VENN DIAGRAMS

DATE OF SOLUTIONS: 15/05/2018
MAXIMUM MARK: 62

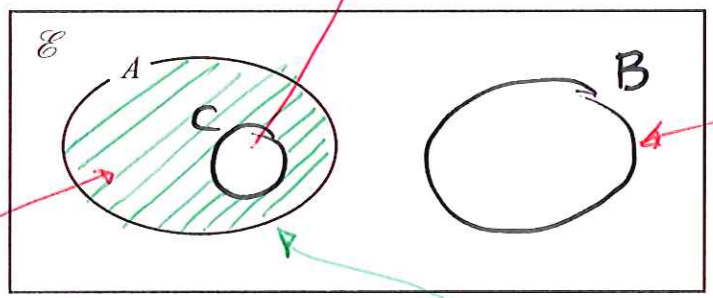
SOLUTIONS

GCSE (+ IGCSE) EXAM QUESTION PRACTICE

1. [Edexcel, 2012] Venn Diagrams **[3 Marks]**

A, B and C are three sets.

$A \cap B = \emptyset$ and $C \subset A$



(a) Complete the Venn diagram to show the sets B and C

(2)

(b) On the Venn diagram, shade the region that represents $A \cap C'$

(1)

2. [Edexcel, 2007] Venn Diagrams **[4 Marks]**

There are 35 students in a group.
18 students play hockey.
12 students play both hockey and tennis.
15 students play neither hockey nor tennis.

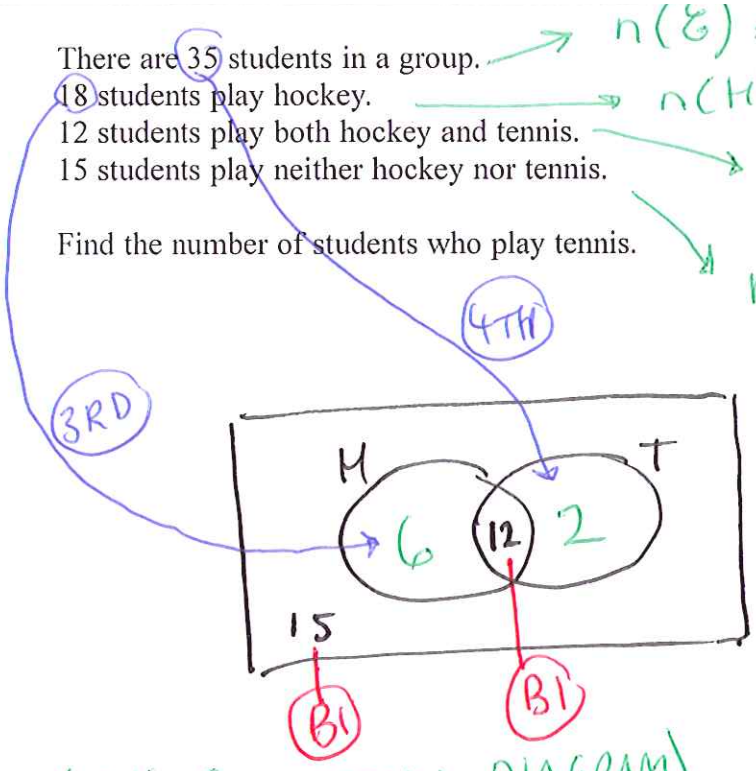
Find the number of students who play tennis.

$n(\mathcal{E}) = 35$

$n(H) = 18$

$n(H \cap T) = 12$ (1ST)

$n(H' \cap T') = 15$ (2ND)



FROM VENN DIAGRAM

$n(T) = 12 + 2$ M1

$= 14$ A1

WORKING (FOR VENN DIAGRAM)

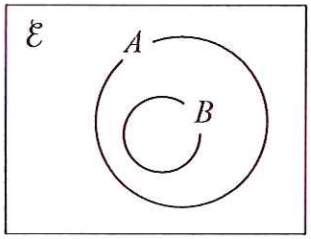
- INTERSECTIONS WERE WRITTEN IN FIRST (12 AND 15)
- HOCKEY ONLY $= 18 - 12$
  $= 6$

  (THIS COMPLETES 3 REGIONS)
- FOURTH REGION $= 35 - (15 + 6 + 12)$
  $= 2$

Statements				
$A \subset B$	$B \subset A$	$A \cup B = \mathcal{E}$	$A \cap B = \emptyset$	$A \cap B = A$

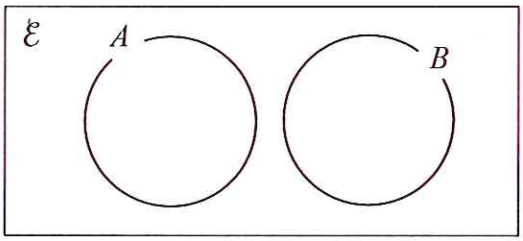
Choose a statement from the box that describes the relationship between sets A and B.

(i)



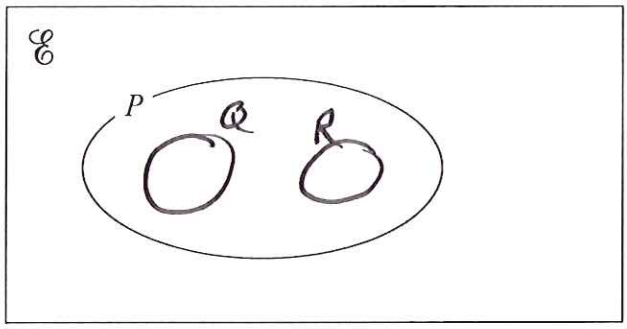
$B \subset A$ (A1)

(ii)



$A \cap B = \phi$ (A1)

4. [Edexcel, 2004] Venn Diagrams **[3 Marks]**



Set P is shown on the Venn Diagram.
Two sets, Q and R, are such that

$R \subset P$ → R IS INSIDE P (A1)

$Q \cap R = \varnothing$ → Q AND R ARE SEPERATE (A1)

$P \cup Q = P$ → Q IS INSIDE P (A1)

Complete the Venn Diagram to show set Q and set R.

SUMMARY:

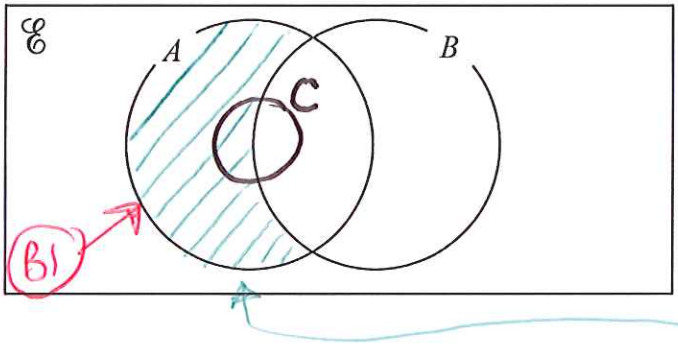
BOTH R AND Q ARE INSIDE P,

BUT THEY DO NOT INTERSECT WITH EACH OTHER

5. [Edexcel, 2004] Venn Diagrams **[4 Marks]**

The universal set, $\mathscr{E}$ = {Whole numbers}
A = {Multiples of 5} 5, 10, 15, 20, 25, ...
B = {Multiples of 3} 3, 6, 9, 12, 15, 18, 21, 24, ...

Sets A and B are represented by the circles in the Venn diagram.



(a) (i) On the diagram, shade the region that represents the set $A \cap B'$. ////

(ii) Write down **three** members of the set $A \cap B'$.

5, 10, 20 A1

(2)

C = {Multiples of 10}.

(b) (i) On the diagram draw a circle to represent the set C.

(ii) Write down **three** members of the set $A \cap B \cap C'$

15, 45, 75 A2

(2)

(-1 eeo)

9 A and B are two sets.

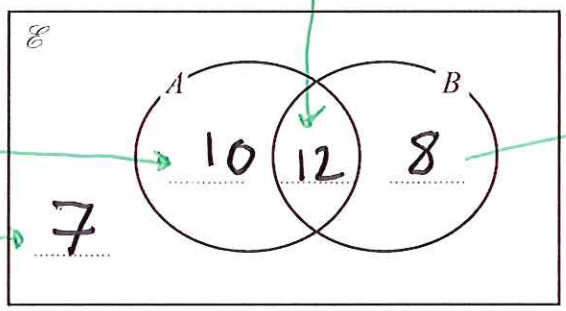
$n(\mathscr{E}) = 37$

$n(A) = 22$ 2ND

$n(A \cap B) = 12$ 1ST

$n(A \cup B) = 30$ 3RD

(a) Complete the Venn Diagram to show the **numbers** of elements.



30 - (10+12)

B2 ALL

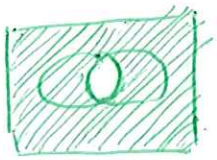
(37-30)

(2)

(b) Find (i) $n(A \cap B')$

10 A1

(ii) $n(A' \cup B')$



7+10+8

25 A1

(2)

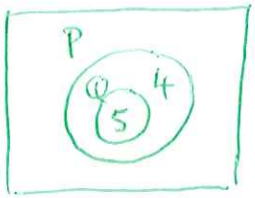
P and Q are two sets.
$n(P) = 9$ and $n(Q) = 5$

(a) Find the value of $n(P \cup Q)$ when $P \cap Q = \emptyset$

$n(P \cup Q) =$ 14 A1

(1)

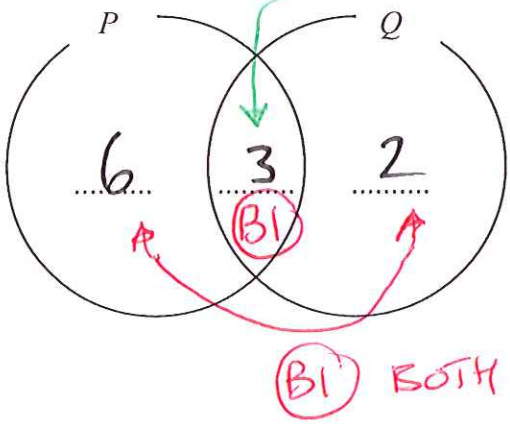
(b) Find the value of $n(P \cup Q)$ when $Q \subset P$



$n(P \cup Q) =$ 9 A1

(1)

(c) (i) Complete the Venn Diagram to show **numbers** of elements when $n(P \cap Q) = 3$



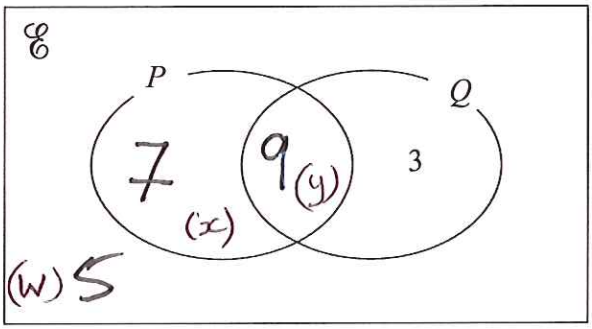
(ii) Find the value of $n(P \cup Q)$ when $n(P \cap Q) = 3$

$6 + 3 + 2$

$n(P \cup Q) =$ 11 A1

(3)

8. [Edexcel, 2005] Venn Diagrams **[6 Marks]**



THIRD:
7+3+5 = 15
24 − 15 = 9

In the Venn diagram, 3, w, x and y represent the **numbers** of elements.

$n(\mathscr{E}) = 24$ $n(P') = 8$ $n((P \cap Q)') = 15$ (2ND)

(THIRD) (1ST)

(a) Find the value of (i) w (ii) x (iii) y

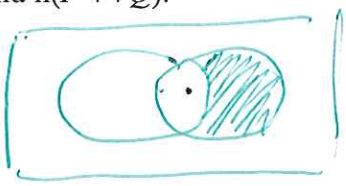
(i) $w =$ 5 A1

(ii) $x =$ 7 A1

(iii) $y =$ 9 A1

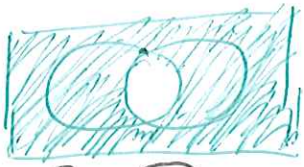
(3)

(b) (i) Find $n(P' \cap Q)$.



3 A1

(ii) Find $n(P' \cup Q')$.



5 + 7 + 3

15 A1

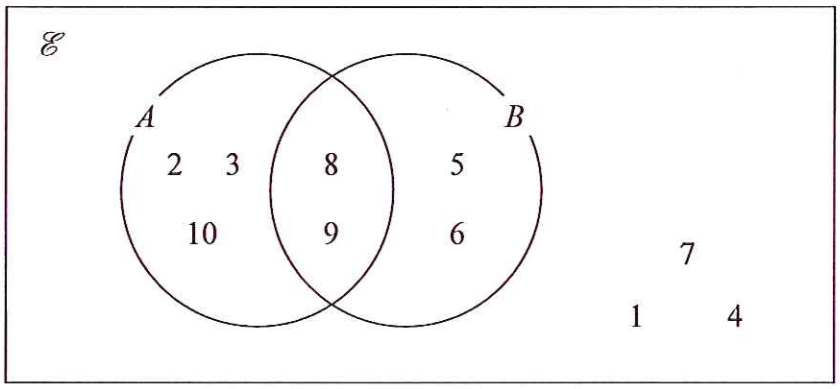
(iii) Find $n(P \cap Q \cap P')$

$P \cap P' = \phi$!!

so $P \cap Q \cap P' = \phi$

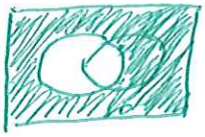
0 A1

(3)



The Venn diagram shows all of the elements in sets A, B and $\mathscr{E}$.

(a) Write down the elements in A'



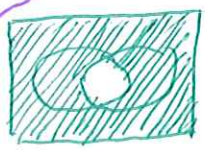
AI

$\{1, 4, 5, 6, 7\}$

(1)

(b) Find $n(A \cap B)'$

'n'



$\{1, 2, 3, 4, 5, 6, 7, 10\}$

AI

8

(1)

(c) Find the elements in $(A \cap B) \cup (A \cup B)'$

$\{8, 9\}$ $\{1, 4, 7\}$

AI

$\{1, 4, 7, 8, 9\}$

(1)

$A \cap C = \emptyset$
$B \cup C = \{5, 6, 7, 8, 9\}$
$n(C) = 3$

C DOESN'T HAVE 2, 3, 8, 9, 10

∴ THE 8, 9 ARE NOT C

SO...

(d) Write down the elements in C.

AI

$C = \{5, 6, 7\}$

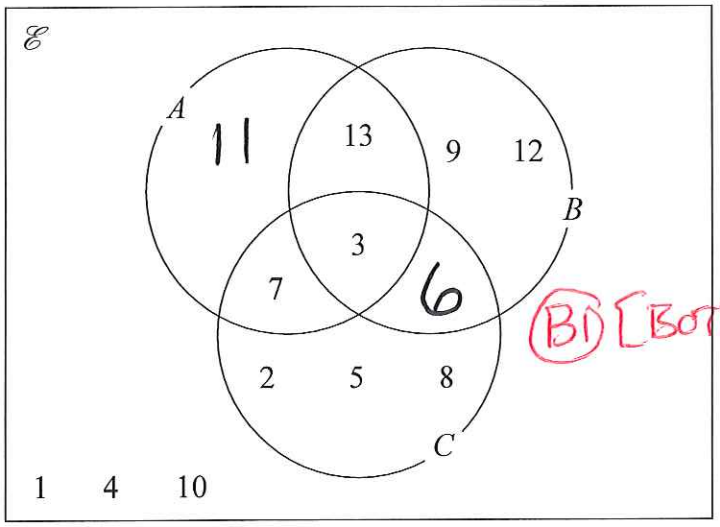
(1)

10. [Edexcel, 2016] Venn Diagrams **[4 Marks]**

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
$A = \{3, 7, 11, 13\}$
$B = \{3, 6, 9, 12, 13\}$
$C = \{2, 3, 5, 6, 7, 8\}$

(a) Complete the Venn diagram.

[MISSING ARE 6, 11]



B1 [BOTH 6 AND 11]

(1)

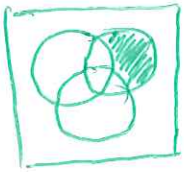
(b) List the members of the set $B' \cap C$

[TWO REGIONS]

$\{7, 2, 5, 8\}$ B1

(1)

(c) List the members of the set $(A \cup C)' \cap B$

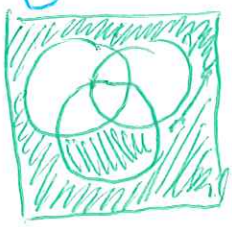


[ONE REGION]

$\{9, 12\}$ B1

(1)

(d) Find $n(A' \cap B')$ ← HOW MANY!



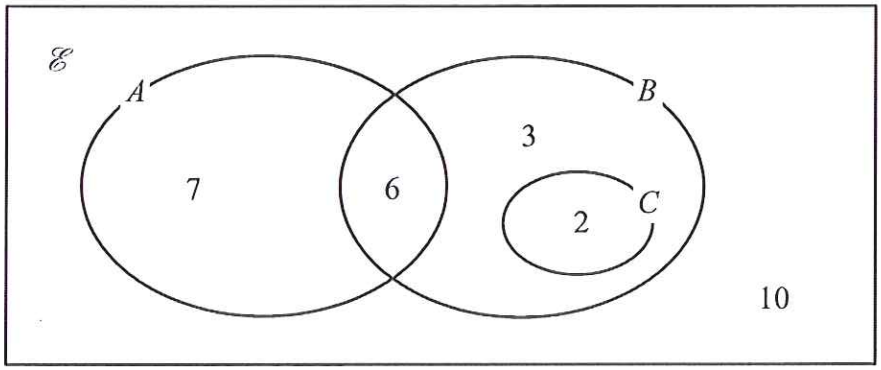
[TWO REGIONS]

$\{1, 4, 10, 2, 5, 8\}$ → 6 B1

(1)

11. [Edexcel, 2013] Venn Diagrams **[4 Marks]**

The Venn diagram shows a universal set $\mathscr{E}$ and three sets A, B and C.



7, 6, 3, 2 and 10 represent the **numbers** of elements.

Find

(i) $n(A \cup B)$

$7 + 6 + 3 + 2$

18 A1

(ii) $n(A')$

$3 + 2 + 10$

15 A1

(iii) $n(B \cap C')$

$6 + 3$

9 A1

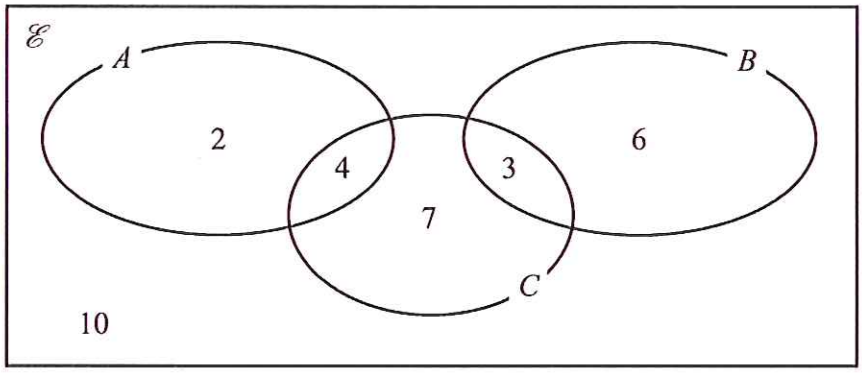
(iv) $n(A' \cup B')$

$3 + 2 + 10 + 7$

22 A1

12. [Edexcel, 2013] Venn Diagrams **[4 Marks]**

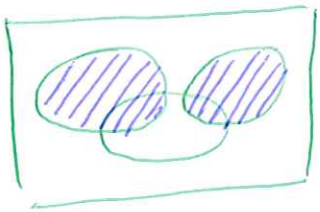
The Venn diagram shows a universal set $\mathscr{E}$ and 3 sets A, B and C.



2, 4, 7, 3, 6 and 10 represent **numbers** of elements.

Find

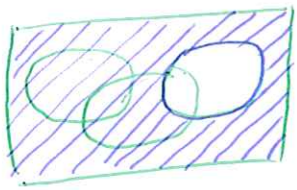
(i) n $(A \cup B)$



2 + 4 + 3 + 6

15 B1

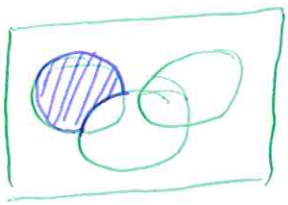
(ii) n (B')



10 + 2 + 4 + 7

23 B1

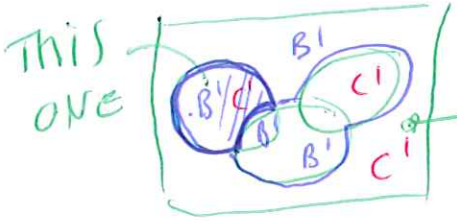
(iii) n $(A \cap C')$



2

2 B1

(iv) n $(B' \cap C')$

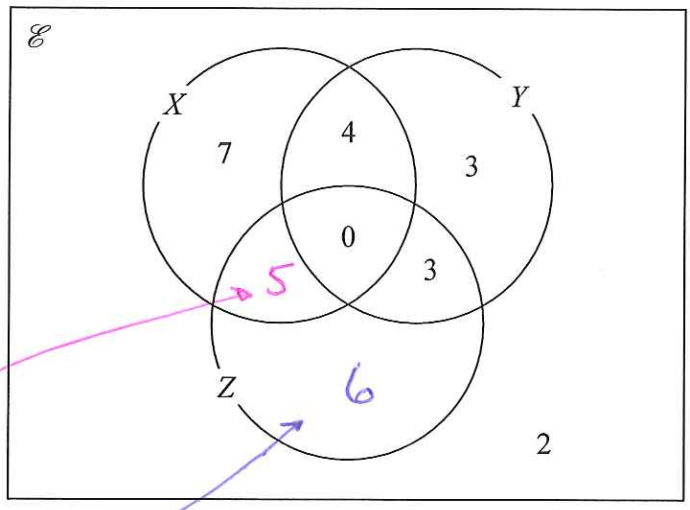


10 + 2

12 B1

13. [Edexcel, 2016] Venn Diagrams **[4 Marks]**

The Venn diagram shows a universal set $\mathscr{E}$ and three sets X, Y and Z.



2ND

B2

The numbers shown represent **numbers** of elements.

$n(X') = 14$

$n(Z) = 14$

1ST

(a) Complete the Venn diagram.

(2)

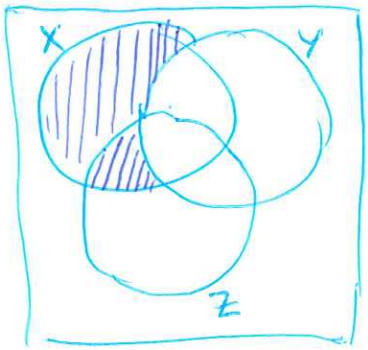
(b) Find the value of

(i) $n(X \cup !Z)$

$7 + 4 + 5 + 0 + 3 + 6$

25 A1

(ii) $n(X \cap Y')$



$7 + 5$

12 A1

(2)

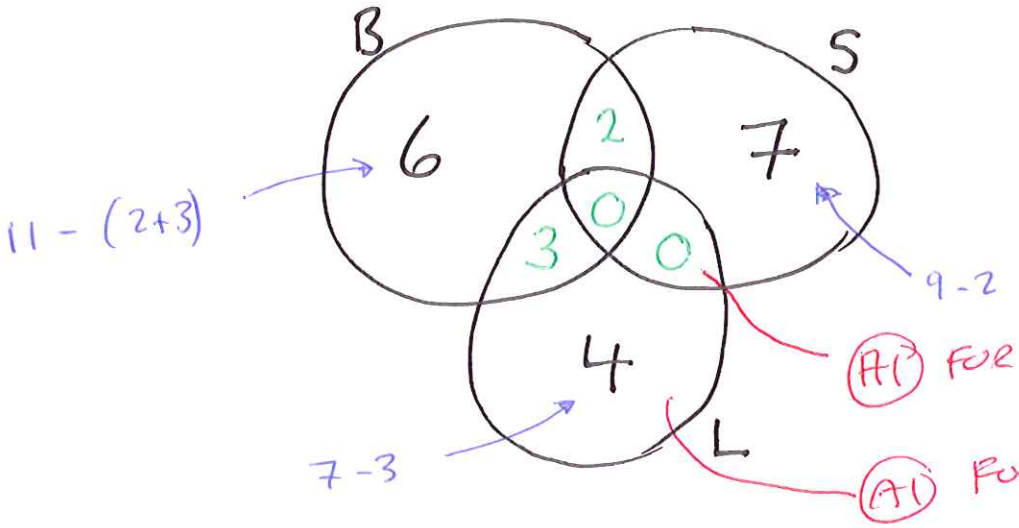
14. [Edexcel, 2013] Venn Diagrams **[3 Marks]**

A garage tests cars for faults.
There are three types of fault – braking, steering and lighting.
A car fails the test if it has one or more of these three types of fault.

3 SETS

Last week, 11 cars had braking faults — $n(B) = 11$
9 cars had steering faults — $n(S) = 9$
7 cars had lighting faults — $n(L) = 7$
no car had both steering faults and lighting faults — $n(S \cap L) = 0$
2 cars had both braking faults and steering faults — $n(B \cap S) = 2$
3 cars had both braking faults and lighting faults. — $n(B \cap L) = 3$

By drawing a Venn Diagram, or otherwise, find the number of cars which failed the test last week.



WORKING

FILL IN INTERSECTIONS FIRST

(A1) FOR 2, 3, 0, 0

(A1) FOR 4, 6, 7

NOTE THE ASSUMPTION THAT NO CAR HAD ALL THREE FAULTS

FROM THE DIAGRAM,

NUMBER OF CARS WITH A FAULT (OR MORE)

AND SO FAILED THE TEST = $6+7+4+2+3$

$= \underline{22}$ (A1)

15. [Edexcel, 2016] Venn Diagrams **[4 Marks]**

Each student in a group of 32 students was asked the following question.

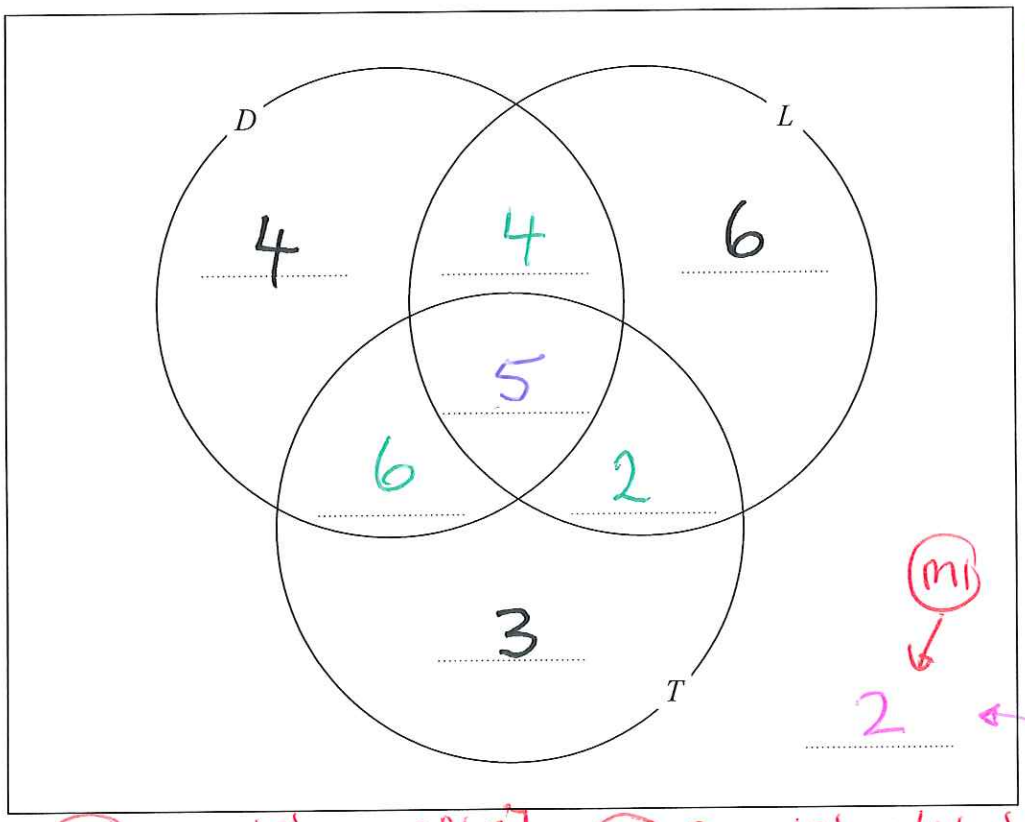
"Do you have a desktop computer (D), a laptop (L) or a tablet (T)?"

Their answers showed that

19 students have a desktop computer
17 students have a laptop
16 students have a tablet
9 students have both a desktop computer and a laptop
11 students have both a desktop computer and a tablet
7 students have both a laptop and a tablet
5 students have all three.

3RD 2ND 1ST

(a) Using this information, complete the Venn diagram to show the number of students in each appropriate subset.



4TH
4
4
6
5
6
2
+3
30

32-30=2

M1

M1 [FOR '5' IN MIDDLE] M1 [FOR '2' OR '4' OR '6' [OUTER MIDDLE]

One of the students with both a desktop computer and a laptop is chosen at random.

(b) Find the probability that this student also has a tablet.

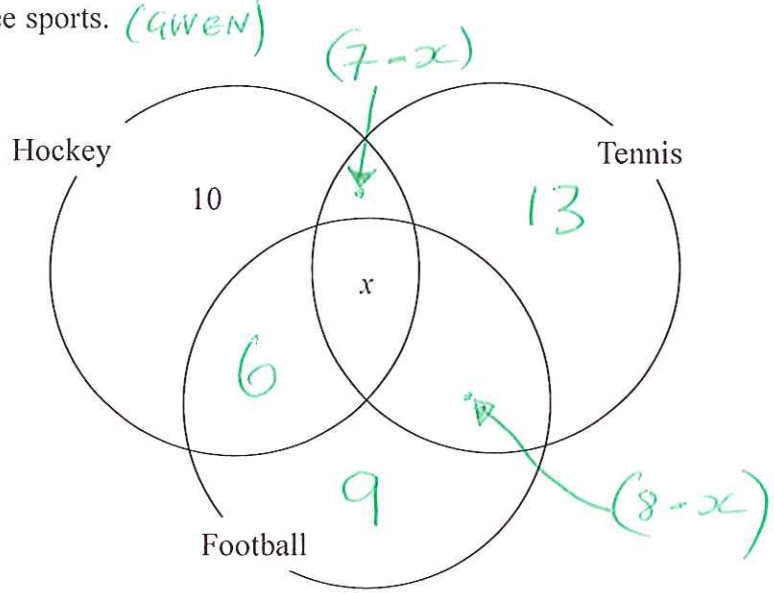
$\frac{5}{9}$ A1

16. [Edexcel, 2006] Venn Diagrams **[4 Marks]**

Each student in a group plays at least one of hockey, tennis and football.

FILL IN VENN DIAGRAM (STARTING INSIDE AND WORKING OUT)

10 students play hockey only (GIVEN)
9 play football only. 1ST
13 play tennis only. 2ND
6 play hockey and football but not tennis. 3RD
7 play hockey and tennis. 4TH
8 play football and tennis.
x play all three sports. (GIVEN)



(a) Write down an expression, in terms of x, for the number of students who play hockey and tennis, but not football.

$7 - x$ (A1)

(1)

There are 50 students in the group.

(b) Find the value of x.

ALL REGIONS MUST ADD TO 50.

$10 + 13 + 9 + 6 + (7 - x) + (8 - x) + x = 50$ (M1)

(M1) OR $53 - x = 50$

$x = 3$

$x =$ 3 (A1)

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There is no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

Please report any errors or omissions that You find*. These solutions will be updated to correct errors that are discovered. It is recommended that You always check that You have the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and there is no claim that the method that is used is necessarily the most efficient or 'best' method. From time to time, a solution to a question might be updated to show a different method if it is judged that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then You should consider to keep using your existing method and not change to the method that is used here. However, the choice of method is always up to You and it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions there is an indication of where marks **might** be awarded for each question. B marks, M marks and A marks have been used in a similar, but **not identical**, way that an exam board uses these marks within their mark schemes. This slight difference in the use of these marking symbols has been done for simplicity and convenience. Sometimes B marks, M marks and A marks have been interchanged, when compared to an examiners' mark scheme and sometimes the marks have been awarded for different aspects of a solution when compared to an examiners' mark scheme.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. Method marks have been shown in places where they might be awarded for the method that is shown. If You use a different method to get a correct answer, then the same number of method marks would be awarded but it is not practical to show all possible methods, and the way in which marks might be awarded for their use, within these particular solutions. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site.

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown a method) and all of the accuracy marks.

Note that some questions contain the words 'show that', 'show your working out', or similar. These questions require working out to be shown. Failure to show sufficient working out is likely to result in no marks being awarded, even if the final answer is correct.

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