UPPER AND LOWER BOUNDS

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 77

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2011] Upper and Lower Bounds **[2 Marks]**

The length of a fence is 137 metres, correct to the nearest metre. → 137 ± 0.5

Write down

(i) the lower bound for the length of the fence,

$137 - 0.5$

136.5 metres A1

(ii) the upper bound for the length of the fence.

$137 + 0.5$

137.5 metres A1

2. [New Question, by Maths4Everyone.com] Upper and Lower Bounds **[5 Marks]**

Zoe's weight is 62 kg, correct to the nearest kilogram. → 62 ± 0.5
Anu's weight is 85 kg, correct to the nearest kilogram. → 85 ± 0.5

(a) Write down the upper bound for the weight of Zoe.

62.5 B1

(1)

(b) Write down the lower bound for the weight of Anu.

84.5 B1

(1)

(c) Work out the upper bound for the difference between Zoe's weight and Anu's weight.

$(85 \pm 0.5) - (62 \pm 0.5)$

$= 85.5 - 61.5$

B1 B1

24 kg A1

(3)

3. [Edexcel, 2011] Upper and Lower Bounds **[2 Marks]**

$y = 1.8$ correct to 1 decimal place. → 1.8 ± 0.05

Calculate the lower bound for the value of $4y + 1$

$y = 1.75$

$4y + 1 = 4(1.75) + 1$

$= \underline{\underline{8}}$

4. [Edexcel, 2005] Upper and Lower Bounds **[3 Marks]**

The volume of oil in a tank is 1000 litres, correct to the nearest 10 litres.
The oil is poured into tins of volume 2.5 litres, correct to one decimal place.

Calculate the upper bound of the number of tins which will be required.

±5 LITRES!

±0.05

NUMBER OF TINS = $\frac{\text{TOTAL VOLUME OF OIL}}{\text{VOLUME OF A TIN}}$

$= \frac{1000 \pm 5}{2.5 \pm 0.05}$ M1 [DIVIDING]

M1 [USING CORRECT BOUNDS]

$= \frac{1000 + 5}{2.5 - 0.05}$ } THIS GIVES THE LARGEST ANSWER

$= 410.204...$

$= 411$ A1 [MUST GO UP, EVEN IF THE LAST TIN ONLY GETS A SMALL AMOUNT OF OIL!]

5. [Edexcel, 2010] Upper and Lower Bounds **[7 Marks]**

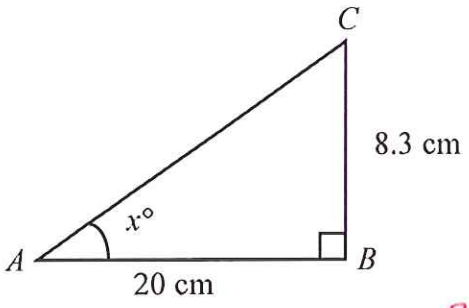


Diagram **NOT** accurately drawn

Triangle ABC is right-angled at B.
$AB = 20$ cm, correct to 1 significant figure. → 20 ± 0.5 (ASSUME NEAREST WHOLE NUMBER!)
$BC = 8.3$ cm, correct to 2 significant figures. → 8.3 ± 0.05

(a) Write down the lower bound for the length of

(i) AB,

$20 - 0.5$

[COULD BE $20 - 5 = 15$]

19.5 cm (A1)

(ii) BC.

$8.3 - 0.05$

8.25 cm (A1)

(2)

(b) Calculate the lower bound for the area of triangle ABC.

$A = \frac{\text{BASE} \times \text{HEIGHT}}{2}$

$= \frac{19.5 \times 8.25}{2}$ (M1) $= 80.4375$

80.4 cm^2 (A1)

(2)

(ACCEPT 61.875!)

(c) Calculate the lower bound for the value of tan $x°$.

(TOA)

$\tan x = \frac{\text{OPP}}{\text{ADJ}}$

$= \frac{8.25}{20.5}$ (B1) (B1)

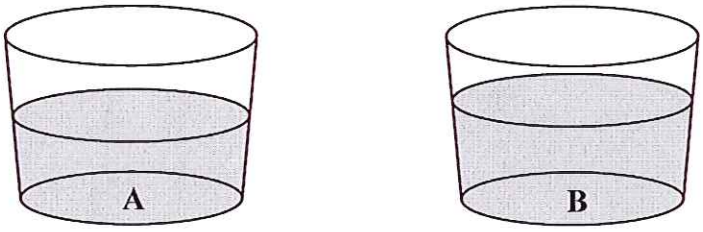
$= 0.40243...$

0.4024 (A1)

(3)

(ACCEPT 0.33)

6. [Edexcel, 2013] Upper and Lower Bounds **[2 Marks]**



Glass **A** contains 122 millilitres of water, correct to the nearest millilitre. — 122 ± 0.5
Glass **B** contains 168 millilitres of water, correct to the nearest millilitre. — 168 ± 0.5

Calculate the upper bound of the difference, in millilitres, between the volume of water in glass **A** and the volume of water in glass **B**.

HIGHEST − LOWEST

168.5 − 121.5

B1 CORRECT BOUNDS

47 A1

..... millilitres

7. [Edexcel, 2015] Upper and Lower Bounds **[3 Marks]**

An athlete runs 400 metres, correct to the nearest metre.
The athlete takes 50.2 seconds, correct to the nearest 0.1 of a second.

Work out the upper bound of the athlete's average speed.
Give your answer correct to 3 significant figures.

400 ± 0.5

50.2 ± 0.05

M1 [ANY!]

$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$$

$$= \frac{400 + 0.5}{50.2 - 0.05}$$ M1 [UPPER / LOWER]

$$= 7.986...$$

7.99 m/s A1

8. [Edexcel, 2015] Upper and Lower Bounds **[3 Marks]**

$$y = \frac{2a}{b - c}$$

$a = 42$ correct to 2 significant figures. → 42 ± 0.5
$b = 24$ correct to 2 significant figures. → 24 ± 0.5
$c = 14$ correct to 2 significant figures. → 14 ± 0.5

(m1) [FOR ANY CORRECT BOUNDS]

Work out the lower bound for the value of y.
Give your answer correct to 2 significant figures.
Show your working clearly.

$$y = \frac{2a}{b-c}$$ [SMALLEST] [LARGEST]

$$= \frac{2 \times 41.5}{24.5 - 13.5}$$ (m1)

$$= 7.5$$ (A1)

9. [Edexcel, 2014] Upper and Lower Bounds **[3 Marks]**

There are 1300 sheets of paper, correct to the nearest 100 sheets, in a pile. → 1300 ± 50
Each sheet is of equal thickness.
The height of the pile is 160 mm, correct to the nearest 10 mm. → 160 ± 5

Calculate the upper bound, in millimetres, for the thickness of one sheet of paper.

$$\text{THICKNESS} = \frac{\text{HEIGHT}}{\text{NO. OF SHEETS}}$$ M1 [CORRECT DIVISION]

$$= \frac{160+5}{1300-50}$$ (UPPER) (LOWER)

$$= \frac{165}{1250}$$ M1 [CORRECT VALUES]

$$= 0.132 \text{ mm}$$ A1

10. [Edexcel, 2007] Upper and Lower Bounds **[4 Marks]**

Some cases have to be lifted by a crane.
Each case has a mass of 68 kg, correct to 2 significant figures. → 68 ± 0.5

(a) Write down the upper bound of the mass of a case.

68.5 kg
(1)

A crane can lift safely a load of 1200 kg, correct to 2 significant figures. → 1200 ± 50

(b) Find the greatest number of cases that the crane can lift safely in one load.

$$\text{CASES} = \frac{\text{LIFTING CAPACITY}}{\text{MASS OF CASE}}$$ | M1 [FOR CAPACITY ÷ MASS]

WE MUST ENSURE SAFETY SO...

$$\text{CASES} = \frac{1150}{68.5}$$ | M1 [FOR CORRECT VALUES]

$$= 16.7883...$$ A1 [UN ROUNDED]

$$= \underline{16 \text{ CASES}}$$ A1 [CORRECTLY ROUNDED DOWN]

11. [Edexcel, 2004] Upper and Lower Bounds **[4 Marks]**

The length of a side of a square is 6.81 cm, correct to 3 significant figures.

→ 6.81 ± 0.005

(a) Work out the lower bound for the perimeter of the square.

$4 \times (6.81 - 0.005)$

$= 4 \times 6.805$ (B1)

27.22 cm (A1)

(2)

(b) Give the perimeter of the square to an appropriate degree of accuracy.
You must show working to explain how you obtained your answer.

UPPER BOUND IS:

$4 \times (6.81 + 0.005)$ (M1)

$= 4 \times 6.815$

$= 27.26$

27 cm (A1)

(2)

$27.22 \leq$ PERIMETER < 27.26

$\therefore$ 27 (BOTH BOUNDS ROUND TO 27)

(a) Make r the subject of the formula $A = \pi r^2$, where r is positive.

$\pi r^2 = A$

$r^2 = \frac{A}{\pi}$ M1

$r = \sqrt{\frac{A}{\pi}}$ A1

(2)

The area of a circle is 14 cm^2, correct to 2 significant figures.

AREA = 14 ± 0.5

(b) (i) Work out the lower bound for the radius of the circle.
Write down all the figures on your calculator display.

$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{13.5}{\pi}}$ ← LOWER BOUND (AREA) M1

2.072964 cm A1

(ii) Give the radius of the circle to an appropriate degree of accuracy.
You must show working to explain how you obtained your answer.

CALCULATE $\sqrt{\frac{14.5}{\pi}}$ ← UPPER BOUND (AREA) M1

$= 2.14836$

2.1 cm A1

(4)

	LOWER	UPPER
3sf	2.07	2.15
2sf	2.1	2.1

← UPPER AND LOWER ARE SAME WHEN ROUNDED TO 2sf.

13. [Edexcel, 2014] Upper and Lower Bounds **[4 Marks]**

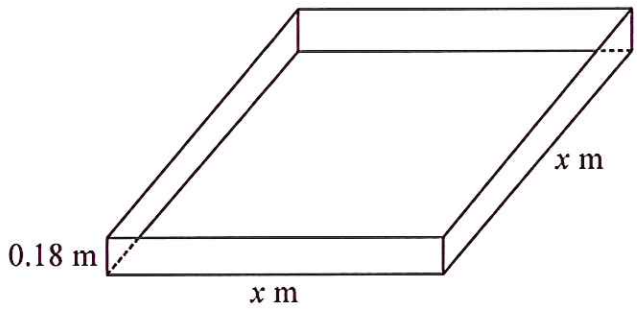


Diagram **NOT** accurately drawn

Trena wants to build a sandpit in the shape of a cuboid.
The volume of sand in the sandpit will be 1.0 m^3, correct to 1 decimal place.
The depth of sand in the sandpit will be 0.18 metres, correct to 2 decimal places.
The sandpit will have a square base with sides of length x metres.

1.0 ± 0.05

0.18 ± 0.005

Find the upper bound for x
Give your answer correct to 3 significant figures.

$$x^2 \times (0.18 \pm 0.005) = 1.0 \pm 0.05$$

$$\Rightarrow x^2 = \frac{1.0 \pm 0.05}{0.18 \pm 0.005}$$

$$= \frac{1.0 + 0.05}{0.18 - 0.005} \quad \text{(M1) [FOR UPPER BOUND]}$$

(M1) [FOR LOWER BOUND]

$$= 6$$

$$x = \sqrt{6} \quad \text{(M1)}$$

$$= 2.44948...$$

$$= \underline{\underline{2.45}} \text{ m} \quad \text{(A1)}$$

14. [Edexcel, 2013] Upper and Lower Bounds **[5 Marks]**

(a) Correct to the nearest millimetre, the length of a side of a regular hexagon is 3.6 cm.

Calculate the upper bound for the perimeter of the regular hexagon.

3.6 ± 0.05

6×3.65 M1

21.9 A1 cm

(2)

(b) Correct to 1 significant figure, the area of a rectangle is 80 cm^2
Correct to 2 significant figures, the length of the rectangle is 12 cm.

80 ± 5

12 ± 0.5

Calculate the lower bound for the width of the rectangle.
Show your working clearly.

AREA = LENGTH × WIDTH.

$\Rightarrow$ WIDTH = $\frac{\text{AREA}}{\text{LENGTH}}$ [LOWER BOUND] [UPPER BOUND]

$= \frac{75}{12.5}$ M1 [DIVIDING] M1 [CORRECT BOUNDS]

$= 6$ A1

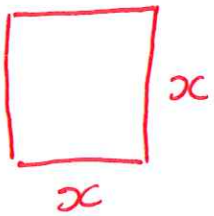
15. [Edexcel, 2009] Upper and Lower Bounds **[3 Marks]**

Correct to 2 significant figures, the area of a square is 230 cm^2.

Calculate the lower bound for the perimeter of the square.

NEAREST 10

230 ± 5



$x^2 = 230 - 5$ M1

$= 225$

$x = \sqrt{225}$

$= 15$ A1

$\therefore$ PERIMETER $= 4 \times 15$

$= 60$ A1

60 cm

16. [Edexcel, 2012] Upper and Lower Bounds **[4 Marks]**

Correct to 2 decimal places, the volume of a solid cube is 42.88 cm^3

Calculate the lower bound for the surface area of the cube.

SMALLEST

$V = 42.88 \pm 0.005$

SMALLEST SIDE LENGTH $= \sqrt[3]{42.875}$ (B1)

LOWER BOUND!

$= 3.5$ (A1)

SURFACE AREA $= 6 \times 3.5^2$ (M1)

$= 73.5 cm^2$ (A1)

..... cm^2

17. [Edexcel, 2015] Upper and Lower Bounds **[3 Marks]**

Correct to 2 significant figures, $a = 58$, $b = 28$ and $c = 18$

Calculate the upper bound for the value of $\frac{a}{b-c}$

Show your working clearly.

$a = 58 \pm 0.5$

$b = 28 \pm 0.5$

$c = 18 \pm 0.5$

UPPER BOUND OF $\frac{a}{b-c}$ [HIGHEST] / [LOWEST]

$= \frac{58.5}{27.5 - 18.5}$ (m1) (m1)

$= \frac{58.5}{9}$

$= \underline{\underline{6.5}}$ (A1)

18. [Edexcel, 2016] Upper and Lower Bounds **[5 Marks]**

A metal cube has sides of length 4.5 cm, correct to the nearest 0.5 cm.

± 0.25

The cube is melted down and the metal is used to make small spheres.
Each sphere has a radius of 3 mm, correct to the nearest millimetre.

$V_{SPHERE} = \frac{4}{3}\pi r^3$

Work out the greatest number of spheres that could be made from the metal.
Show your working clearly.

GREATEST VOLUME FOR CUBE
SMALLEST VOLUME FOR SPHERES

GREATEST VOLUME FOR CUBE

$4.75^3 = 107.171875$ (B1)

SMALLEST VOLUME OF SPHERES

$\frac{4}{3}\pi \times 0.25^3 = 0.0654498...$ (M1) [EITHER CORRECT]

CHANGED TO cm! (M1)

GREATEST NUMBER OF SPHERES $= \frac{107.17875}{0.0654498...}$ (M1)

$= 1637.466...$

$= 1637$ (A1)

19. [Edexcel, 2004] Upper and Lower Bounds **[5 Marks]**

NEAREST WHOLE NUMBER

Correct to 1 significant figure, $x = 7$ and $y = 9$

$x = 7 \pm 0.5$

$y = 9 \pm 0.5$

(a) Calculate the lower bound for the value of xy

$(7 - 0.5) \times (9 - 0.5)$

$= 6.5 \times 8.5$ (M1)

$= 55.25$

55.25 (A1)

(2)

(b) Calculate the upper bound for the value of $\frac{x}{y}$

$\frac{7 + 0.5}{9 - 0.5} = \frac{7.5}{8.5}$ (B1) (B1)

$= \frac{15}{17}$

0.88235... (A1)

(3)

20. [Edexcel, 2009] Upper and Lower Bounds **[4 Marks]**

In a race, Paula runs 25 laps of a track.
Each lap of the track is 400 m, correct to the nearest metre. → 400 ± 0.5
Paula's average speed is 5.0 m/s, correct to one decimal place. → 5.0 ± 0.05

Calculate the upper bound for the time that Paula takes to run the race.
Give your answer in minutes and seconds, correct to the nearest second.

$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$$

$$\Rightarrow \text{TIME} = \frac{\text{DISTANCE}}{\text{SPEED}}$$

$$= \frac{400.5 \times 25}{4.95} \quad \text{(M1) [FOR UPPER ÷ LOWER]}$$

(M1) [FOR DIVIDING]

$$= 2023 \text{ SECONDS} \quad \text{(M1)}$$

$$= 33 \text{ MIN } 43 \text{ SECS} \quad \text{(A1)}$$

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Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners' mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners' mark scheme.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site.

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words 'show that', 'show your working out', or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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