

VECTORS

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 76

SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2015]

Column Vectors [5 Marks]

$$\mathbf{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 7 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} -7 \\ 0 \end{pmatrix}$$

(a) Write, as a column vector, $2\mathbf{a}$

$$2\mathbf{a} = 2 \times \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 10 \\ -4 \end{pmatrix}$$

(1)

(b) Write, as a column vector, $3\mathbf{b} - \mathbf{c}$

$$3 \times \begin{pmatrix} 1 \\ 7 \end{pmatrix} - \begin{pmatrix} -7 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 21 \end{pmatrix} - \begin{pmatrix} -7 \\ 0 \end{pmatrix}$$

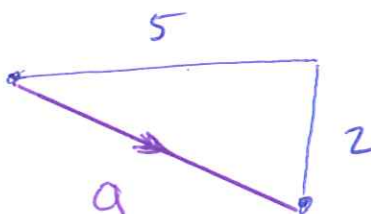
Handwritten notes: "3 - -7" above the 3, "21 - 0" below the 21, and a circled "m" with an arrow pointing to the 21.

$$\begin{pmatrix} 10 \\ 21 \end{pmatrix}$$

(2)

(c) Work out the magnitude of \mathbf{a}
Give your answer as a surd.

USE PYTHAGORAS



$$\begin{aligned} a^2 &= 5^2 + 2^2 \\ &= 25 + 4 \\ &= 29 \end{aligned}$$

$$\Rightarrow a = \underline{\underline{\sqrt{29}}}$$

MAGNITUDE
MEANS
'SIZE'

$ABCD$ is a parallelogram.

$$\vec{BC} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} \quad \vec{DC} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

Find \vec{BD} as a column vector.

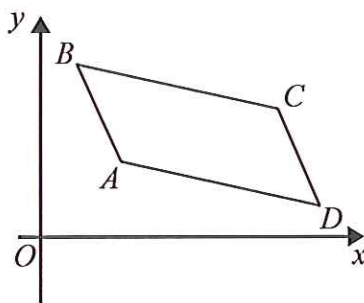


Diagram NOT
accurately drawn

$$\begin{aligned} \vec{BD} &= \vec{BC} + \vec{CD} \\ &= \begin{pmatrix} 5 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} \end{aligned}$$

$$\begin{pmatrix} 7 \\ -4 \end{pmatrix} \begin{matrix} (1) \\ (1) \end{matrix}$$

A is the point with coordinates $(2, 3)$.

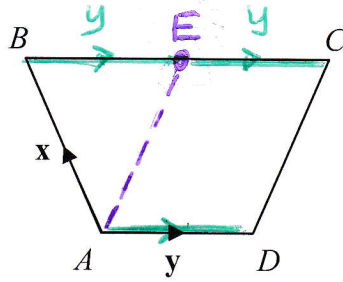
$$\vec{AB} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}.$$

$$(2+5, 3-4)$$

Find the coordinates of B .

$$\left(\frac{7}{(B)}, \frac{-1}{(B)} \right)$$

The diagram shows a trapezium $ABCD$.



$$\vec{BC} = 2\vec{AD}. \quad [= 2y]$$

$$\vec{AB} = x. \quad \vec{AD} = y.$$

(a) Find, in terms of x and y ,

$$(i) \vec{AC} = \vec{AB} + \vec{BC}$$

$$= x + 2y$$

$$(ii) \vec{DC} = \vec{DA} + \vec{AC}$$

$$= -y + (x + 2y)$$

$$x + 2y \quad (B1)$$

$$x + y \quad (B1)$$

(2)

(b) The point E is such that $\vec{AE} = x + y$.

Use your answer to part (a)(ii) to explain why $AECD$ is a parallelogram.

$$\vec{AE} = \vec{DC}, \text{ so } AE \text{ AND } DC \text{ ARE PARALLEL} \quad (B1)$$

$$\vec{EC} = \frac{1}{2} \vec{BC}$$

$$= \frac{1}{2} \times 2y$$

$$= y$$

$$= \vec{AD}, \text{ so } EC \text{ AND } AD \text{ ARE PARALLEL}$$

(2)

(A1)

[OTHER METHODS MIGHT ALSO SHOW PARALLELNESS]

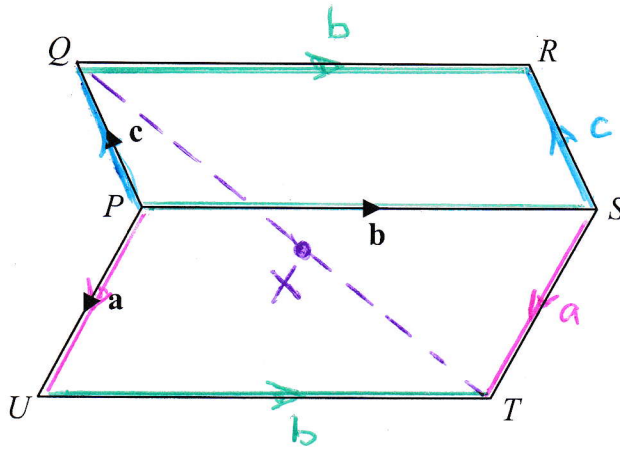


Diagram NOT
accurately drawn

$PQRS$ and $PSTU$ are parallelograms.

$$\vec{PU} = \mathbf{a} \quad \vec{PS} = \mathbf{b} \quad \vec{PQ} = \mathbf{c}$$

Find, in terms of \mathbf{a} , \mathbf{b} and \mathbf{c}

$$\begin{aligned} \text{(i) } \vec{TQ} &= \vec{TU} + \vec{UP} + \vec{PQ} \\ &= -\mathbf{b} - \mathbf{a} + \mathbf{c} \end{aligned}$$

$$\rightarrow \underline{\underline{\mathbf{c} - \mathbf{a} - \mathbf{b}}}$$

$$\text{(ii) } \vec{PX} \text{ where } X \text{ is the midpoint of } TQ.$$

(B1)

Simplify your answer as much as possible.

$$\begin{aligned} \vec{PX} &= \vec{PU} + \vec{UT} + \frac{1}{2}\vec{TQ} \\ &= \mathbf{a} + \mathbf{b} + \frac{1}{2}(\mathbf{c} - \mathbf{a} - \mathbf{b}) \quad \text{(M1)} \\ &= \mathbf{a} + \mathbf{b} + \frac{1}{2}\mathbf{c} - \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} \\ &= \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{c} \end{aligned}$$

$$\rightarrow \underline{\underline{\frac{1}{2}(\mathbf{a} + \mathbf{b} + \mathbf{c})}} \quad \text{(A1)}$$

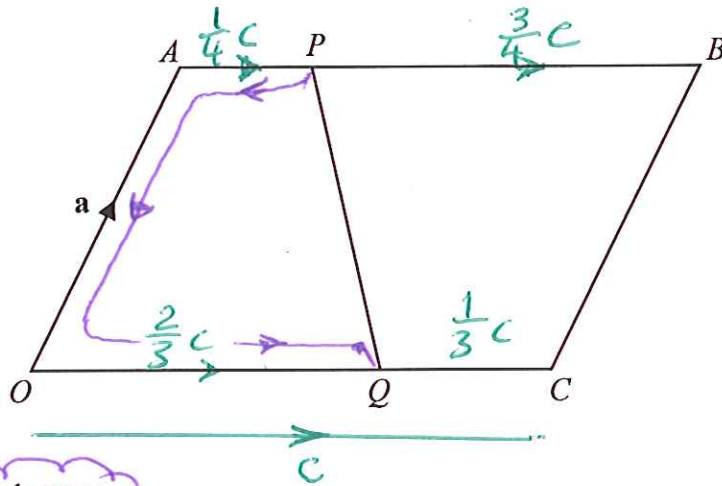


Diagram NOT
accurately drawn

$OABC$ is a parallelogram.

$\vec{OA} = \mathbf{a}$ and $\vec{OC} = \mathbf{c}$

P is the point on AB such that $AP = \frac{1}{4}AB$.

Q is the point on OC such that $OQ = \frac{2}{3}OC$.

Find, in terms of \mathbf{a} and \mathbf{c} , \vec{PQ} .

Give your answer in its simplest form.

$$\vec{PQ} = \vec{PA} + \vec{AO} + \vec{OQ}$$

$$= -\frac{1}{4}\mathbf{c} - \mathbf{a} + \frac{2}{3}\mathbf{c} \quad (\text{M1})$$

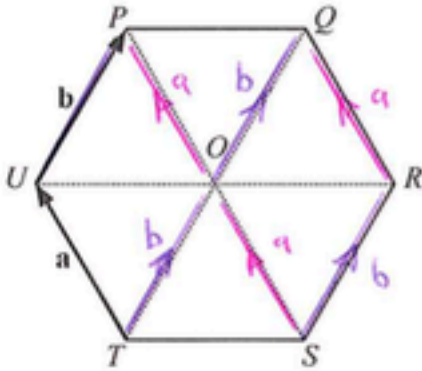
$$= \left(\frac{2}{3} - \frac{1}{4}\right)\mathbf{c} - \mathbf{a}$$

$$= \frac{5}{12}\mathbf{c} - \mathbf{a} \quad (\text{A1})$$

(A1)

$PQRSTU$ is a regular hexagon, centre O .

The hexagon is made from six equilateral triangles of side 2.5 cm.



I'VE WRITTEN ON ALL PARALLEL VECTORS!

$$\vec{TU} = \mathbf{a}, \vec{UP} = \mathbf{b}.$$

(a) Find, in terms of \mathbf{a} and/or \mathbf{b} , the vectors

$$\begin{aligned} \text{(i) } \vec{TP} &= \vec{TU} + \vec{UP} \\ &= \mathbf{a} + \mathbf{b} \end{aligned}$$

$$\frac{\mathbf{a} + \mathbf{b}}{\text{(1)}} \quad \text{(AI)}$$

(ii) \vec{PO}

$$\frac{-\mathbf{a}}{\text{(1)}} \quad \text{(AI)}$$

$$\begin{aligned} \text{(iii) } \vec{UO} &= \vec{UP} + \vec{PO} \\ &= \mathbf{b} - \mathbf{a} \end{aligned}$$

$$\frac{\mathbf{b} - \mathbf{a}}{\text{(1)}} \quad \text{(AI)}$$

(b) Find the modulus (magnitude) of \vec{UR} .

$$2 \times 2.5$$

$$\frac{5}{\text{cm}} \quad \text{(1)} \quad \text{(AI)}$$

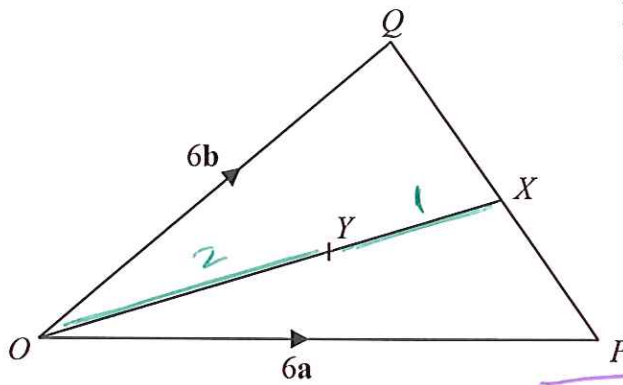


Diagram NOT
accurately drawn

In triangle OPQ , $\vec{OP} = 6\mathbf{a}$ and $\vec{OQ} = 6\mathbf{b}$

X is the midpoint of PQ .

IST

$$\begin{aligned}\vec{QP} &= \vec{QO} + \vec{OP} \\ &= -6\mathbf{b} + 6\mathbf{a}\end{aligned}$$

- (a) Find, in terms of \mathbf{a} and \mathbf{b} , the vector \vec{OX}
Give your answer in its simplest form.

$$\begin{aligned}\vec{OX} &= \vec{OQ} + \frac{1}{2}\vec{QP} \\ &= 6\mathbf{b} + \frac{1}{2}(-6\mathbf{b} + 6\mathbf{a}) \\ &= 6\mathbf{b} - 3\mathbf{b} + 3\mathbf{a}\end{aligned}$$

}

(M1) [EITHER]

$$\frac{3\mathbf{a} + 3\mathbf{b}}{(2)}$$

A1

Y is the point on OX such that $OY : YX = 2 : 1$

- (b) Find, in terms of \mathbf{a} and \mathbf{b} , the vector \vec{QY}
Give your answer in its simplest form.

$$\begin{aligned}\vec{QY} &= \vec{QO} + \vec{OY} \\ &= \vec{QO} + \frac{2}{3}\vec{OX} \\ &= -6\mathbf{b} + \frac{2}{3}(3\mathbf{a} + 3\mathbf{b}) \\ &= -6\mathbf{b} + 2\mathbf{a} + 2\mathbf{b}\end{aligned}$$

}

(M1) [EITHER]

$$\frac{2\mathbf{a} - 4\mathbf{b}}{(2)}$$

A1

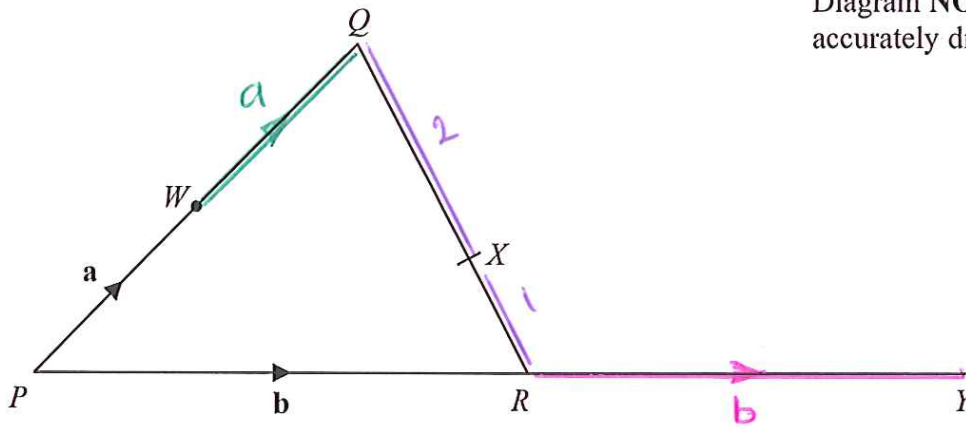


Diagram NOT
accurately drawn

PQR is a triangle.

The midpoint of PQ is W .

X is the point on QR such that $QX : XR = 2 : 1$

PRY is a straight line.

$$\vec{PW} = \mathbf{a} \quad \vec{PR} = \mathbf{b}$$

(a) Find, in terms of \mathbf{a} and \mathbf{b} ,

$$(i) \quad \vec{QR} \\ = \vec{QP} + \vec{PR} = \underline{\underline{-2\mathbf{a} + \mathbf{b}}}$$

$$\underline{\underline{\mathbf{b} - 2\mathbf{a}}} \quad \text{(A1)}$$

$$(ii) \quad \vec{QX} \\ = \frac{2}{3} \vec{QR} = \underline{\underline{\frac{2}{3}(\mathbf{b} - 2\mathbf{a})}}$$

$$\underline{\underline{\frac{2}{3}\mathbf{b} - \frac{4}{3}\mathbf{a}}} \quad \text{(A1)}$$

$$(iii) \quad \vec{WX} \\ = \vec{WQ} + \vec{QX} = \mathbf{a} + \left[\frac{2}{3}\mathbf{b} - \frac{4}{3}\mathbf{a} \right]$$

$$\underline{\underline{\frac{2}{3}\mathbf{b} - \frac{1}{3}\mathbf{a}}} \quad \text{(A1)}$$

(3)

R is the midpoint of the straight line PRY .

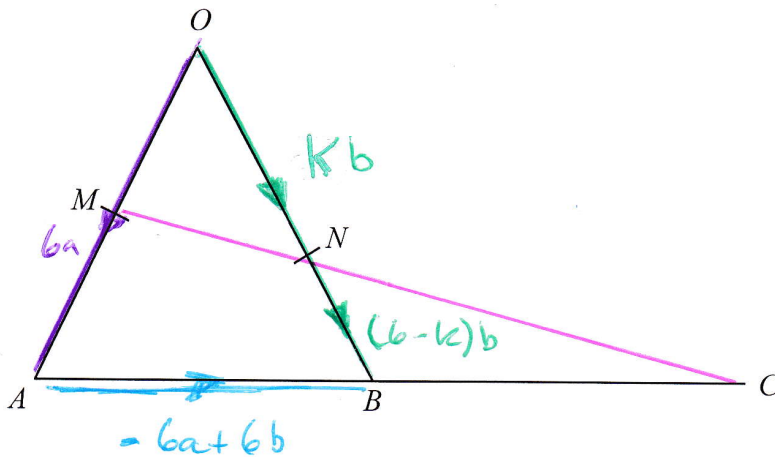
(b) Use a vector method to show that WXY is a straight line.

$$\begin{aligned} \vec{XY} &= \vec{XR} + \vec{RY} \\ &= \frac{1}{3} \vec{QR} + \vec{RY} \\ &= \frac{1}{3} [\mathbf{b} - 2\mathbf{a}] + \mathbf{b} \\ &= \frac{1}{3}\mathbf{b} - \frac{2}{3}\mathbf{a} + \mathbf{b} \\ &= \underline{\underline{\frac{4}{3}\mathbf{b} - \frac{2}{3}\mathbf{a}}} \end{aligned}$$

$$= \underline{\underline{\frac{2}{3}(2\mathbf{b} - \mathbf{a})}} \quad \text{(B1)}$$

$$\vec{WX} = \frac{1}{3}(2\mathbf{b} - \mathbf{a}) \quad \text{(B1)}$$

NOTE THAT
BOTH ARE MULTIPLES OF
($2\mathbf{b} - \mathbf{a}$) \therefore SAME DIRECTION
ALSO, THEY BOTH GO THROUGH
COMMON POINT X
(\therefore STRAIGHT LINE)



OMA , ONB and ABC are straight lines.

M is the midpoint of OA .

B is the midpoint of AC .

$\vec{OA} = 6\mathbf{a}$ $\vec{OB} = 6\mathbf{b}$ $\vec{ON} = k\mathbf{b}$ where k is a scalar quantity.

Given that MNC is a straight line, find the value of k .

$$\vec{MN} = -3\mathbf{a} + k\mathbf{b} \quad (m1)$$

$$\begin{aligned} \vec{MC} &= 3\mathbf{a} + 2(-6\mathbf{a} + 6\mathbf{b}) \\ &= -9\mathbf{a} + 12\mathbf{b} \quad (m2) \end{aligned}$$

[NOTE THAT \vec{MN} MIGHT BE COMPARED WITH \vec{NC} INSTEAD OF \vec{MC} !]

IF MNC IS A STRAIGHT LINE, THEN

$$MC = n \times MN \quad [\text{WHERE } n \text{ IS A SCALAR}]$$

$$\Rightarrow -9\mathbf{a} + 12\mathbf{b} = n(-3\mathbf{a} + k\mathbf{b}) \quad (m3)$$

$$\Rightarrow -9\mathbf{a} + 12\mathbf{b} = -3n\mathbf{a} + kn\mathbf{b}$$

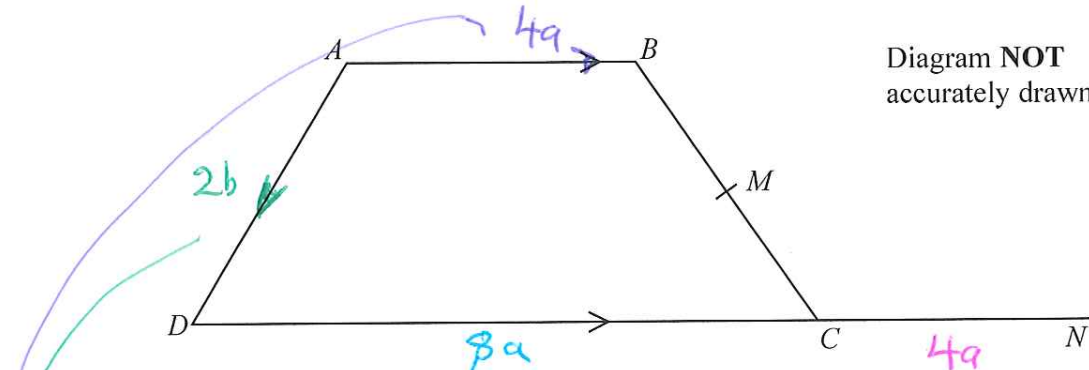
COMPARING THE 'a' COEFFICIENTS...

$$-3n = -9$$

$$\Rightarrow n = \underline{\underline{3}} \quad (m4)$$

COMPARING THE 'b' COEFFICIENTS

$$kn = 12 \Rightarrow k \times 3 = 12 \Rightarrow k = \underline{\underline{4}} \quad (A1)$$



AB is parallel to DC
 DC = 2AB
 M is the midpoint of BC

$\vec{AD} = 2b$
 $\vec{AB} = 4a$

$\vec{BM} = \frac{1}{2} \vec{BC}$

- (a) Find \vec{BM} in terms of a and b .
 Give your answer in its simplest form.

$\vec{BC} = -4a + 2b + 8a$ (m) [FOR \vec{BC}]
 $= 4a + 2b$

$\Rightarrow \vec{BM} = \frac{1}{2} (4a + 2b)$

$2a + b$ (A)
 (2)

- N is the point such that DCN is a straight line and $DC : CN = 2 : 1 \rightarrow \vec{CN} = 4a$
 (b) Show that AMN is a straight line.

$\vec{AM} = \vec{AB} + \vec{BM}$
 $= 4a + (2a + b) = 6a + b$

$\vec{AN} = \vec{AD} + \vec{DC} + \vec{CN}$
 $= 2b + 8a + 4a$ (m) [BOTH]
 $= 12a + 2b$
 $= 2(6a + b)$
 $= 2\vec{AM}$

EXPLANATION
 SINCE $\vec{AM} = k \times \vec{AN}$
 (m) [EXPLANATION]
 AM AND AN MUST BE PARALLEL AND SINCE THEY BOTH HAVE POINT A IN COMMON, FORM A STRAIGHT LINE. (NEEDED!)

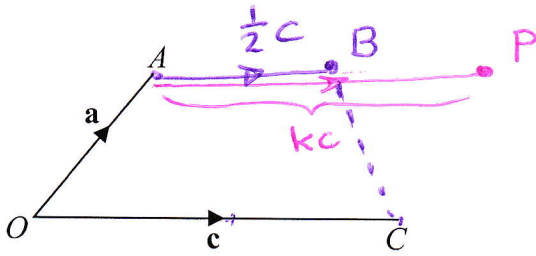


Diagram NOT
accurately drawn

In the diagram $\vec{OA} = \mathbf{a}$ and $\vec{OC} = \mathbf{c}$.

(a) Find \vec{CA} in terms of \mathbf{a} and \mathbf{c} .

$$\begin{aligned}\vec{CA} &= \vec{CO} + \vec{OA} \\ &= -\mathbf{c} + \mathbf{a}\end{aligned}$$

$$\dots\dots\dots \mathbf{a} - \mathbf{c} \quad (1)$$

(b) The point B is such that $\vec{AB} = \frac{1}{2}\mathbf{c}$.

Give the mathematical name for the quadrilateral $OABC$.

[AB IS PARALLEL TO OC!]

TRAPEZIUM (1)

(c) The point P is such that $\vec{OP} = \mathbf{a} + k\mathbf{c}$, where $k \geq 0$

State the two conditions relating to $\mathbf{a} + k\mathbf{c}$ that must be true for $OAPC$ to be a rhombus.

IF A RHOMBUS,

$$AP = OC \Rightarrow \underline{k=1} \quad (1)$$

ALSO

$$\underline{\underline{\text{LENGTH OF } \mathbf{a} = \text{LENGTH OF } \mathbf{c}}} \quad (1)$$

(2)

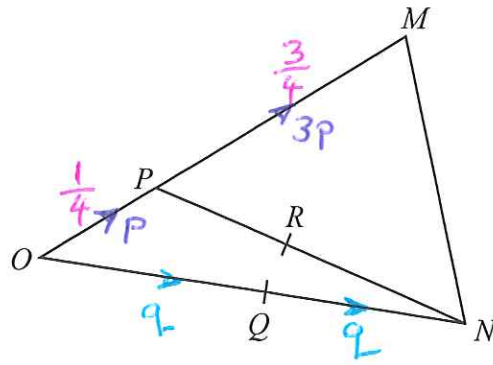


Diagram NOT
accurately drawn

OMN is a triangle.

P is the point on OM such that $OP = \frac{1}{4} OM$

Q is the midpoint of ON

R is the midpoint of PN

$\vec{OP} = \mathbf{p}$ $\vec{OQ} = \mathbf{q}$

(a) Find, in terms of \mathbf{p} and \mathbf{q} ,

(i) \vec{MN}

$$\vec{MN} = \vec{MO} + \vec{ON} = -3\mathbf{p} - \mathbf{p} + \mathbf{q} + \mathbf{q} = 2\mathbf{q} - 4\mathbf{p} \quad \text{(BI)}$$

(ii) \vec{PR}

$$= \frac{1}{2} \vec{PN} = \frac{1}{2} (-\mathbf{p} + 2\mathbf{q}) \rightarrow \mathbf{q} - \frac{1}{2} \mathbf{p} \quad \text{(BI)} \quad (2)$$

(b) Use a vector method to prove that QR is parallel to OP

$$\vec{QR} = \vec{QO} + \vec{OP} + \vec{PR} = -\mathbf{q} + \mathbf{p} + \left[\mathbf{q} - \frac{1}{2}\mathbf{p}\right]$$

$$= \mathbf{p} - \frac{1}{2}\mathbf{p}$$

$$= \frac{1}{2}\mathbf{p} \quad \text{(BI)} \quad \text{SINCE } \vec{QR} = k \vec{OP} \quad \text{(BI)} \quad \text{[STATEMENT]}$$

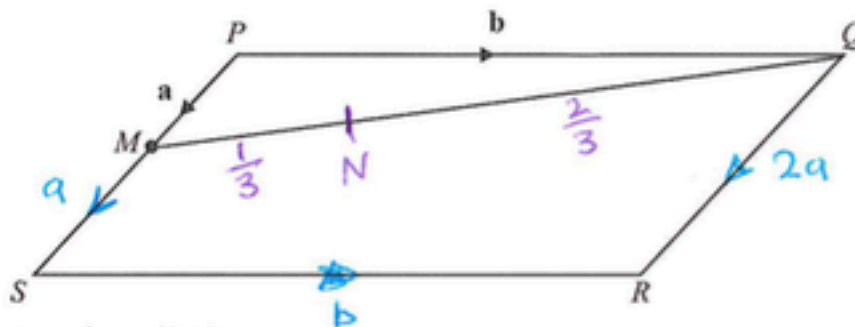
QR IS PARALLEL TO OP .

The diagram shows a parallelogram, $PQRS$.

M is the midpoint of PS .

$$\vec{PM} = \mathbf{a} \quad \vec{PQ} = \mathbf{b}$$

Diagram NOT
accurately drawn



(a) Find, in terms of \mathbf{a} and/or \mathbf{b} ,

(i) \vec{PS}

$$\underline{2\mathbf{a}} \quad \text{(A1)}$$

(ii) $\vec{PR} = \vec{PQ} + \vec{QR}$
 $= \mathbf{b} + 2\mathbf{a}$

$$\underline{2\mathbf{a} + \mathbf{b}} \quad \text{(A1)}$$

(iii) $\vec{MQ} = \vec{MP} + \vec{PQ}$
 $= -\mathbf{a} + \mathbf{b}$

$$\underline{-\mathbf{a} + \mathbf{b}} \quad \text{(A1)}$$

(3)

N is the point on MQ such that $MN = \frac{1}{3}MQ$

(b) Use a vector method to prove that PNR is a straight line.

$$\begin{aligned} \vec{PN} &= \vec{PM} + \vec{MN} \\ &= \vec{PM} + \frac{1}{3}\vec{MQ} \end{aligned}$$

$$= \mathbf{a} + \frac{1}{3}(-\mathbf{a} + \mathbf{b})$$

$$= \mathbf{a} - \frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$$

$$= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$$

$$= \frac{1}{3}(2\mathbf{a} + \mathbf{b})$$

(B1)

$$= \frac{1}{3}\vec{PR}$$

SINCE $\vec{PN} = k\vec{PR}$

PN AND PR ARE PARALLEL.

THEY ALSO SHARE A COMMON POINT, AND SO

PNR IS A STRAIGHT LINE

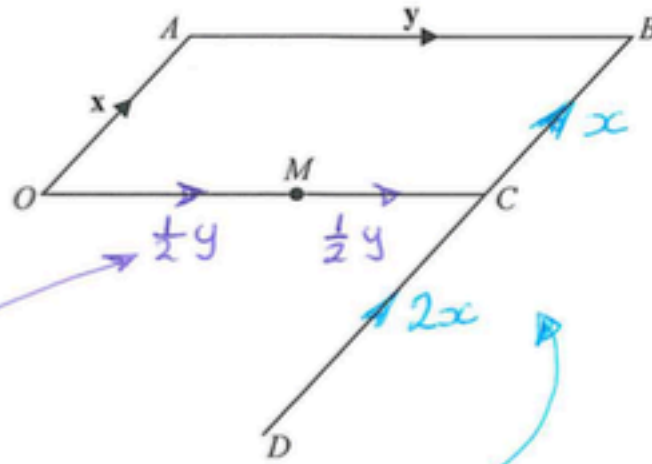


Diagram NOT
accurately drawn

$OACB$ is a parallelogram.

BCD is a straight line.

$BD = 3BC$.

M is the midpoint of OC .

$$\vec{OA} = x \quad \vec{AB} = y$$

(a) Find, in terms of x and y ,

(i) \vec{AM}

$$\begin{aligned} &= \vec{AO} + \vec{OM} \\ &= -x + \frac{1}{2}y \end{aligned}$$

$$\frac{-x + \frac{1}{2}y}{\text{---}}$$

(ii) \vec{OD}

$$\begin{aligned} &= \vec{OC} + \vec{CD} \\ &= y - 2x \end{aligned}$$

$$= \frac{1}{2}(-2x + y)$$

$$\frac{-2x + y}{\text{---}}$$

(2)

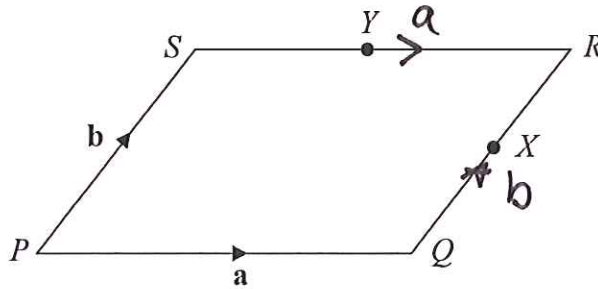
(b) Use your answers to (a)(i) and (ii) to write down two different geometric facts about the lines AM and OD .

SINCE $\vec{AM} = \frac{1}{2}\vec{OD}$, AM AND OD ARE PARALLEL

AM IS HALF THE LENGTH OF OD

(A1)

(2)



$PQRS$ is a parallelogram.

X is the midpoint of QR and Y is the midpoint of SR .

$\vec{PQ} = \mathbf{a}$ and $\vec{PS} = \mathbf{b}$.

(a) Write down, in terms of \mathbf{a} and \mathbf{b} , expressions for

$$\begin{aligned} \text{(i) } \vec{PX} &= \vec{PQ} + \vec{QX} \\ &= \mathbf{a} + \frac{1}{2}\mathbf{b} \end{aligned}$$

$$\underline{\underline{\mathbf{a} + \frac{1}{2}\mathbf{b}}} \quad \text{(A1)}$$

$$\begin{aligned} \text{(ii) } \vec{PY} &= \vec{PS} + \vec{SY} \\ &= \mathbf{b} + \frac{1}{2}\mathbf{a} \end{aligned}$$

$$\underline{\underline{\mathbf{b} + \frac{1}{2}\mathbf{a}}} \quad \text{(A1)}$$

$$\begin{aligned} \text{(iii) } \vec{QS} &= \vec{QP} + \vec{PS} \\ &= -\mathbf{a} + \mathbf{b} \end{aligned}$$

$$\underline{\underline{\mathbf{b} - \mathbf{a}}} \quad \text{(A1)}$$

(3)

(b) Use a vector method to show that XY is parallel to QS and that $XY = \frac{1}{2}QS$.

$$\vec{XY} = \vec{XP} + \vec{PY}$$

$$= (-\mathbf{a} - \frac{1}{2}\mathbf{b}) + (\mathbf{b} + \frac{1}{2}\mathbf{a}) \quad \text{(M1)}$$

$$= -\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$$

$$= \frac{1}{2}(\mathbf{b} - \mathbf{a}) \quad \text{(A1)}$$

$$= \frac{1}{2}\vec{QS} \quad \left(\text{SINCE } \vec{XY} = k\vec{QS}, \right.$$

XY and QS ARE PARALLEL)

(2)

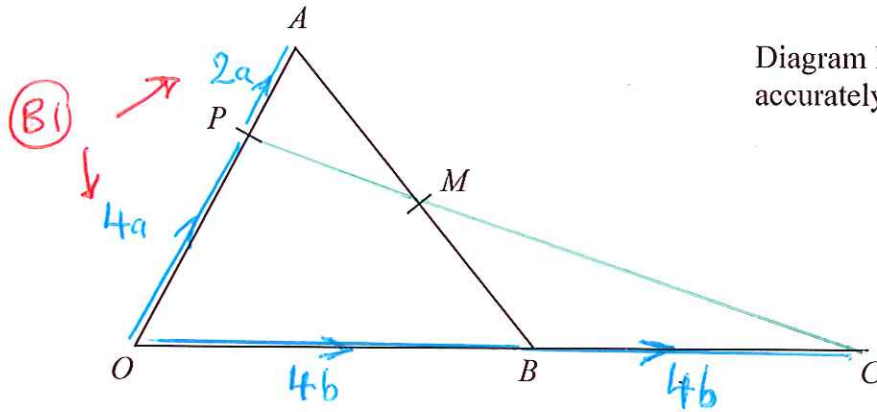


Diagram NOT
accurately drawn

OAB is a triangle.

P is the point on OA such that $OP:PA = 2:1 = 4a:2a$

C is the point such that B is the midpoint of OC .

M is the midpoint of AB .

$$\vec{OA} = 6a$$

$$\vec{OB} = 4b$$

Show that PMC is a straight line.

MUST SHOW THAT

$$PM = k \times MC$$

$$\text{OR } PM = k \times PC$$

$$\text{OR } MC = k \times PC$$

$$\vec{AB} = -6a + 4b$$

$$\therefore \vec{AM} = \frac{1}{2}(-6a + 4b)$$

$$= -3a + 2b$$

(A1) [FOR \vec{AM} OR \vec{BM}]

$$\vec{PM} = \vec{PA} + \vec{AM}$$

$$= 2a + (-3a + 2b)$$

$$= \underline{\underline{-a + 2b}} \quad \text{(M1)}$$

$$\vec{PC} = -4a + 8b$$

$$= 4(-a + 2b) \quad \text{(M1)}$$

$$= \underline{\underline{4\vec{PM}}}$$

SINCE $\vec{PC} = k\vec{PM}$
THEY ARE PARALLEL

SINCE THEY ALSO BOTH
GO THROUGH POINT P
THEY MUST FORM A
STRAIGHT LINE. (A1)

$OABC$ is a parallelogram.

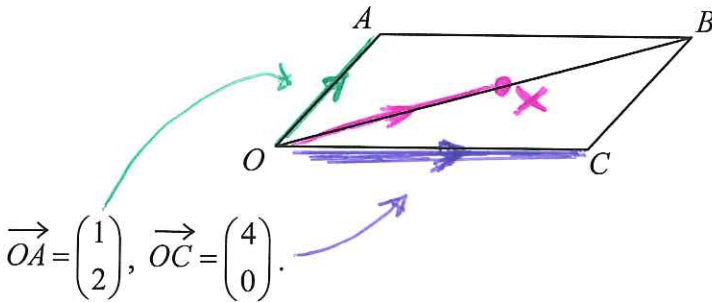


Diagram NOT accurately drawn

$$\vec{OA} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \vec{OC} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

(a) Find the vector \vec{OB} as a column vector.

$$\vec{OB} = \vec{OA} + \vec{AB} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \quad \text{(1)}$$

X is the point on OB such that $\vec{OX} = k\vec{OB}$, where $0 < k < 1$

(b) Find, in terms of k , the vectors

$$(i) \vec{OX} = k \times \vec{OB} = k \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 5k \\ 2k \end{pmatrix} \quad \text{(A1)}$$

$$(ii) \vec{AX} = \vec{AO} + \vec{OX} = -\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 5k \\ 2k \end{pmatrix} = \begin{pmatrix} 5k-1 \\ 2k-2 \end{pmatrix} \quad \text{(A1)}$$

$$(iii) \vec{XC} = \vec{XO} + \vec{OC} = -\begin{pmatrix} 5k \\ 2k \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4-5k \\ -2k \end{pmatrix} \quad \text{(A1)}$$

(c) Find the value of k for which $\vec{AX} = \vec{XC}$.

$$\begin{pmatrix} 5k-1 \\ 2k-2 \end{pmatrix} = \begin{pmatrix} 4-5k \\ -2k \end{pmatrix} \Rightarrow 5k-1 = 4-5k \quad \text{(M1)} \\ \Rightarrow 10k = 5 \rightarrow k = \frac{1}{2} \quad \text{(A1)}$$

(d) Use your answer to part (c) to show that the diagonals of the parallelogram $OABC$ bisect one another.

SINCE $k = \frac{1}{2}$, X IS MIDPOINT OF \vec{OB} (M1)

$$\vec{AX} = \begin{pmatrix} 5 \times 0.5 - 1 \\ 2 \times 0.5 - 2 \end{pmatrix} = \begin{pmatrix} 1.5 \\ -1 \end{pmatrix}$$

$$\vec{XC} = \begin{pmatrix} 4 - 5 \times 0.5 \\ -2 \times 0.5 \end{pmatrix} = \begin{pmatrix} 1.5 \\ -1 \end{pmatrix}$$

SAME SO X IS MIDPOINT OF AC (M1)

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Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners’ mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners’ mark scheme.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site.

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words ‘show that’, ‘show your working out’, or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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