image'.**

TRANSLATION

A translation is a 'shift' from one location to another.

A translation is described by a **vector**.

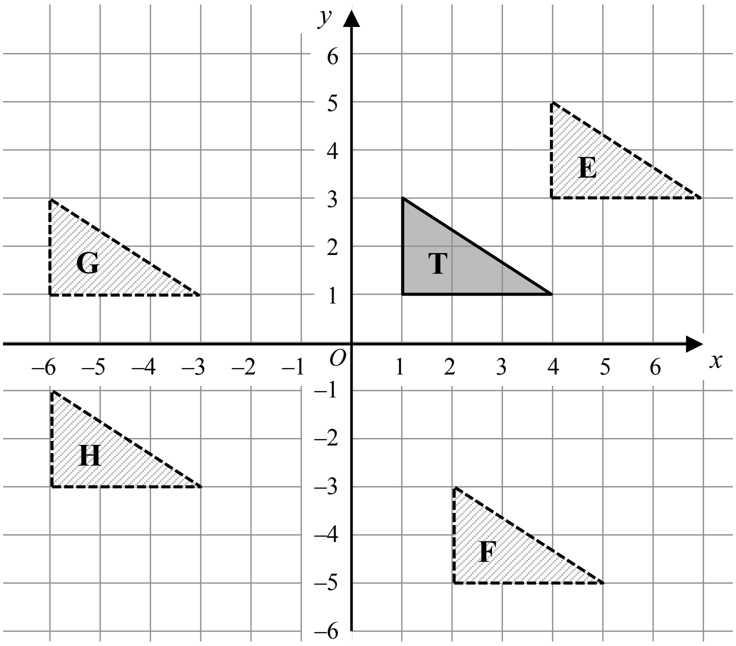


The following table shows some of the translations from the grid above:

Mapping	Description
T to **A**	Translation, by vector $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$
T to **B**	Translation, by vector $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$
T to **C**	Translation, by vector $\begin{pmatrix} -6 \\ 6 \end{pmatrix}$
T to **D**	Translation, by vector $\begin{pmatrix} -9 \\ -1 \end{pmatrix}$
D to **T**	Translation, by vector $\begin{pmatrix} 9 \\ 1 \end{pmatrix}$

SPEED ACTIVITY 1

The diagram shows several translations of triangle **T**:



1. Complete the table to describe the translations:

Mapping	Vector
T onto **E**	
T onto **F**	
T onto **G**	
T onto **H**	

2. On the grid:

(a) Translate triangle **T** by the vector $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$
Label the new triangle **P**

(b) Translate triangle **T** by the vector $\begin{pmatrix} -6 \\ 3 \end{pmatrix}$
Label the new triangle **Q**

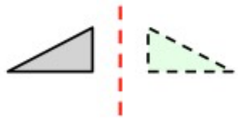
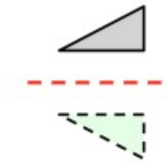
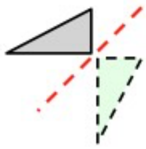
(c) Translate triangle **T** by the vector $\begin{pmatrix} -5 \\ -4 \end{pmatrix}$
Label the new triangle **R**

REFLECTION

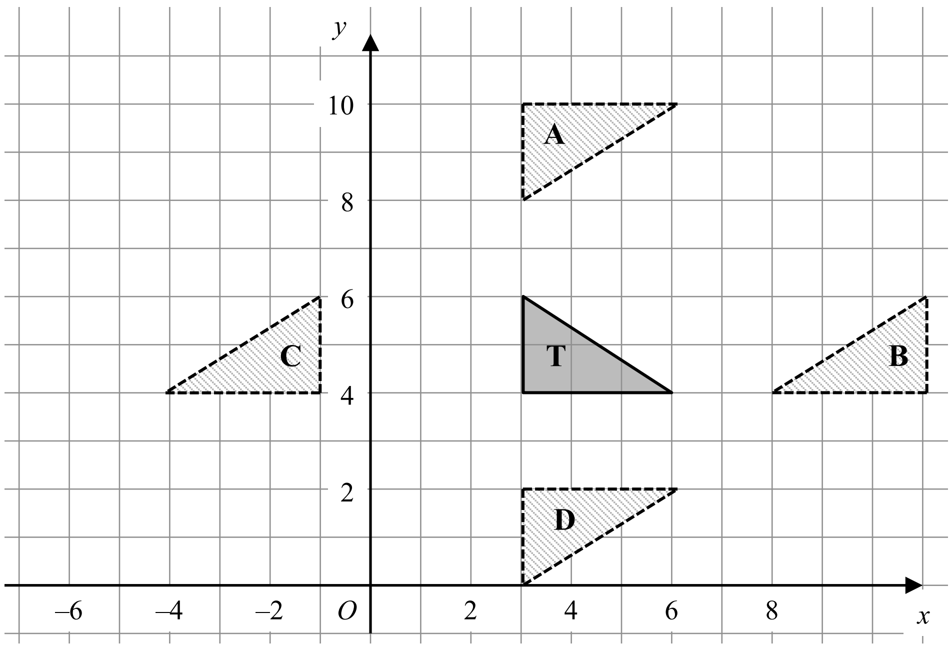
Reflection is when a shape is ‘flipped’ to the other side of a mirror line.

Reflections are described by stating the **equation of the mirror line**.

Note that in GCSE the mirror line may vertical, horizontal or at a 45º slope:

Type of mirror line	Type of reflection	Format of equation
Vertical line	Shape flips back-to-front 	$x = \ldots$
Horizontal line	Shape flips upside-down 	$y = \ldots$
Diagonal line – almost certainly a 45º line	Difficult to describe – sometimes looks a bit like a rotation, but it’s not! 	$y = x$

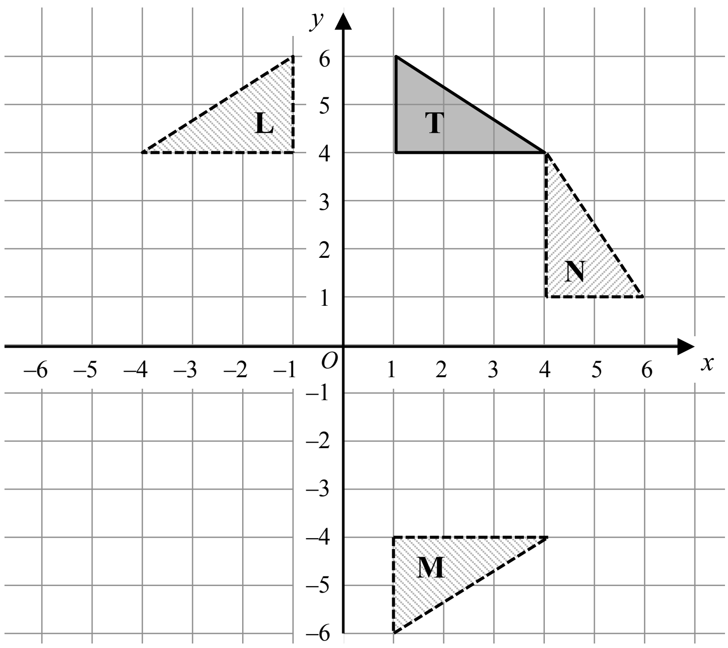
REFLECTIONS IN HORIZONTAL AND VERTICAL LINES



The following table shows reflections from the grid on the previous page:

Mapping	Mirror line
T to **A**	$y = 7$
T to **B**	$x = 7$
T to **C**	$x = 1$
T to **D**	$y = 3$

REFLECTIONS IN THE AXES AND IN THE LINE *Y* = *X*.

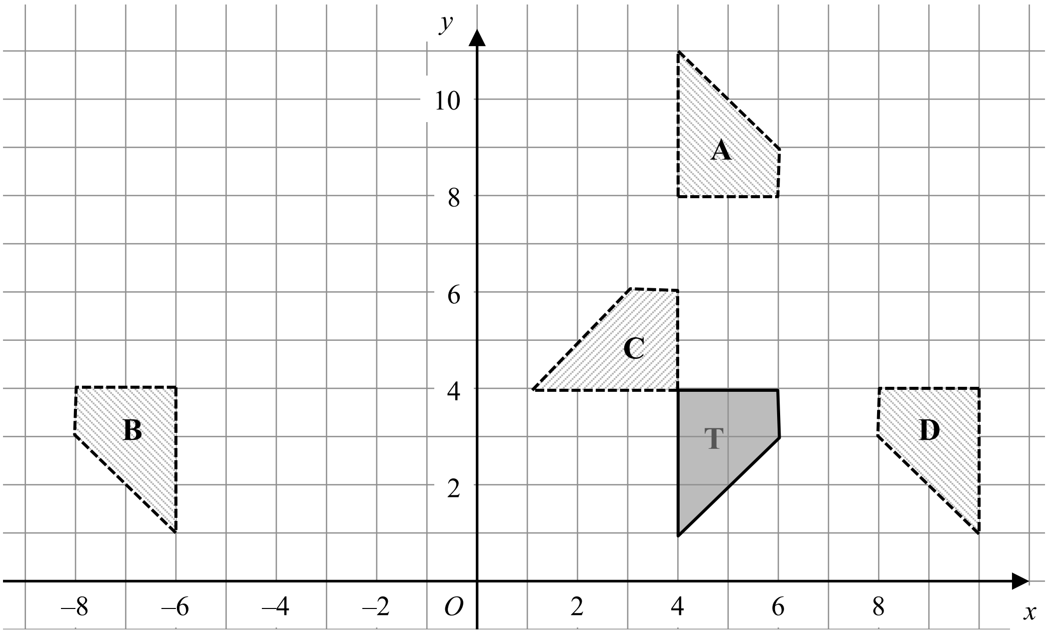


The following table shows reflections from the grid above:

Mapping	Description
T to **L**	Reflection, in line $x = 0$ (aka 'the y-axis')
T to **M**	Reflection, in line $y = 0$ (aka 'the x-axis')
T to **N**	Reflection, in line $y = x$

SPEED ACTIVITY 2

Look at the diagram:



1. Complete the table to describe the reflections:

Mapping	Mirror line
T onto **A**	
T onto **B**	
T onto **C**	
T onto **D**	

2. On the grid:

(a) Reflect trapezium **A** in the line $x = 7$
Label the new trapezium **E**.

(b) Reflect trapezium **B** in the line $x = -4$
Label the new trapezium **F**.

(c) Reflect trapezium **B** in the line $y = 4$
Label the new trapezium **G**.

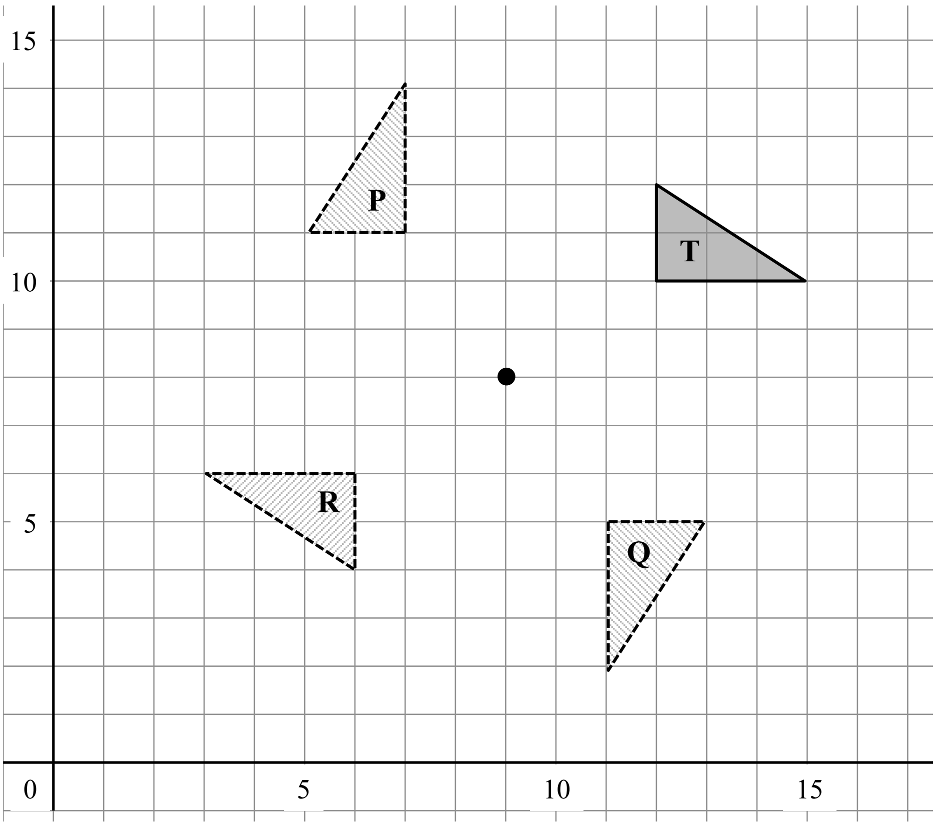
(d) Reflect trapezium **A** in the y-axis
Label the new trapezium **H**.

ROTATION

A rotation is when a shape is turned around a point.

Rotations are described by the **angle turned**,
the **direction of turn** and
the **point/coordinates** at which the rotation is centred)

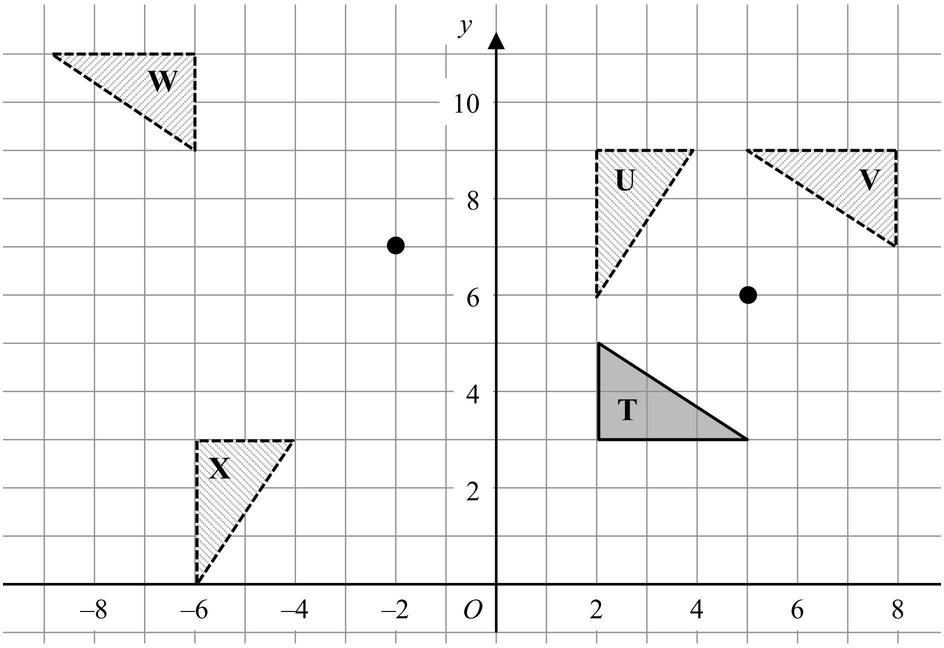
Examples:
The images below have been created by rotating triangle **T** around the point (9, 8)



Mapping	Description
T to **P**	Rotation, 90° anti-clockwise
T to **Q**	Rotation, 90° clockwise
T to **R**	Rotation, 180°

USE TRACING PAPER TO HELP YOU WORK OUT ROTATIONS

More examples:

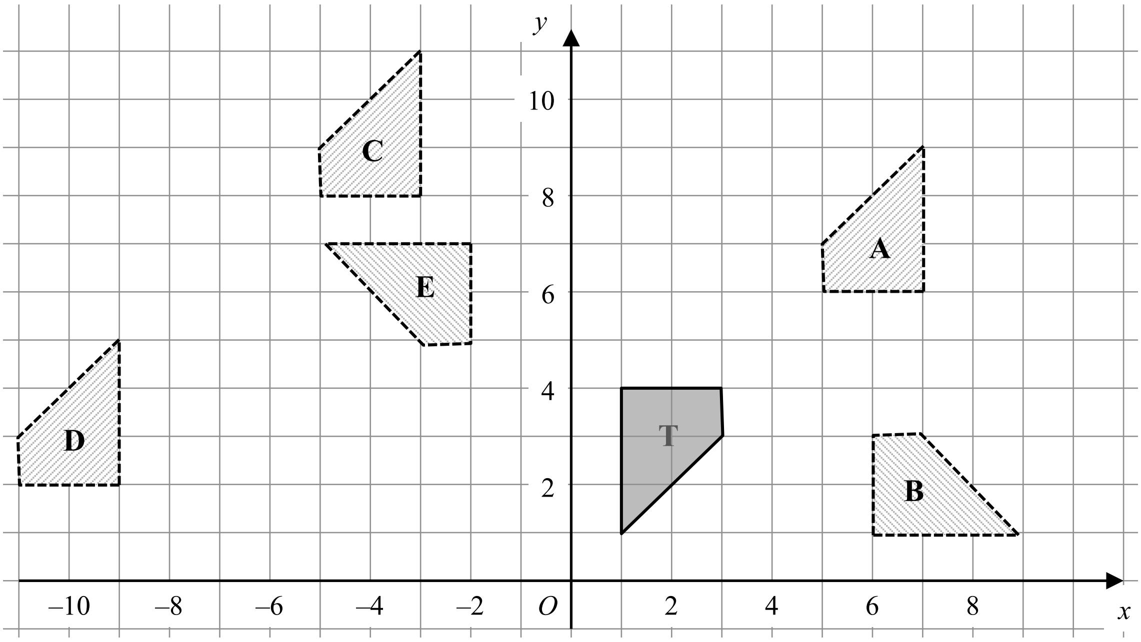


The transformations above can be described in the following ways:

Mapping	Description
T to **U**	Rotation, 90º clockwise, centre (5, 6)
T to **V**	Rotation, 180º, centre (5, 6)
T to **W**	Rotation, 180º, centre (–2, 7)
T to **X**	Rotation, 90º anti-clockwise, centre (–2, 7)

SPEED ACTIVITY 3

Look at the diagram:



1. Complete the table to describe the rotations:

Mapping	Angle	Direction	Centre
T onto **A**			
T onto **B**			
T onto **C**			
T onto **D**			
T onto **E**			

2. On the grid:

(a) Rotate trapezium **C** 90º anti-clockwise about the point (–6, 6).
Label the new trapezium **P**.

(b) Rotate trapezium **B** 90º clockwise about the point (10, 3)
Label the new trapezium **Q**.

(c) Rotate trapezium **C** 180º about the point (–5, 6)
Label the new trapezium **R**.

(d) Rotate trapezium **T** 90º clockwise about the point (5, 6)
Label the new trapezium **S**.

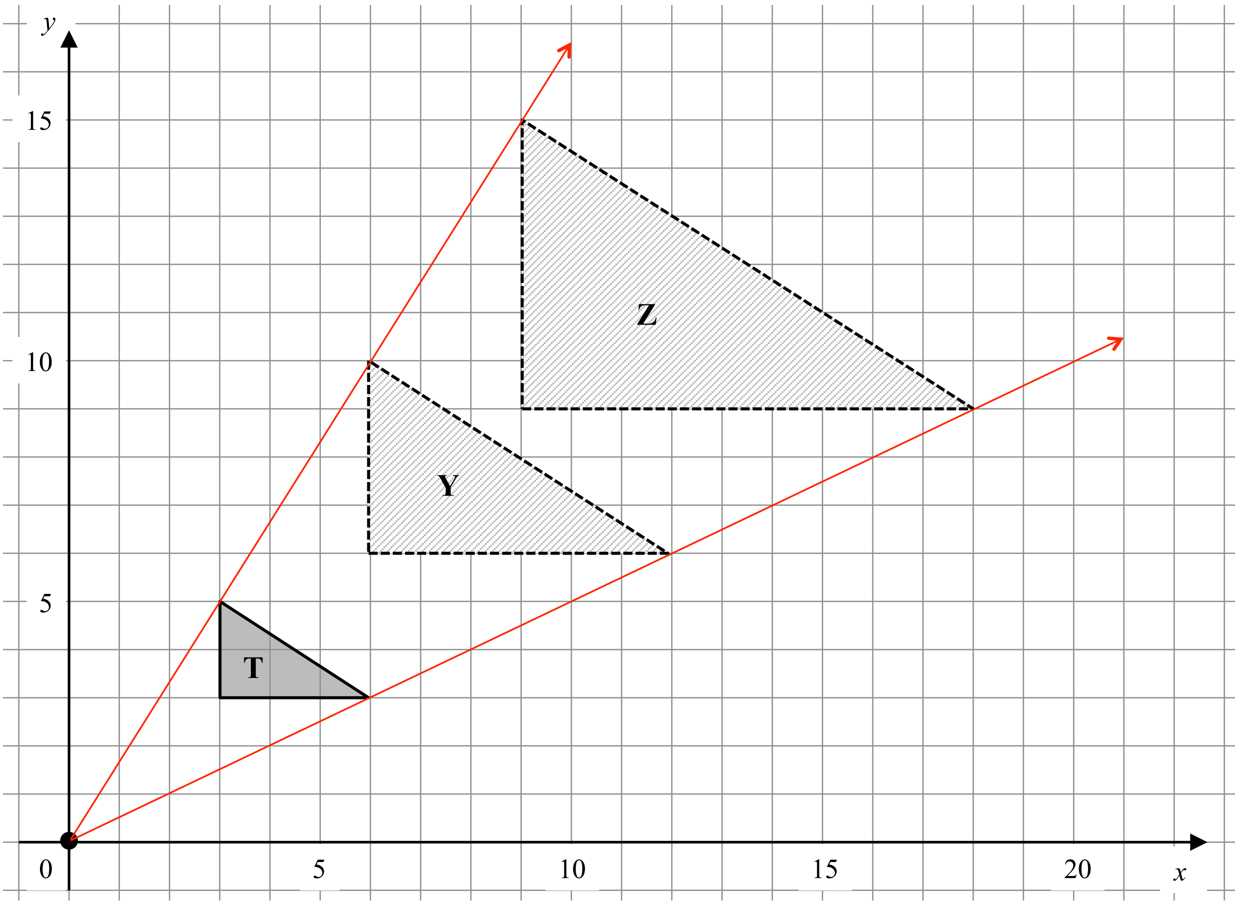
ENLARGEMENT

An enlargement is a change in size of a shape.

Although we usually think of an enlargement as making a shape bigger, an enlargement could be a fractional change that makes the shape smaller!

An enlargement is described by its **scale factor** and the **point** from which the shape enlarges (known as the **centre**)

Example 1:



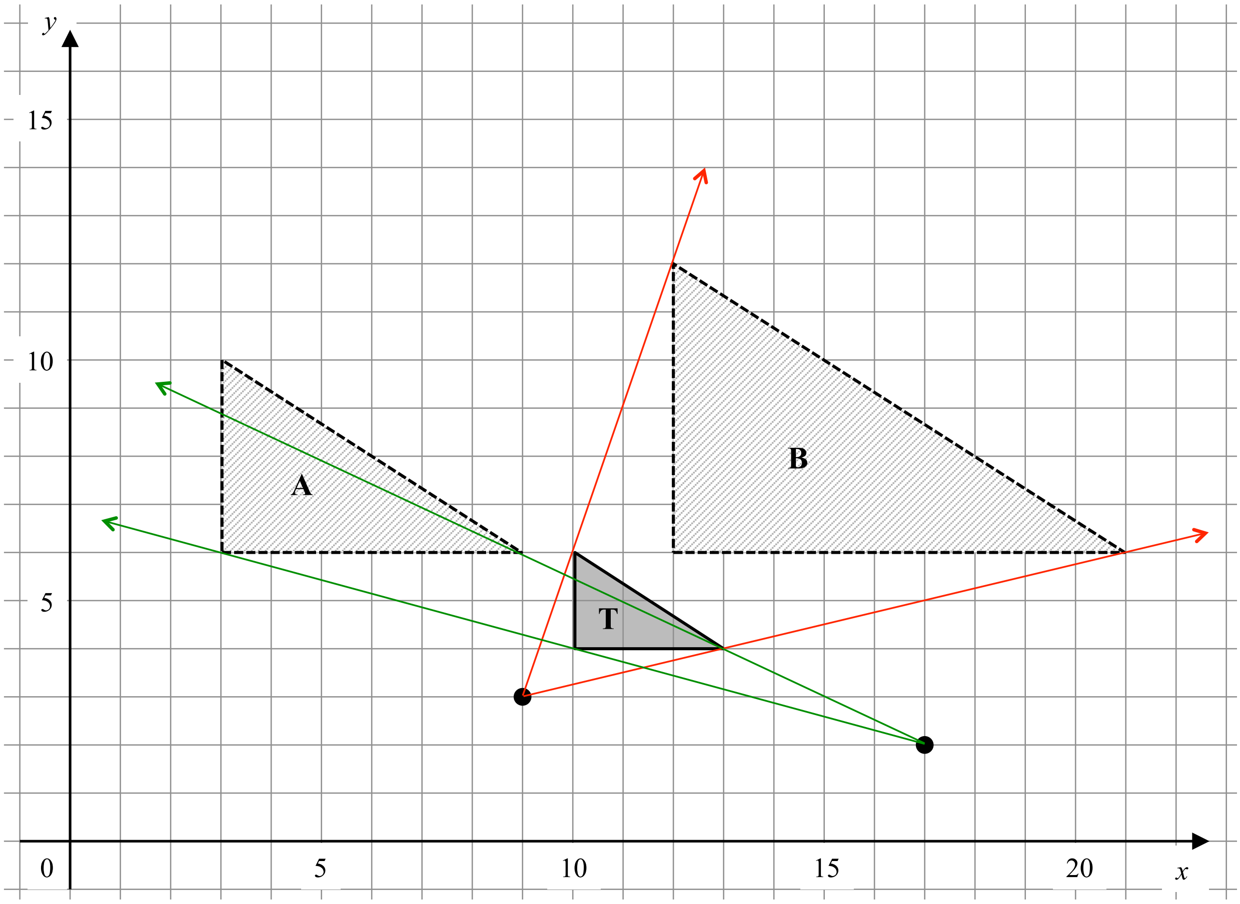
When a shape is enlarged, the original and the enlargement are said to be '**similar**':

- the angles stay the same;
- the sides all increase by the scale factor;
- the distance of each of the vertices from the centre increases by the scale factor.

The transformations in the grid above can be described in the following ways:

Mapping	Description
T to **Y**	Enlargement, scale factor 2, centre (0, 0)
T to **Z**	Enlargement, scale factor 3, centre (0, 0)

Example 2:

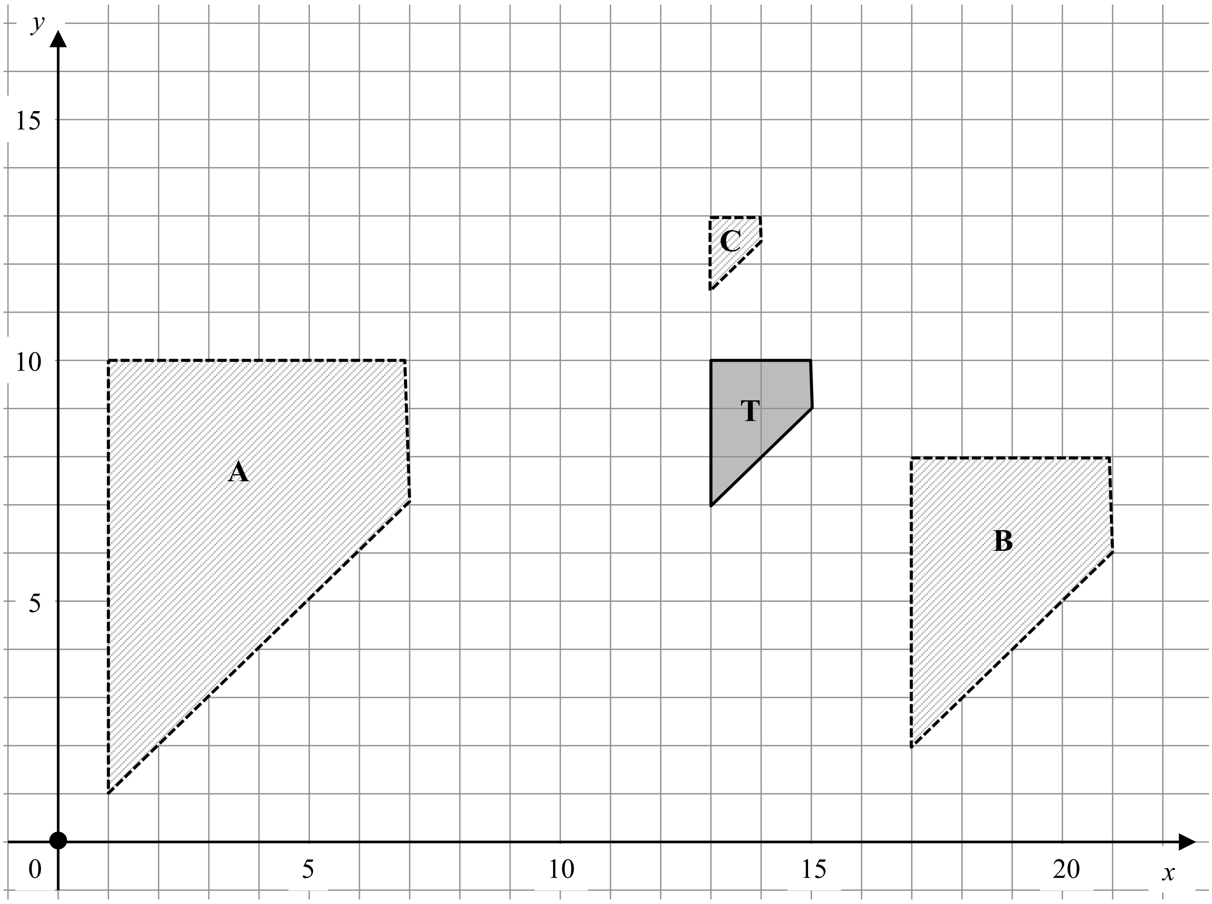


The transformations in the grid above can be described in the following ways:

Mapping	**Description**
T to **A**	Enlargement, scale factor 2, centre (17, 2)
T to **B**	Enlargement, scale factor 3, centre (9, 3)

SPEED ACTIVITY 4

Look at the diagram:



1. Complete the table to describe the enlargements:

Mapping	Scale factor	Centre
T onto **A**		
T onto **B**		
T onto **C**		

2. On the grid:

(a) Enlarge shape **T** with scale factor 2 and centre (16, 13).
Label the new shape **X**.

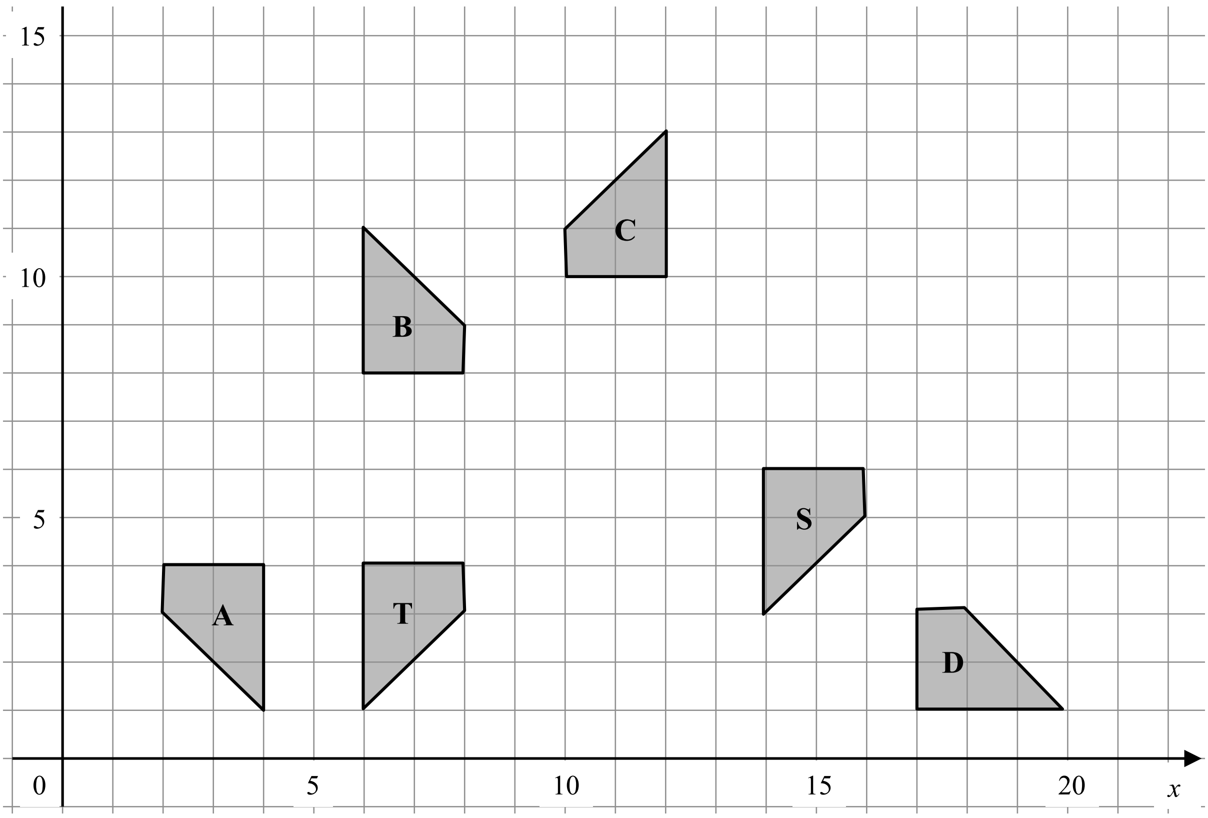
(b) Enlarge shape **T** with scale factor $\frac{1}{2}$ and centre (7, 12).
Label the new shape **Y**.

MIXED TRANSFORMATIONS

DESCRIBING TRANSFORMATIONS

SAMPLE QUESTIONS 1

1. The diagram shows some shapes on a coordinate grid:



Describe fully each of the following transformations (remember to state the type of transformation):

(a) **T** to **A**

(b) **T** to **B**

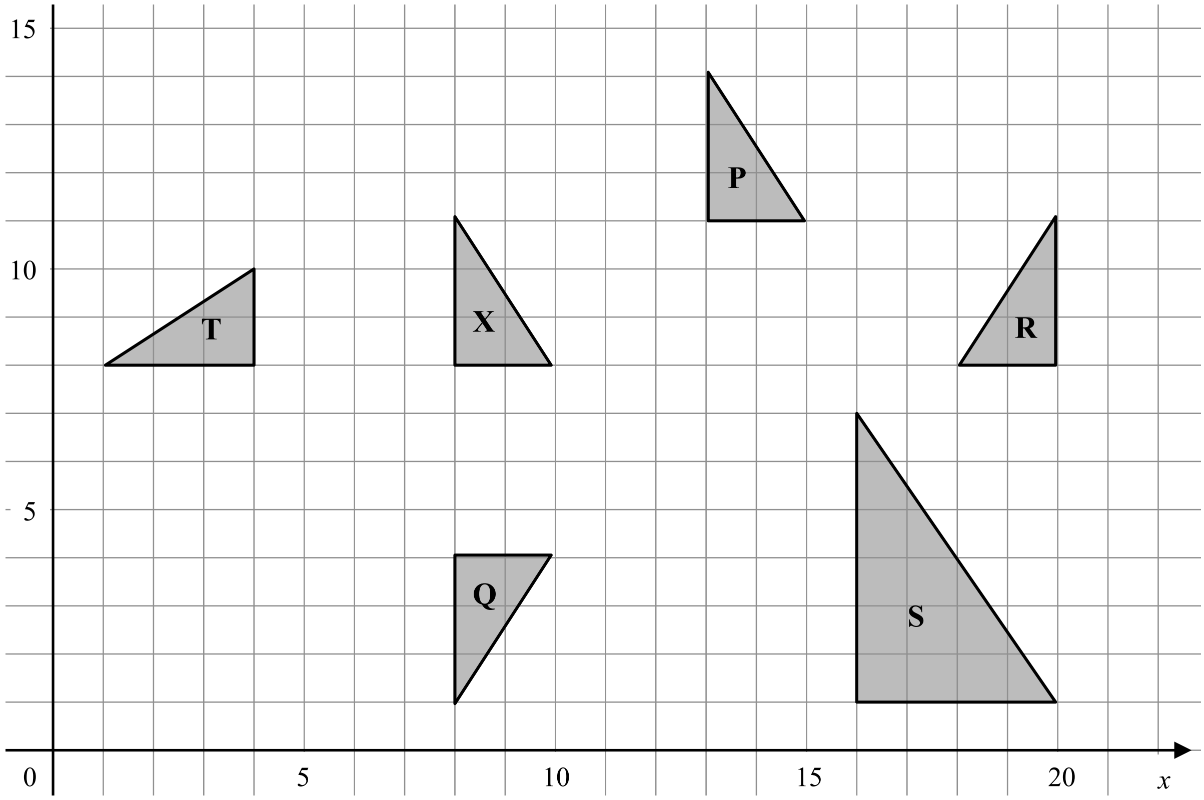
(c) **S** to **C**

(d) **S** to **D**

(e) **T** to **S**

(f) **D** to **T**

2. The diagram shows some triangles on a coordinate grid:



Describe fully each of the following transformations:

(a) **X** to **P**

(b) **X** to **Q**

(c) **X** to **R**

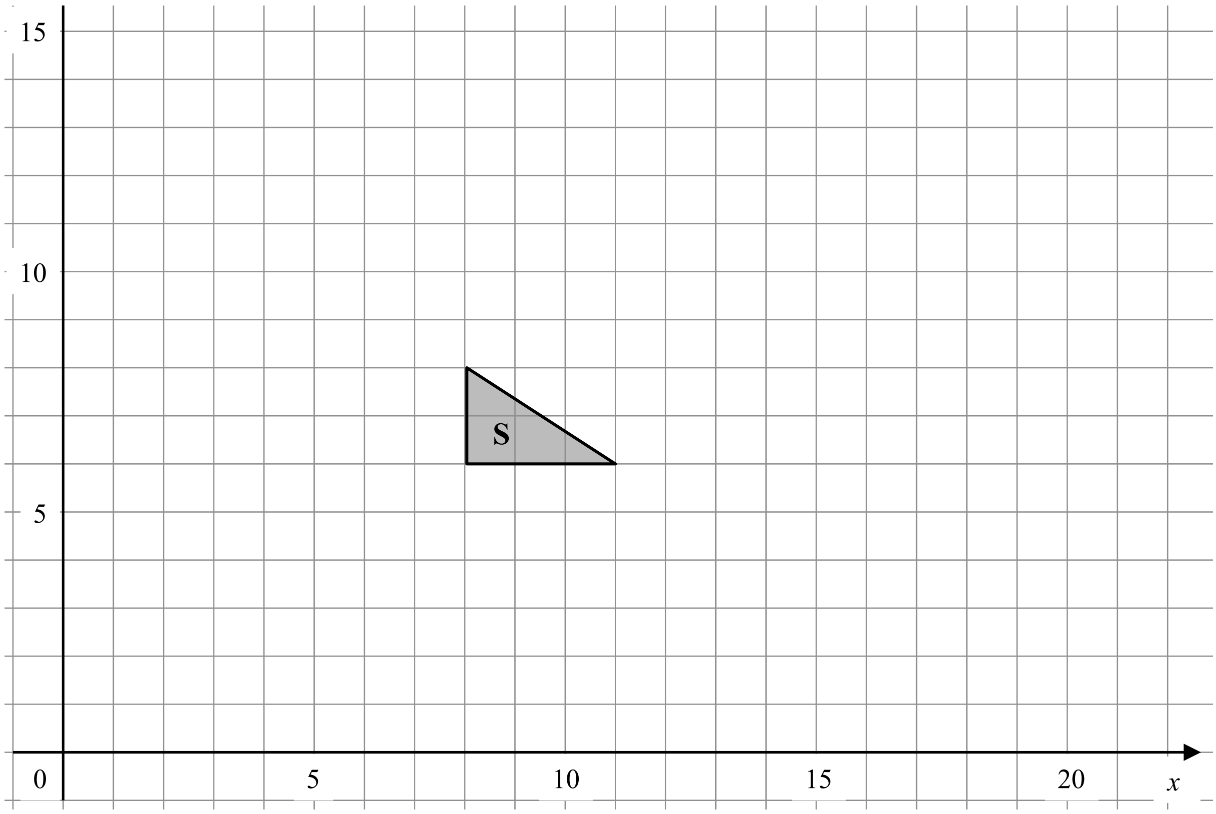
(d) **X** to **S**

(e) **X** to **T**

DRAWING TRANSFORMATIONS

SAMPLE QUESTIONS 2

1. Below is a coordinate grid showing a triangle:



On the grid:

(a) Reflect triangle **S** in the line $y = 4$
Label the new triangle **A**

(b) Reflect triangle **S** in the line $x = 6$
Label the new triangle **B**

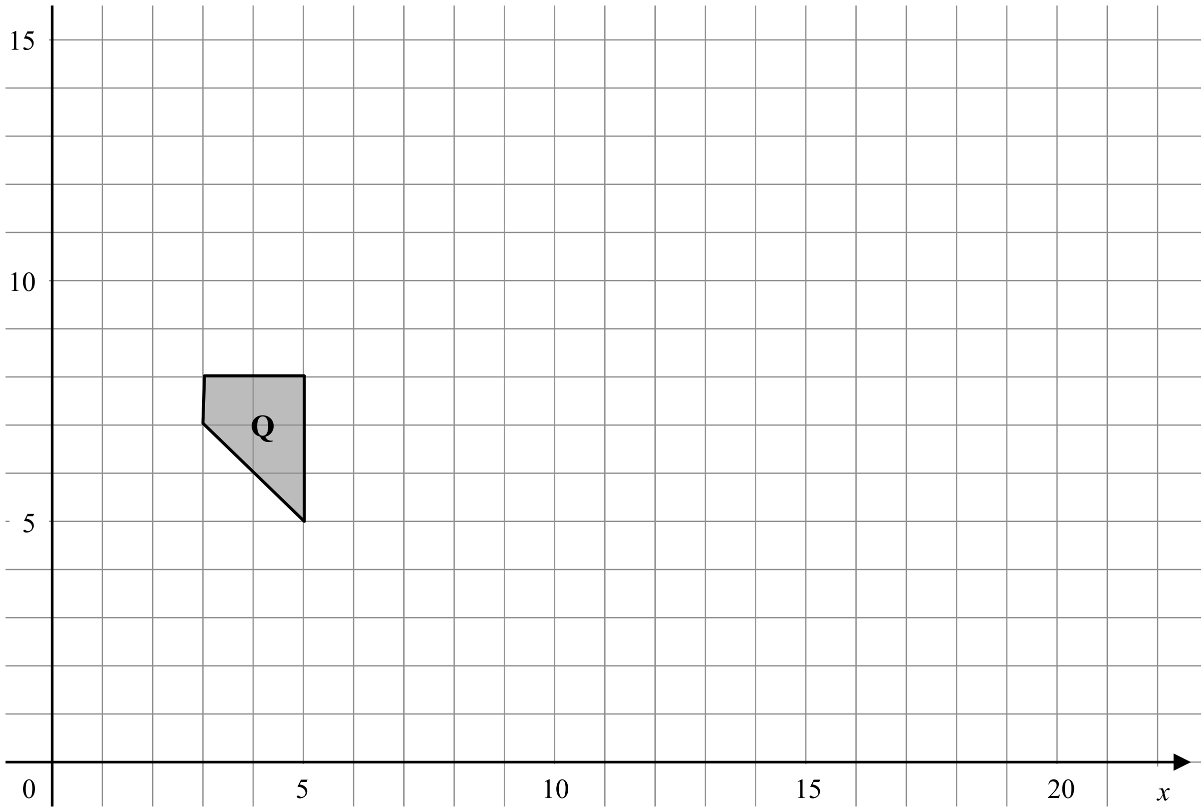
(c) Rotate triangle **S** 90º clockwise around the point (14, 6)
Label the new triangle **C**

(d) Translate triangle **S** by the vector $\begin{pmatrix} 8 \\ -4 \end{pmatrix}$
Label the new triangle **D**

(e) Reflect triangle **S** in the line $y = x$
Label the new triangle **E**

(f) Enlarge triangle **S** with scale factor 2 and centre (6, 4)
Label the new triangle **F**

2. Below is a coordinate grid showing a quadrilateral:



On the grid:

(a) Reflect quadrilateral **Q** in the line $y = x$
Label the new triangle **R**

(b) Reflect quadrilateral **Q** in the line $y = 10$
Label the new triangle **S**

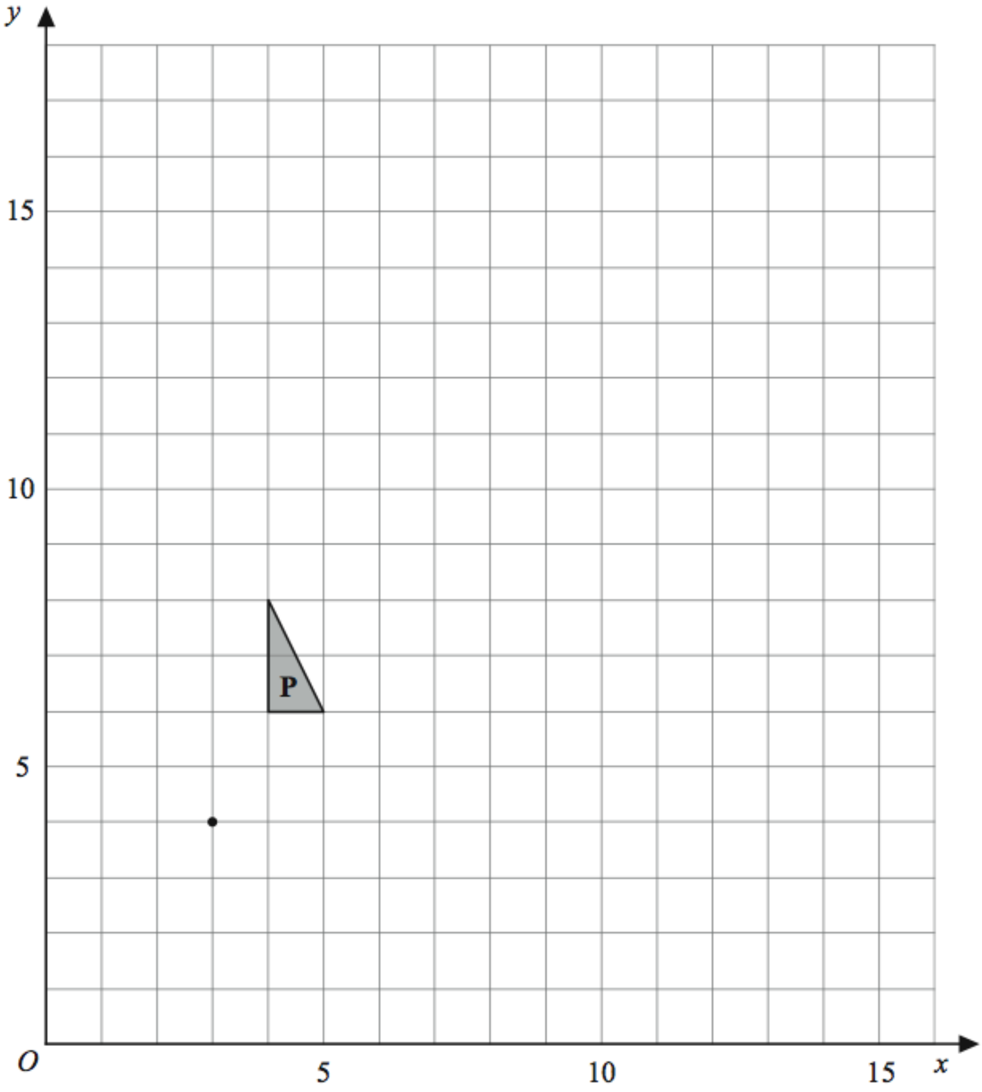
(c) Enlarge quadrilateral **Q** with scale factor 3 and centre (0, 6)
Label the new triangle **T**

(d) Translate quadrilateral **Q** by the vector $\begin{pmatrix} 15 \\ 6 \end{pmatrix}$
Label the new triangle **U**

(e) Rotate quadrilateral **Q** 90º anticlockwise around the point (13, 12)
Label the new triangle **V**

(f) Reflect quadrilateral **Q** in the line $x = 12$
Label the new triangle **W**

3.



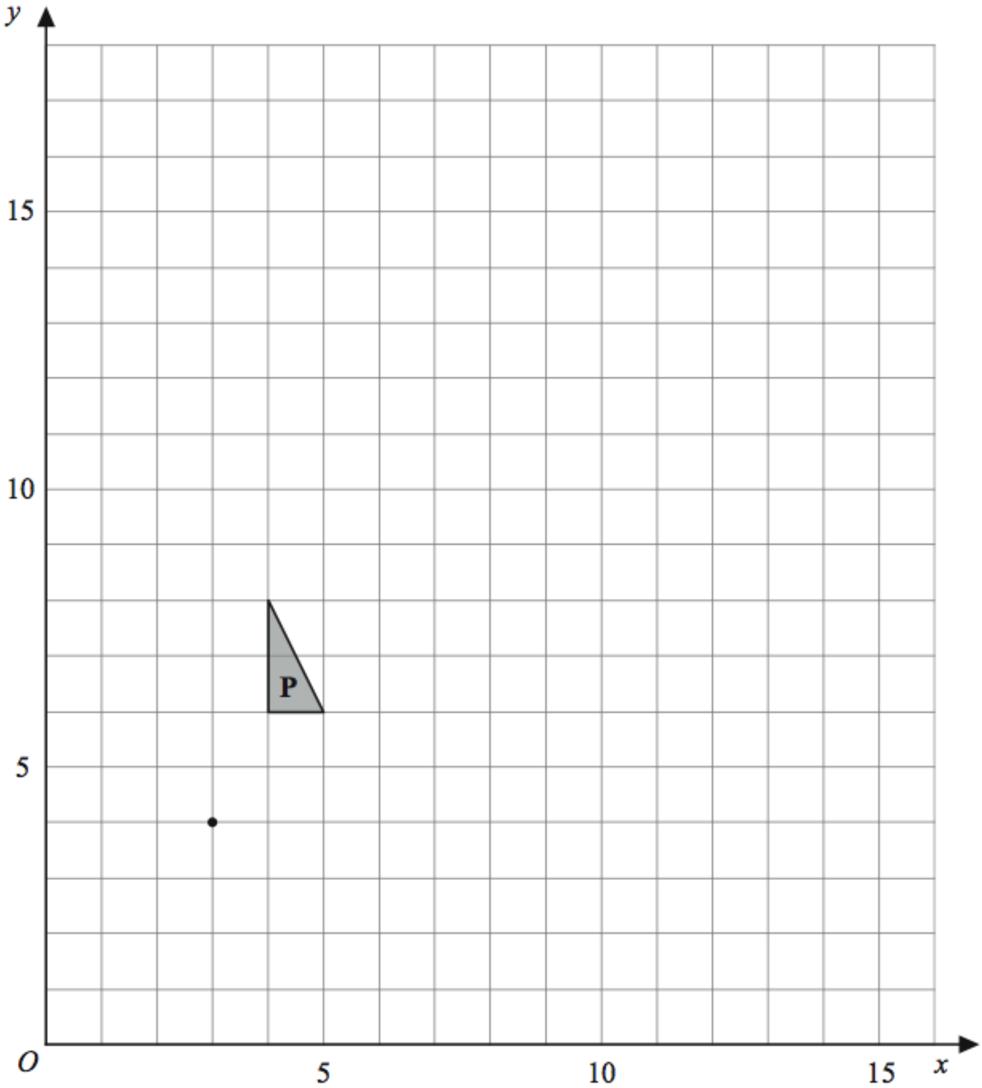
(a) On the grid, enlarge triangle **P** with a scale factor 3 and centre (3, 4)
Label the new triangle **Q**.

(b) On the grid, translate triangle **Q** by the vector $\begin{pmatrix} 4 \\ -8 \end{pmatrix}$
Label the new triangle **R**.

(c) Describe fully the single transformation that maps triangle **P** onto triangle **R**.

(d) On the grid, reflect triangle **Q** in the line $x = 9$
Label the new triangle **S**.

4.



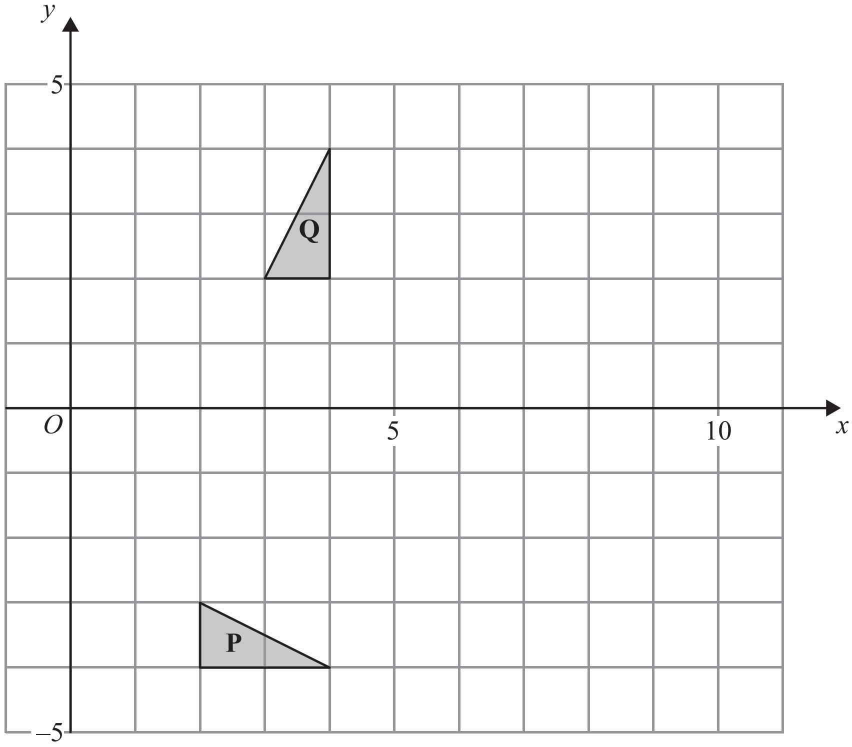
(a) On the grid, reflect triangle **P** in the line $y = 11$
Label the new triangle **Q**.

(b) On the grid, reflect triangle **P** in the line $y = x$
Label the new triangle **R**.

(c) On the grid, rotate triangle **Q** through 90º clockwise about the point (8, 12).
Label the new triangle **S**.

(d) Describe fully the single transformation that maps triangle **R** onto triangle **S**.

5.



(a) Describe fully the single transformation, which maps triangle **P** onto triangle **Q**.

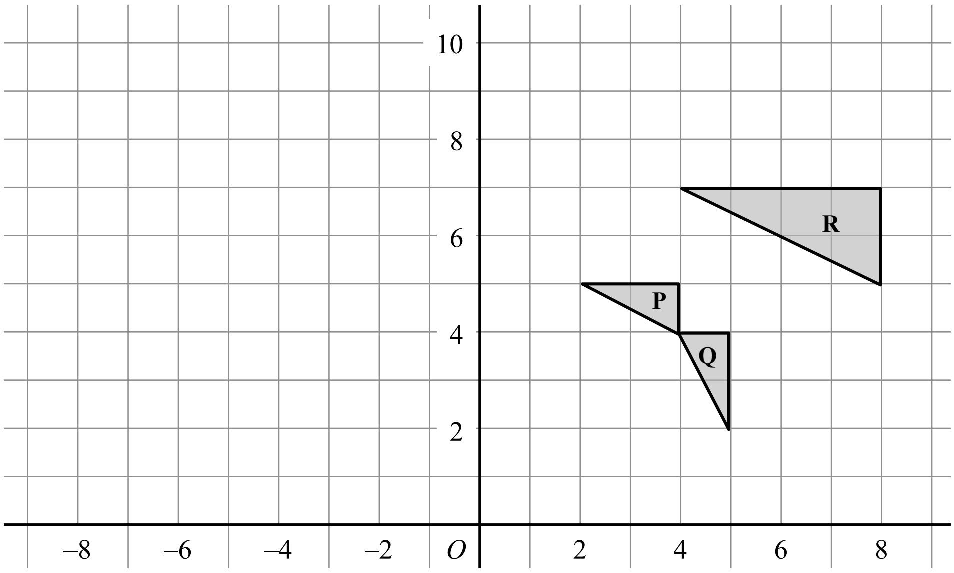
(b) On the grid, translate triangle **Q** by the vector $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$
Label the new triangle **R**.

(c) Describe fully the single transformation, which maps triangle **P** onto triangle **R**.

(d) On the grid, enlarge triangle **P** with scale factor 2 and centre (1, –5).
Label the new triangle **S**.

(e) On the grid, rotate triangle **Q** through 180º about the point (2, 1).
Label the new triangle **T**.

6.



(a) Describe fully the single transformation that maps triangle **P** onto triangle **Q**.

(b) Describe fully the single transformation that maps triangle **P** onto triangle **R**.

(c) On the grid, reflect triangle **R** in the *y*-axis.
Label the new triangle **S**.

(d) On the grid, rotate triangle **P** 90º anti-clockwise about the point (–1, 3).
Label the new shape **T**.

7. A shape, **P**, is enlarged by scale factor 3 to give shape **Q**.
Which of the following statements are true?

	True	False
The angles in **P** and **Q** are the same.	☐	☐
The lengths in **P** and **Q** are the same.	☐	☐
Shapes **P** and **Q** are congruent.	☐	☐
Shapes **P** and **Q** are similar.	☐	☐